Can Partial Cooperation Between Developed and Developing Countries Be Stable?

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Workshop on Dynamic Games and Applications
October 27-28, 2022
Paris

Outline

- Introduction
- Model
- Stability concepts
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Introduction

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Model

- $N = \{1, 2, 3\}$ — players (neighboring industries or countries).
- The players are of two types:
  - I — vulnerable player (or developed country)
  - II — nonvulnerable player (or developing country)
- One nonvulnerable and two vulnerable players
- Emissions $e_i(t)$ — strategy of player i
- Pollution stock S dynamics:

$$\dot{S}(t) = \mu \sum_{i \in N} e_i(t) - \varepsilon S(t), \;\; S(0) = S_0, \tag{1}$$

where $\mu > 0$, $\varepsilon > 0$.

Model

- The nonvulnerable player maximizes

$$W_i = \int_0^\infty e^{-\rho t}(\alpha_i e_i(t) - \frac{1}{2}e_i^2(t))dt. \qquad (2)$$

- The vulnerable player's objective function is

$$W_i = \int_0^\infty e^{-\rho t}(\alpha_i e_i(t) - \frac{1}{2}e_i^2(t) - \frac{1}{2}\beta_i S^2(t))dt, \qquad (3)$$

  where $\alpha_i > 0$, $\beta_i > 0$.
- We use objective function (3) but any player with the parameters:
  - $\beta_i > 0$ for vulnerable player
  - $\beta_i = 0$ for nonvulnerable player
- Two vulnerable players may be asymmetric.

Different Scenarios

1. *Noncooperative scenario*, $\pi_1 = \{\{I\}, \{I\}, \{II\}\}$;
2. *Cooperative scenario*, $\pi_2 = \{\{I, I, II\}\}$;
3. *Partially cooperative scenarios*:
    1. Case 1 (two developed countries cooperate): $\pi_3 = \{\{I, I\}, \{II\}\}$;
    2. Case 2 (one developing and one developed country cooperate): $\pi_4 = \{\{I, II\}, \{I\}\}$.
       Two variants: $\pi_{4_1} = \{\{1, 2\}, \{3\}\}$ and $\pi_{4_2} = \{\{1, 3\}, \{2\}\}$.
       Hereinafter, we refer to a general form of the coalition structure π_4 if the result is true for both structures π_{4_1} and π_{4_2}.

NE under noncooperative scenario

Proposition 1

In the noncooperative scenario $\pi_1 = \{\{I\}, \{I\}, \{II\}\}$, assuming an interior solution, the feedback-Nash equilibrium is given by

$$e_1^{nc}(t) = \alpha_1, \tag{4}$$
$$e_j^{nc}(t) = \alpha_j + \mu(x_j S^{nc}(t) + y_j),\ j = 2, 3, \tag{5}$$

where x_j, y_j, z_j for $j = 2, 3$ satisfy the following system (given in the paper). The corresponding equilibrium state trajectory is

$$S^{nc}(t) = \frac{\mu\alpha_{123} + \mu^2 y_{23}}{\mu^2 x_{23} - \varepsilon}(e^{(\mu^2 x_{23} - \varepsilon)t} - 1) + e^{(\mu^2 x_{23} - \varepsilon)t} S_0, \tag{6}$$

where $y_{23} = y_2 + y_3$.

NE under cooperative scenario

Proposition 2

In the cooperative scenario, when $\pi_2 = \{I, I, II\}$, the players' optimal feedback strategies are given by

$$e_i^c(t) = \alpha_i + \mu(x_c S^c(t) + y_c), \;\; i \in N, \tag{7}$$

where x_c, y_c are given in the paper. The cooperative state trajectory is

$$S^c(t) = \frac{\mu\alpha_{123} + 3\mu^2 y_c}{3\mu^2 x_c - \varepsilon}(e^{(3\mu^2 x_c - \varepsilon)t} - 1) + e^{(3\mu^2 x_c - \varepsilon)t} S_0. \tag{8}$$

The steady-state emission stock is

$$S_\infty^c = \frac{(\rho + \varepsilon)\mu\alpha_{123}}{(\varepsilon - 3\mu^2 x_c)(\rho + \varepsilon - 3\mu^2 x_c)}, \tag{9}$$

which is globally asymptotically stable.

Partially Cooperative Scenarios

Case 1: $\{\{I, I\}, \{II\}\}$ **or** $\{\{2, 3\}, \{1\}\}$

Player 1 aims to maximize

$$\max_{e_1} W_1(e_1, e_2, e_3)$$

The objective of coalition $\{2, 3\}$ is given by

$$\max_{e_2, e_3} \Big(W_2(e_1, e_2, e_3) + W_3(e_1, e_2, e_3) \Big)$$

s.t. state dynamics (1) with $S(0) = S_0$.

Partially Cooperative Scenarios

Proposition 3

Under partially cooperative scenario with coalition structure $\pi_3 = \{\{1\}, \{2,3\}\}$, the feedback-Nash equilibrium is given by

$$e_1^{pc_1}(t) = \alpha_1, \tag{10}$$
$$e_i^{pc_1}(t) = \alpha_i + \mu(x_{c_1} S^{pc_1}(t) + y_{c_1}), \quad i = 2, 3, \tag{11}$$

where $x_{c_1}, y_{c_1}, z_{c_1}$ are given in the paper.
The corresponding Nash equilibrium trajectory under p.c.s. (case 1) is

$$S^{pc_1}(t) = \frac{\mu\alpha_{123} + 2\mu^2 y_{c_1}}{2\mu^2 x_{c_1} - \varepsilon}(e^{(2\mu^2 x_{c_1} - \varepsilon)t} - 1) + e^{(2\mu^2 x_{c_1} - \varepsilon)t} S_0. \tag{12}$$

The steady-state emission stock is

$$S_\infty^{pc_1} = \frac{(\rho + \varepsilon)\mu\alpha_{123}}{(\varepsilon - 2\mu^2 x_{c_1})(\rho + \varepsilon - 2\mu^2 x_{c_1})}, \tag{13}$$

which is globally asymptotically stable if $2\mu^2 x_{c_1} - \varepsilon < 0$.

Partially Cooperative Scenarios

Case 2: $\{\{I, II\}, \{I\}\}$ **or** $\{\{1,2\},\{3\}\}$

Player 3 aims to maximize

$$\max_{e_3} W_3(e_1, e_2, e_3),$$

and an optimization problem of coalition $\{1,2\}$ is

$$\max_{e_1, e_2} \Big(W_1(e_1, e_2, e_3) + W_2(e_1, e_2, e_3) \Big),$$

where the payoff function $W_i, i = 1, 2$ is given by (2) and (3) respectively, s.t. state dynamics (1) with $S(0) = S_0$.

Partially Cooperative Scenarios

Proposition 4

In the partial-cooperative scenario (case 2) with coalition structure $\pi_{4_1} = \{\{1,2\},\{3\}\}$, the feedback-Nash equilibrium is given by

$$e_i^{pc_2}(t) = \alpha_i + \mu(x_{c_2} S^{pc_2}(t) + y_{c_2}), \quad i = 1,2 \tag{14}$$

$$e_3^{pc_2}(t) = \alpha_3 + \mu(x_{3_{c2}}^{pc_2} S^{pc_2}(t) + y_{3_{c2}}), \tag{15}$$

where $x_{c_2}, x_{3_{c_2}}, y_{c_2}, y_{3_{c_2}}, z_{c_2}, z_{3_{c_2}}$ are given in the paper. The state trajectory is

$$S^{pc_2}(t) = \frac{\mu\alpha_{123} + \mu^2(2y_{c_2} + y_{3_{c_2}})}{\mu^2(2x_{c_2} + x_{3_{c_2}}) - \varepsilon}\left(e^{[\mu^2(2x_{c_2}+x_{3_{c_2}})-\varepsilon]t} - 1\right) + e^{[\mu^2(2x_{c_2}+x_{3_{c_2}})-\varepsilon]t} S_0. \tag{16}$$

The steady-state emission stock is

$$S_\infty^{c_2} = \frac{\mu\alpha_{123} + \mu^2 y_{3_{c_2}} + 2\mu^2 y_{c_2}}{\varepsilon - 2\mu^2 x_{c_2} - \mu^2 x_{3_{c_2}}}, \tag{17}$$

which is globally asymptotically stable if $\mu^2(2x_{c_2} + x_{3_{c_2}}) - \varepsilon < 0$.

Two stability concepts

Scenario or coalition structure is **stable** when any player **will not increase her payoff** if she changes this structure in an individual way.

We consider two possibilities for a deviating player:

- she can join any possible coalition without any restrictions (Nash stability)
- the coalition to which the deviating player would like to join can block the deviation if there exists at least one member who can lose by accepting the deviator (individual stability)

Definition 1

A coalition structure $\pi = \{B_1, \dots, B_m\}$ is Nash stable (or simply, stable) if for any player $i \in N$ it holds that

$$W_i^{\pi} \geq W_i^{\pi'} \text{ for all } \pi' = \{B(i)\backslash\{i\}, B_j \cup \{i\}, \pi_{-B(i)\cup B_j}\},$$

where $B_j \in \pi \cup \varnothing$, $B_j \neq B(i)$, $\pi_{-B(i)\cup B_j} = \pi\backslash\{B(i) \cup B_j\}$, and W^{π} and $W^{\pi'}$ are vectors of players' payoffs under π and π' respectively.

Definition 2

A coalition structure $\pi = \{B_1, \dots, B_m\}$ is individually stable if for any player $i \in N$ it holds that

$$W_i^{\pi} \geq W_i^{\pi''} \text{ for all } \pi'' = \{B(i)\backslash\{i\}, B_j \cup \{i\}, \pi_{-B(i)\cup B_j}\} \text{ such that}$$
$$W_k^{\pi''} \geq W_k^{\pi} \text{ for all } k \in B_j,$$

where $B_j \in \pi \cup \varnothing$, $B_j \neq B(i)$, $\pi_{-B(i)\cup B_j} = \pi\backslash\{B(i) \cup B_j\}$, and W^{π}, $W^{\pi''}$ are vectors of players' payoffs under π and π'' respectively.

Proposition 4 (Nash stability conditions)

In the differential game given by (1)–(3), the following coalition structures or scenarios are stable or Nash stable if and only if the corresponding conditions given in the table are satisfied:

	Nonvul. Player 1	Vul. Player 2	Vul. Player 3
π_1	$\begin{cases} W_1^{\pi_1} \geq W_1^{\pi_{4_1}} \\ W_1^{\pi_1} \geq W_1^{\pi_{4_2}} \end{cases}$	$\begin{cases} W_2^{\pi_1} \geq W_2^{\pi_3} \\ W_2^{\pi_1} \geq W_2^{\pi_{4_1}} \end{cases}$	$\begin{cases} W_3^{\pi_1} \geq W_3^{\pi_3} \\ W_3^{\pi_1} \geq W_3^{\pi_{4_2}} \end{cases}$
π_2	$W_1^{\pi_2} \geq W_1^{\pi_3}$	$W_2^{\pi_2} \geq W_2^{\pi_{4_1}}$	$W_3^{\pi_2} \geq W_3^{\pi_{4_2}}$
π_3	$W_1^{\pi_3} \geq W_1^{\pi_2}$	$\begin{cases} W_2^{\pi_3} \geq W_2^{\pi_1} \\ W_2^{\pi_3} \geq W_2^{\pi_{4_1}} \end{cases}$	$\begin{cases} W_3^{\pi_3} \geq W_3^{\pi_1} \\ W_3^{\pi_3} \geq W_3^{\pi_{4_2}} \end{cases}$
π_{4_1}	$\begin{cases} W_1^{\pi_{4_1}} \geq W_1^{\pi_1} \\ W_1^{\pi_{4_1}} \geq W_1^{\pi_{4_2}} \end{cases}$	$\begin{cases} W_2^{\pi_{4_1}} \geq W_2^{\pi_1} \\ W_2^{\pi_{4_1}} \geq W_2^{\pi_3} \end{cases}$	$W_3^{\pi_{4_1}} \geq W_3^{\pi_2}$
π_{4_2}	$\begin{cases} W_1^{\pi_{4_2}} \geq W_1^{\pi_1} \\ W_1^{\pi_{4_2}} \geq W_1^{\pi_{4_1}} \end{cases}$	$W_2^{\pi_{4_2}} \geq W_2^{\pi_2}$	$\begin{cases} W_3^{\pi_{4_2}} \geq W_3^{\pi_1} \\ W_3^{\pi_{4_2}} \geq W_3^{\pi_3} \end{cases}$

Proposition 5 (Individual stability conditions)

In the differential game given by (1)–(3), the following coalition structures or scenarios are individually stable if and only if the corresponding conditions given in the table are satisfied:

	Nonvul. Player 1	Vul. Player 2	Vul. Player 3
π_1	$\left\{\begin{array}{l} \left[\begin{array}{l} \begin{cases} W_2^{\pi_1} < W_2^{\pi_{4_1}} \\ W_1^{\pi_1} \geq W_1^{\pi_{4_1}} \end{cases} \\ \text{or } W_2^{\pi_1} \geq W_2^{\pi_{4_1}} \end{array}\right. \\ \left[\begin{array}{l} \begin{cases} W_3^{\pi_1} < W_3^{\pi_{4_2}} \\ W_1^{\pi_1} \geq W_1^{\pi_{4_2}} \end{cases} \\ \text{or } W_3^{\pi_1} \geq W_3^{\pi_{4_2}} \end{array}\right. \end{array}\right.$	$\left\{\begin{array}{l} \left[\begin{array}{l} \begin{cases} W_3^{\pi_1} < W_3^{\pi_3} \\ W_2^{\pi_1} \geq W_2^{\pi_3} \end{cases} \\ \text{or } W_3^{\pi_1} \geq W_3^{\pi_3} \end{array}\right. \\ \left[\begin{array}{l} \begin{cases} W_1^{\pi_1} < W_1^{\pi_{4_1}} \\ W_2^{\pi_1} \geq W_2^{\pi_{4_1}} \end{cases} \\ \text{or } W_1^{\pi_1} \geq W_1^{\pi_{4_1}} \end{array}\right. \end{array}\right.$	$\left\{\begin{array}{l} \left[\begin{array}{l} \begin{cases} W_2^{\pi_1} < W_2^{\pi_3} \\ W_3^{\pi_1} \geq W_3^{\pi_3} \end{cases} \\ \text{or } W_2^{\pi_1} \geq W_2^{\pi_3} \end{array}\right. \\ \left[\begin{array}{l} \begin{cases} W_1^{\pi_1} < W_1^{\pi_{4_2}} \\ W_3^{\pi_1} \geq W_3^{\pi_{4_2}} \end{cases} \\ \text{or } W_1^{\pi_1} \geq W_1^{\pi_{4_2}} \end{array}\right. \end{array}\right.$
π_2	$W_1^{\pi_2} \geq W_1^{\pi_3}$	$W_2^{\pi_2} \geq W_2^{\pi_{4_1}}$	$W_3^{\pi_2} \geq W_3^{\pi_{4_2}}$
π_3	$\left[\begin{array}{l} \begin{cases} W_2^{\pi_3} < W_2^{\pi_2} \\ W_3^{\pi_3} < W_3^{\pi_2} \\ W_1^{\pi_3} \geq W_1^{\pi_2} \end{cases} \\ \text{or } W_2^{\pi_3} \geq W_2^{\pi_2} \\ \text{or } W_3^{\pi_3} \geq W_3^{\pi_2} \end{array}\right.$	$\left\{\begin{array}{l} W_2^{\pi_3} \geq W_2^{\pi_1} \\ \left[\begin{array}{l} \begin{cases} W_1^{\pi_3} < W_1^{\pi_{4_1}} \\ W_2^{\pi_3} \geq W_2^{\pi_{4_1}} \end{cases} \\ \text{or } W_1^{\pi_3} \geq W_1^{\pi_{4_1}} \end{array}\right. \end{array}\right.$	$\left\{\begin{array}{l} W_3^{\pi_3} \geq W_3^{\pi_1} \\ \left[\begin{array}{l} \begin{cases} W_1^{\pi_3} < W_1^{\pi_{4_2}} \\ W_3^{\pi_3} \geq W_3^{\pi_{4_2}} \end{cases} \\ \text{or } W_1^{\pi_3} \geq W_1^{\pi_{4_2}} \end{array}\right. \end{array}\right.$

Proposition 5 (Individual stability conditions)

	Nonvul. Player 1	Vul. Player 2	Vul. Player 3
$\pi 4_1$	$\begin{cases} W_1^{\pi 4_1} \geq W_1^{\pi 1} \\ \left[\begin{array}{l} \begin{cases} W_3^{\pi 4_1} < W_3^{\pi 4_2} \\ W_1^{\pi 4_1} \geq W_1^{\pi 4_2} \end{cases} \\ \text{or } W_3^{\pi 4_1} \geq W_3^{\pi 4_2} \end{array}\right. \end{cases}$	$\begin{cases} W_2^{\pi 4_1} \geq W_2^{\pi 1} \\ \left[\begin{array}{l} \begin{cases} W_3^{\pi 4_1} < W_3^{\pi 3} \\ W_2^{\pi 4_1} \geq W_2^{\pi 3} \end{cases} \\ \text{or } W_3^{\pi 4_1} \geq W_3^{\pi 3} \end{array}\right. \end{cases}$	$\left[\begin{array}{l} \begin{cases} W_1^{\pi 4_1} < W_1^{\pi 2} \\ W_2^{\pi 4_1} < W_2^{\pi 2} \\ W_3^{\pi 4_1} \geq W_3^{\pi 2} \end{cases} \\ \text{or } W_1^{\pi 4_1} \geq W_1^{\pi 2} \\ \text{or } W_2^{\pi 4_1} \geq W_2^{\pi 2} \end{array}\right.$
$\pi 4_2$	$\begin{cases} W_1^{\pi 4_2} \geq W_1^{\pi 1} \\ \left[\begin{array}{l} \begin{cases} W_2^{\pi 4_2} < W_2^{\pi 4_1} \\ W_1^{\pi 4_2} \geq W_1^{\pi 4_1} \end{cases} \\ \text{or } W_2^{\pi 4_2} \geq W_2^{\pi 4_1} \end{array}\right. \end{cases}$	$\left[\begin{array}{l} \begin{cases} W_1^{\pi 4_2} < W_1^{\pi 2} \\ W_3^{\pi 4_2} < W_3^{\pi 2} \\ W_2^{\pi 4_2} \geq W_2^{\pi 2} \end{cases} \\ \text{or } W_1^{\pi 4_2} \geq W_1^{\pi 2} \\ \text{or } W_3^{\pi 4_2} \geq W_3^{\pi 2} \end{array}\right.$	$\begin{cases} W_3^{\pi 4_2} \geq W_3^{\pi 1} \\ \left[\begin{array}{l} \begin{cases} W_2^{\pi 4_2} < W_2^{\pi 3} \\ W_3^{\pi 4_2} \geq W_3^{\pi 3} \end{cases} \\ \text{or } W_2^{\pi 4_2} \geq W_2^{\pi 3} \end{array}\right. \end{cases}$

Example

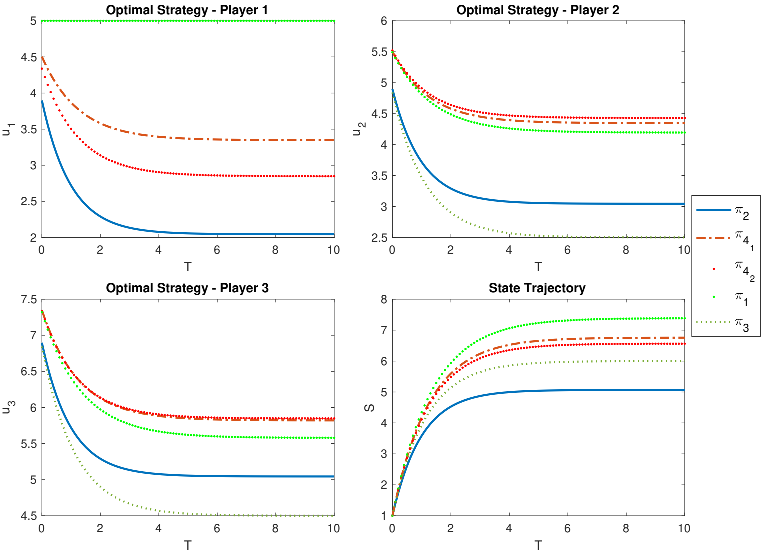
$$\beta_1 = 0,\ \beta_2 = 3,\ \beta_3 = 4,$$
$$\alpha_1 = 5,\ \alpha_2 = 6,\ \alpha_3 = 8,$$
$$\varepsilon = 0.6,\ \mu = 0.3,\ S_0 = 1.$$

Players' payoffs under different scenarios:

	Player 1 Nonvul. player	Player 2 Vul. player	Player 3 Vul. player
$\pi_1 = \{\{1\},\{2\},\{3\}\}$	**4.167**	2.772	6.306
$\pi_2 = \{\{1,2,3\}\}$	3.734	**3.205**	**7.085**
$\pi_3 = \{\{1\},\{2,3\}\}$	**4.167**	2.810	6.581
$\pi_{4_1} = \{\{1,2\},\{3\}\}$	4.069	2.976	6.596
$\pi_{4_2} = \{\{1,3\},\{2\}\}$	3.995	3.043	1.994

- No Nash-stable scenario
- $\pi_3 = \{\{I, I\}, \{II\}\}$ is unique individually stable scenario
- Without transfers payments inside a coalition, we are able to find an individually stable scenario in the game

Example (cntd)



Dynamically stable scenarios (subgames at $\bar{t} = 1, 5, 10$)

$\bar{t} = 1$	Player 1 Nonvul. player	Player 2 Vul. player	Player 3 Vul. player
$\pi_1 = \{\{1\}, \{2\}, \{3\}\}$	4.167	-6.681	-6.382
$\pi_2 = \{\{1, 2, 3\}\}$	0.144	-0.307	-0.255
$\pi_3 = \{\{1\}, \{2, 3\}\}$	4.167	-0.495	-0.499
$\pi_{4_1} = \{\{1, 2\}, \{3\}\}$	0.190	-0.569	-4.736
$\pi_{4_2} = \{\{1, 3\}, \{2\}\}$	0.177	-5.082	-0.786

$\bar{t} = 5$	Player 1 Nonvul. player	Player 2 Vul. player	Player 3 Vul. player
$\pi_1 = \{\{1\}, \{2\}, \{3\}\}$	4.167	-20.911	-25.467
$\pi_2 = \{\{1, 2, 3\}\}$	0	0	0
$\pi_3 = \{\{1\}, \{2, 3\}\}$	4.167	0	0
$\pi_{4_1} = \{\{1, 2\}, \{3\}\}$	0	0	-19.807
$\pi_{4_2} = \{\{1, 3\}, \{2\}\}$	0	-15.464	0

$\bar{t} = 10$	Player 1 Nonvul. player	Player 2 Vul. player	Player 3 Vul. player
$\pi_1 = \{\{1\}, \{2\}, \{3\}\}$	4.167	-21.817	-26.682
$\pi_2 = \{\{1, 2, 3\}\}$	0	0	0
$\pi_3 = \{\{1\}, \{2, 3\}\}$	4.167	0	0
$\pi_{4_1} = \{\{1, 2\}, \{3\}\}$	0	0	-20.553
$\pi_{4_2} = \{\{1, 3\}, \{2\}\}$	0	-15.936	0

Cooperative scenario: Nash and individual stability

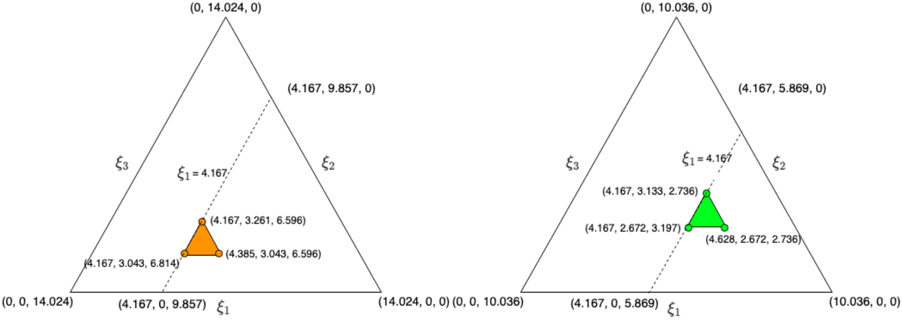
- The conditions are the same for Nash and individual stability
- The stability conditions:

$$\begin{cases} \xi_1 + \xi_2 + \xi_3 = \sum_{i=1}^{3} W_i^{\pi_2}, \\ \xi_1 \geq W_1^{\pi_3}, \\ \xi_2 \geq W_2^{\pi_{4_2}}, \\ \xi_3 \geq W_3^{\pi_{4_1}}. \end{cases} \tag{18}$$

- If there exists a solution of system (18), then the transfer payment to player $i \in N$ is defined by

$$\theta_i^{\pi_2} = \xi_i - W_i^{\pi_2}. \tag{19}$$

Cooperative scenario: Nash and individual stability



(a) The first run (b) The second run

Figure 1: The set of payments to the players (ξ_1, ξ_2, ξ_3) satisfying conditions (18)

Nash stability of partial cooperation scenario

Players' payoffs under different scenarios with transfers:

	Player 1 Nonvul. player	Player 2 Vul. player	Player 3 Vul. player
$\pi_1 = \{\{1\},\{2\},\{3\}\}$	4.167	2.772	6.306
$\pi_2 = \{\{1,2,3\}\}$	$\xi_1^{\pi_2}$	$\xi_2^{\pi_2}$	$14.024 - \xi_1^{\pi_2} - \xi_1^{\pi_2}$
$\pi_3 = \{\{1\},\{2,3\}\}$	4.167	$\xi_2^{\pi_3}$	$10.290 - \xi_2^{\pi_3}$
$\pi_{4_1} = \{\{1,2\},\{3\}\}$	$\xi_1^{\pi_{4_1}}$	$7.045 - \xi_1^{\pi_{4_1}}$	6.596
$\pi_{4_2} = \{\{1,3\},\{2\}\}$	$\xi_1^{\pi_{4_2}}$	3.043	$5.991 - \xi_1^{\pi_{4_2}}$

Conditions to make scenario π_{4_1} stable (including 5 variables):

$$\xi_1^{\pi_{4_1}} \geq 4.167,$$

$$\xi_1^{\pi_{4_1}} \geq \xi_1^{\pi_{4_2}},$$

$$7.045 - \xi_1^{\pi_{4_1}} \geq 2.772,$$

$$7.045 - \xi_1^{\pi_{4_1}} \geq \xi_2^{\pi_3},$$

$$6.596 \geq 14.024 - \xi_1^{\pi_2} - \xi_2^{\pi_2}.$$

Nash stability of partial cooperation scenario

ES-value:

	Player 1 Nonvul. player	Player 2 Vul. player	Player 3 Vul. player
$\pi_1 = \{\{1\},\{2\},\{3\}\}$	4.167	2.772	6.306
$\pi_2 = \{\{1,2,3\}\}$	4.240	3.116	6.669
$\pi_3 = \{\{1\},\{2,3\}\}$	4.167	3.378	6.912
$\pi_{4_1} = \{\{1,2\},\{3\}\}$	4.220	2.825	6.596
$\pi_{4_2} = \{\{1,3\},\{2\}\}$	1.758	3.043	-0.247

Conditions to make scenario π_{4_1} stable (including 5 variables):

$$\xi_1^{\pi_{4_1}} \geq 4.167,$$
$$\xi_1^{\pi_{4_1}} \geq \xi_1^{\pi_{4_2}},$$
$$7.045 - \xi_1^{\pi_{4_1}} \geq 2.772,$$
$$7.045 - \xi_1^{\pi_{4_1}} \geq \xi_2^{\pi_3},$$
$$6.596 \geq 14.024 - \xi_1^{\pi_2} - \xi_2^{\pi_2}.$$

Nash stability of partial cooperation scenario

	Player 1 Nonvul. player	Player 2 Vul. player	Player 3 Vul. player
$\pi_1 = \{\{1\},\{2\},\{3\}\}$	4.167	2.772	6.306
$\pi_2 = \{\{1,2,3\}\}$	4.240	3.189 $\uparrow$	6.596 $\downarrow$
$\pi_3 = \{\{1\},\{2,3\}\}$	4.167	2.825 $\downarrow$	7.465 $\uparrow$
$\pi_{4_1} = \{\{1,2\},\{3\}\}$	4.220	2.825	6.596
$\pi_{4_2} = \{\{1,3\},\{2\}\}$	1.758	3.043	-0.247

Conditions to make scenario π_{4_1} stable (including 5 variables):

$$\begin{aligned}
&\xi_1^{\pi_{4_1}} \geq 4.167,\\
&\xi_1^{\pi_{4_1}} \geq \xi_1^{\pi_{4_2}},\\
&7.045 - \xi_1^{\pi_{4_1}} \geq 2.772,\\
&7.045 - \xi_1^{\pi_{4_1}} \geq \xi_2^{\pi_3},\\
&6.596 \geq 14.024 - \xi_1^{\pi_2} - \xi_2^{\pi_2}.
\end{aligned}$$

Conclusions

- Definition of the transfer payment scheme based on individual stability approach.
- Extension of the transfer payment schemes for more than 3 players.
- Transitions from one scenario to another one may be costly (job change, divorce, etc.)
- Mechanism design of the transfer distribution over time, based on the "nice" properties (time consistency, individual rationality, proportional stability, etc.)
- Existence of stable coalition structures in some classes of differential games.

Thank you!