Sustainable agreements in a dynamic game of river pollution with network externalities

Artem Sedakov

Saint Petersburg State University

Workshop on Dynamic Games and Applications
October 27, 2022

Motivation

[Dotan et al., 2017]: a study on endocrine disrupting compounds entering surface waters in two transboundary streams between Israel and the Palestinian West Bank.

- Socio-economic asymmetry.
- Streams receive raw Palestinian wastewater and are only treated when entering the Israeli side.
- A coordinated strategy and joint water management would yield greater benefit for both parties.
- A joint Israeli–Palestinian commission established to develop effective strategies ceased to function a short time later.

Motivation

[Sedakov et al., 2021]: a dynamic game of river pollution (firms are located along the river, represented by a graph)

- Equilibrium behavior is more harmful to the environment than cooperation.
- There must be effective mechanisms stimulating firms to reduce pollution by lowering outputs.
- To encourage cooperation, one can give firms more benefits in the allocation of the cooperative profit (individual rationality).

Literature

- The acid rain differential game: [Mäler, 1989]
- Transboundary pollution between two countries:
  - [Kaitala et al., 1992] (Finland and the USSR)
  - [Mäler and de Zeeuw, 1998] (U.K. and Ireland)
  - [Fernandez, 2008] (U.S. and Mexico)
- A game involving waste disposal: [Jørgensen, 2010] (three neighboring regions)
- A model of river pollution: [Sedakov et al., 2021] (n regions connected in a graph)

Literature

- The acid rain differential game: [Mäler, 1989]
- Transboundary pollution between two countries:
  - [Kaitala et al., 1992] (Finland and the USSR)
  - [Mäler and de Zeeuw, 1998] (U.K. and Ireland)
  - [Fernandez, 2008] (U.S. and Mexico)
- A game involving waste disposal: [Jørgensen, 2010] (three neighboring regions)
- A model of river pollution: [Sedakov et al., 2021] (n regions connected in a graph)

Mechanisms supporting cooperation:

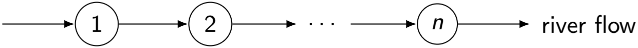
- [Petrosyan, 1979], [Belitskaya and Petrosyan, 2012]: IDP for TU games
- [Petrosyan and Yeung, 2014]: PDP for NTU games

Outline

A model of river pollution

Notations

- $N = \{1, \ldots, n\}$: the set of competing firms which produce homogeneous goods and sell them in a market.
- $\mathcal{T} = \{0, 1, \ldots, T\}$: the set of periods.
- The firms are located along a river.



- The production of the goods is associated with water pollution.
- Single pollutant.
- $u_i(t)$: the amount of the pollutant of firm i in period t.
- $x_i(t)$: the amount of the pollutant in the water within the region administered by firm i at period t.
- Upstream firms influence the water pollution levels in the regions of downstream firms — we have a **directed** network g.

State dynamics and profits

- The relationship between the states:

$$x_i(t+1) = \alpha x_i(t) + u_i(t) + \sum_{j \in N_i^{in}(g)} \delta^{\omega_{ji}(g)} u_j(t).$$

  - $\alpha \in [0,1]$: the natural decline in pollutant concentration;
  - $\delta \in (0,1]$: a decay rate [Jackson and Wolinsky, 1996];
  - $N_i^{in}(g) = \{j \in N : j \xrightarrow{g} i\}$;
  - $\omega_{ji}(g)$: the length of the shortest path $j \xrightarrow{g} i$.

State dynamics and profits

- The relationship between the states:

$$x_i(t+1) = \alpha x_i(t) + u_i(t) + \sum_{j \in N_i^{in}(g)} \delta^{\omega_{ji}(g)} u_j(t).$$

  - $\alpha \in [0,1]$: the natural decline in pollutant concentration;
  - $\delta \in (0,1]$: a decay rate [Jackson and Wolinsky, 1996];
  - $N_i^{in}(g) = \{j \in N : j \xrightarrow{g} i\}$;
  - $\omega_{ji}(g)$: the length of the shortest path $j \xrightarrow{g} i$.

- The firm's stage profit (corresponds to the Cournot competition):

$$h_{it}(x_i(t), u(t)) = p_i(u(t))\, u_i(t) - cu_i(t) - dx_i(t), \quad t \neq T,$$
$$h_{iT}(x_i(T)) = -dx_i(T).$$

  - $cu_i(t)$: production cost functions, $c > 0$;
  - $dx_i(t)$: environmental damage, $d > 0$;
  - $p_i(u(t)) \triangleq a - u_i(t) - b\sum_{j \neq i} u_j(t)$: inverse demand function, $a > c$, $b \geqslant 0$.

State dynamics and profits

- The relationship between the states:

$$x_i(t+1) = \alpha x_i(t) + u_i(t) + \sum_{j \in N_i^{in}(g)} \delta^{\omega_{ji}(g)} u_j(t).$$

  - $\alpha \in [0,1]$: the natural decline in pollutant concentration;
  - $\delta \in (0,1]$: a decay rate [Jackson and Wolinsky, 1996];
  - $N_i^{in}(g) = \{j \in N : j \xrightarrow{g} i\}$;
  - $\omega_{ji}(g)$: the length of the shortest path $j \xrightarrow{g} i$.

- The firm's stage profit (corresponds to the Cournot competition):

$$h_{it}(x_i(t), u(t)) = p_i(u(t))\, u_i(t) - cu_i(t) - dx_i(t), \quad t \neq T,$$
$$h_{iT}(x_i(T)) = -dx_i(T).$$

  - $cu_i(t)$: production cost functions, $c > 0$;
  - $dx_i(t)$: environmental damage, $d > 0$;
  - $p_i(u(t)) \triangleq a - u_i(t) - b\sum_{j \neq i} u_j(t)$: inverse demand function, $a > c$, $b \geqslant 0$.

- The firm's total profit:

$$J_i(x_0, u) = \sum_{t=0}^{T-1} \varrho^t h_{it}(x_i(t), u(t)) + \varrho^T h_{iT}(x_i(T)), \quad \varrho \in (0,1].$$

A general linear-state game with network externalities

A general model

- An **arbitrary** directed network g.

- State equation:

$$x_i(t+1) = b_{i0}x_i(t) + b_{ii}u_i(t) + \sum_{j \in N_i^{in}(g)} b_{ij}u_j(t).$$

with the initial condition $x_i(0) = x_{i0}$.

- Stage payoffs:

$$h_{it}(x_i(t), u(t)) = a_{i0}u_i(t) + a_{i1}u_i^2(t) + a_{i2}u_i(t)\sum_{j \neq i} u_j(t) + a_{i3}x_i(t),$$
$$h_{iT}(x_i(T)) = a_{i4}x_i(T).$$

Solution concepts

Solution concepts

- **Pareto solution** $u^P = (u_1^P, \ldots, u_n^P)$:

$$u^P = \arg\max_u \sum_{i \in N} \theta_i J_i(x_0, u), \quad \theta_i > 0, \; \sum_{i \in N} \theta_i = 1.$$

Solution concepts

- **Pareto solution** $u^P = (u_1^P, \ldots, u_n^P)$:

$$u^P = \arg\max_u \sum_{i \in N} \theta_i J_i(x_0, u), \quad \theta_i > 0,\ \sum_{i \in N} \theta_i = 1.$$

- A **cooperative** strategy profile $u^C = (u_1^C, \ldots, u_n^C)$:

$$u^C = \arg\max_u \sum_{i \in N} J_i(x_0, u).$$

  - Equal surplus division value (ES-value);
  - Shapley value

    An equilibrium profile for the characteristic function (γ-approach [Chander and Tulkens, 1997]):

$$\begin{cases} u_S^{N,S} & = \arg\max_{u_S} \sum_{i \in S} J_i(x_0, (u_S, u_{-S}^{N,S})), \quad i \in S, \\ u_i^{N,S} & = \arg\max_{u_i} J_i(x_0, (u_i, u_{-i}^{N,S})), \quad i \notin S. \end{cases}$$

Solution concepts

- **Pareto solution** $u^P = (u_1^P, \ldots, u_n^P)$:

$$u^P = \arg\max_u \sum_{i\in N} \theta_i J_i(x_0, u), \quad \theta_i > 0,\ \sum_{i\in N} \theta_i = 1.$$

- A **cooperative** strategy profile $u^C = (u_1^C, \ldots, u_n^C)$:

$$u^C = \arg\max_u \sum_{i\in N} J_i(x_0, u).$$

  - Equal surplus division value (ES-value);
  - Shapley value

    An equilibrium profile for the characteristic function (γ-approach [Chander and Tulkens, 1997]):

$$\begin{cases} u_S^{N,S} & = \arg\max_{u_S} \sum_{i\in S} J_i(x_0, (u_S, u_{-S}^{N,S})), \quad i \in S, \\ u_i^{N,S} & = \arg\max_{u_i} J_i(x_0, (u_i, u_{-i}^{N,S})), \quad i \notin S. \end{cases}$$

- **Nash bargaining solution** $u^B = (u_1^B, \ldots, u_n^B)$:

$$u^B = \arg\max_u \prod_{i\in N} (J_i(x_0, u) - J_i^N),$$

$J^N = (J_1^N, \ldots, J_n^N)$ is a disagreement point.

Pareto solution

$$u_i^P(t,\theta) = \sum_{\ell \in N} \left(\frac{\left(\frac{\tilde{a}_{i2} \sum_{j \in N} \frac{\tilde{a}_{j2}}{\theta_j a_{j2}}}{2+\sum_{j \in N} \tilde{a}_{j2}} - \frac{\tilde{a}_{i2}}{\theta_i a_{i2}} \right) \left(\frac{\sum_{j \in N} \theta_j a_{j2} \tilde{a}_{j2}}{2+\sum_{j \in N} \tilde{a}_{j2}} - \theta_\ell a_{\ell 2} \right)}{2+\sum_{j \in N} \tilde{a}_{j2} - \frac{\sum_{j \in N} \theta_j a_{j2} \tilde{a}_{j2} \sum_{j \in N} \frac{\tilde{a}_{j2}}{\theta_j a_{j2}}}{2+\sum_{j \in N} \tilde{a}_{j2}}} + \frac{\tilde{a}_{i2}}{2+\sum_{j \in N} \tilde{a}_{j2}} \right)$$

$$\times \frac{\kappa_\ell(\bar{N}_\ell^{out}(g), t, \theta)}{2\theta_\ell(a_\ell - a_{\ell 2})} - \frac{\kappa_i(\bar{N}_i^{out}(g), t, \theta)}{2\theta_i(a_{i1} - a_{i2})}, \quad i \in N,$$

where

$$\bar{N}_i^{out}(g) = \{j \in N : i \xrightarrow{g} j\} \cup \{i\},$$

$$\kappa_i(S, t, \theta) = \theta_i a_{i0} + \varrho \sum_{j \in S} \theta_j a_{j3} b_{ji} \chi_j(t).$$

Pareto solution

$$u_i^P(t,\theta) = \sum_{\ell\in N}\left(\frac{\left(\frac{\tilde{a}_{i2}\sum_{j\in N}\frac{\tilde{a}_{j2}}{\theta_j a_{j2}}}{2+\sum_{j\in N}\tilde{a}_{j2}} - \frac{\tilde{a}_{i2}}{\theta_i a_{i2}}\right)\left(\frac{\sum_{j\in N}\theta_j a_{j2}\tilde{a}_{j2}}{2+\sum_{j\in N}\tilde{a}_{j2}} - \theta_\ell a_{\ell 2}\right)}{2+\sum\limits_{j\in N}\tilde{a}_{j2} - \frac{\sum_{j\in N}\theta_j a_{j2}\tilde{a}_{j2}\sum_{j\in N}\frac{\tilde{a}_{j2}}{\theta_j a_{j2}}}{2+\sum_{j\in N}\tilde{a}_{j2}}} + \frac{\tilde{a}_{i2}}{2+\sum_{j\in N}\tilde{a}_{j2}} \right)$$

$$\times \frac{\kappa_\ell(\bar{N}_\ell^{out}(g),t,\theta)}{2\theta_\ell(a_\ell - a_{\ell 2})} - \frac{\kappa_i(\bar{N}_i^{out}(g),t,\theta)}{2\theta_i(a_{i1} - a_{i2})}, \quad i\in N,$$

where

$$\bar{N}_i^{out}(g) = \{j\in N : i \xrightarrow{g} j\}\cup\{i\},$$

$$\kappa_i(S,t,\theta) = \theta_i a_{i0} + \varrho\sum_{j\in S}\theta_j a_{j3} b_{ji}\chi_j(t).$$

- Nash bargaining solution: $u_i^B(t) = u_i^P(t,\theta^*)$, $i\in N$.

Pareto solution

$$u_i^P(t,\theta) = \sum_{\ell\in N}\left(\frac{\left(\frac{\tilde{a}_{i2}\sum_{j\in N}\frac{\tilde{a}_{j2}}{\theta_j a_{j2}}}{2+\sum_{j\in N}\tilde{a}_{j2}} - \frac{\tilde{a}_{i2}}{\theta_i a_{i2}}\right)\left(\frac{\sum_{j\in N}\theta_j a_{j2}\tilde{a}_{j2}}{2+\sum_{j\in N}\tilde{a}_{j2}} - \theta_\ell a_{\ell 2}\right)}{2+\sum\limits_{j\in N}\tilde{a}_{j2} - \frac{\sum_{j\in N}\theta_j a_{j2}\tilde{a}_{j2}\sum_{j\in N}\frac{\tilde{a}_{j2}}{\theta_j a_{j2}}}{2+\sum_{j\in N}\tilde{a}_{j2}}} + \frac{\tilde{a}_{i2}}{2+\sum_{j\in N}\tilde{a}_{j2}}\right)$$

$$\times\ \frac{\kappa_\ell(\bar{N}_\ell^{out}(g),t,\theta)}{2\theta_\ell(a_\ell - a_{\ell 2})} - \frac{\kappa_i(\bar{N}_i^{out}(g),t,\theta)}{2\theta_i(a_{i1}-a_{i2})}, \quad i\in N,$$

where

$$\bar{N}_i^{out}(g) = \{j\in N : i \xrightarrow{g} j\}\cup\{i\},$$

$$\kappa_i(S,t,\theta) = \theta_i a_{i0} + \varrho\sum_{j\in S}\theta_j a_{j3} b_{ji}\chi_j(t).$$

- Nash bargaining solution: $u_i^B(t) = u_i^P(t,\theta^*)$, $i\in N$.
- Cooperative strategy profile: $u_i^C(t) = u_i^P(t,\frac{1}{n})$, $i\in N$.

Equilibrium profile for the characteristic function

For $i \in S$:

$$u_i^{N,S}(t) = \sum_{\ell \in S} \Big[\quad \Big] \kappa_\ell(\bar{N}_\ell^{out}(g) \cap S, t, \tfrac{1}{n}) + \sum_{j \in N \setminus S} \Big[\quad \Big] \kappa_j(j, t, \tfrac{1}{n})$$
$$- \Big[\quad \Big] \kappa_i(\bar{N}_i^{out}(g) \cap S, t, \tfrac{1}{n}).$$

For $i \notin S$:

$$u_i^{N,S}(t) = \sum_{\ell \in S} \Big[\quad \Big] \kappa_\ell(\bar{N}_\ell^{out}(g) \cap S, t, \tfrac{1}{n}) + \sum_{j \in N \setminus S} \Big[\quad \Big] \kappa_j(j, t, \tfrac{1}{n}) - \Big[\quad \Big] \kappa_i(i, t, \tfrac{1}{n}).$$

- Nash equilibrium: $u_i^N(t) = u_i^{N,\{i\}}(t)$, $i \in N$.

A special case: cross-product term $a_{i2} = 0$, $i \in N$

Pareto solution:

$$u_i^P(t,\theta) = -\frac{\kappa_i(\bar{N}_i^{out}(g), t, \theta)}{2\theta_i a_{i1}} = -\frac{\theta_i a_{i0} + \varrho \sum_{j \in \bar{N}_i^{out}(g)} \theta_j a_{j3} b_{ji} \chi_j(t)}{2\theta_i a_{i1}}.$$

Equilibrium profile for the characteristic function:

$$u_i^{N,S}(t) = \begin{cases} -\frac{\kappa_i(\bar{N}_i^{out}(g) \cap S, t, \frac{1}{n})}{\frac{2}{n} a_{i1}} = -\frac{a_{i0} + \varrho \sum_{j \in \bar{N}_i^{out}(g) \cap S} a_{j3} b_{ji} \chi_j(t)}{2a_{i1}}, & i \in S, \\ -\frac{\kappa_i(\{i\}, t, \frac{1}{n})}{\frac{2}{n} a_{i1}} = -\frac{a_{i0} + \varrho a_{i3} b_{ii} \chi_i(t)}{2a_{i1}}, & i \notin S, \end{cases}$$

A special case: cross-product term $a_{i2} = 0$, $i \in N$

Pareto solution:

$$u_i^P(t,\theta) = -\frac{\kappa_i(\bar{N}_i^{out}(g), t, \theta)}{2\theta_i a_{i1}} = -\frac{\theta_i a_{i0} + \varrho \sum_{j \in \bar{N}_i^{out}(g)} \theta_j a_{j3} b_{ji} \chi_j(t)}{2\theta_i a_{i1}}.$$

Equilibrium profile for the characteristic function:

$$u_i^{N,S}(t) = \begin{cases} -\frac{\kappa_i(\bar{N}_i^{out}(g) \cap S, t, \frac{1}{n})}{\frac{2}{n} a_{i1}} = -\frac{a_{i0} + \varrho \sum_{j \in \bar{N}_i^{out}(g) \cap S} a_{j3} b_{ji} \chi_j(t)}{2a_{i1}}, & i \in S, \\ -\frac{\kappa_i(\{i\}, t, \frac{1}{n})}{\frac{2}{n} a_{i1}} = -\frac{a_{i0} + \varrho a_{i3} b_{ii} \chi_i(t)}{2a_{i1}}, & i \notin S, \end{cases}$$

Nash equilibrium:

$$u_i^N(t) = -\frac{\kappa_i(\{i\}, t, \frac{1}{n})}{\frac{2}{n} a_{i1}} = -\frac{a_{i0} + \varrho a_{i3} b_{ii} \chi_i(t)}{2a_{i1}}.$$

Allocation procedures

PDP [Petrosyan and Yeung, 2014] and IDP [Petrosyan, 1979]

- PDP for the Nash bargaining solution:

$$PDP_i^B(t) = \begin{cases} \dfrac{1}{\varrho^t} \dfrac{J_i^B - J_i^N}{T+1} + J_i^N(t) - \varrho J_i^N(t+1), & t \neq T, \\ \dfrac{1}{\varrho^T} \dfrac{J_i^B - J_i^N}{T+1} + J_i^N(T), & t = T. \end{cases}$$

- IDP for the Shapley value:

$$IDP_i^{Sh}(t) = \begin{cases} Sh_i(t) - \varrho Sh_i(t+1), & t \neq T, \\ Sh_i(T), & t = T. \end{cases}$$

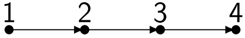
- IDP for the ES-value:

$$IDP_i^{ES}(t) = \begin{cases} ES_i(t) - \varrho ES_i(t+1), & t \neq T, \\ ES_i(T), & t = T. \end{cases}$$

Example

Linear network

- Four firms, $N = \{1, 2, 3, 4\}$.



- Model parameters:
  - $T = 10$;
  - $\varrho = 0.95$;
  - $a = 3$, $b = 0.5$;
  - $c = 1$, $d = 0.1$;
  - $\alpha = 0.65$, $\delta = 2/3$.

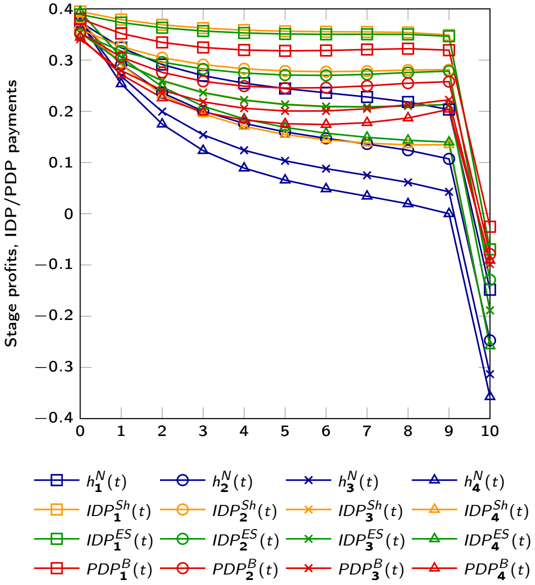
- The initial stock of the pollutant is zero.

Firms' profits

i	1	2	3	4	$\sum$
J_i^N	2.0900	1.5077	1.1195	0.8607	5.5779
J_i^C	1.6046	1.8639	2.2888	2.8498	8.6070
J_i^{B*}	2.6593	2.1470	1.8470	1.7119	8.3651
ES_i	2.8473	2.2650	1.8768	1.6180	
Sh_i	2.8881	2.3248	1.8819	1.5122	

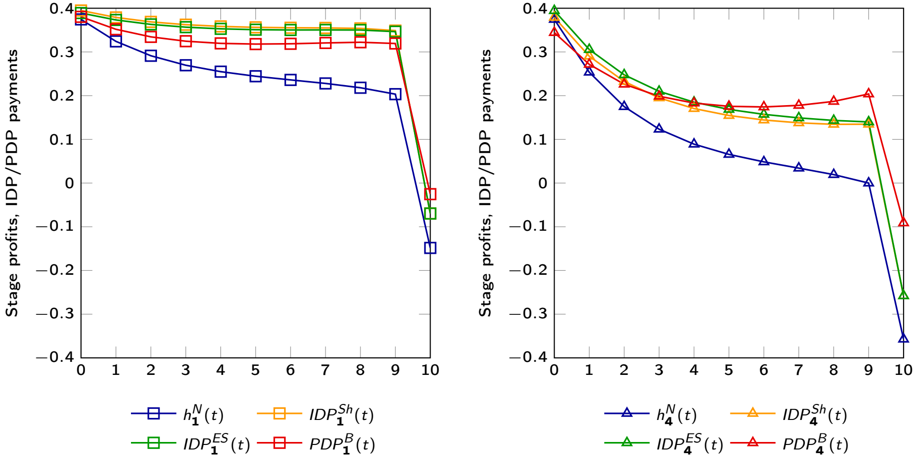
$^* \ \theta = (0.2992, 0.2665, 0.2342, 0.2001)$.

Linear network



Linear network

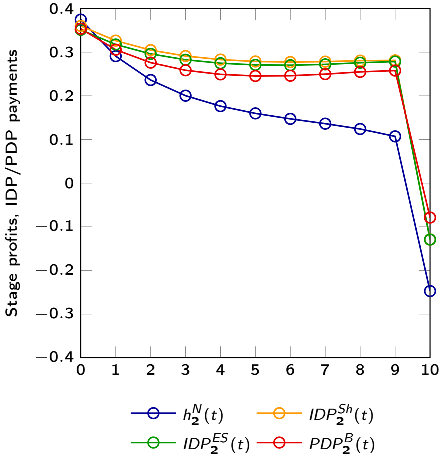
Firms 1 and 4

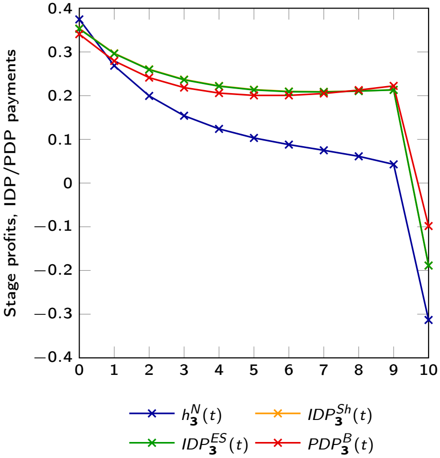


i	J_i^N	J_i^B	ES_i	Sh_i
1	2.0900	2.6593	2.8473	2.8881
4	0.8607	1.7119	1.6180	1.5122

Linear network

Firms 2 and 3





i	J_i^N	J_i^B	ES_i	Sh_i
2	1.5077	2.1470	2.2650	2.3248
3	1.1195	1.8470	1.8768	1.8819

Conclusions

- The principal contribution is the analysis of an oligopolistic competition model of river pollution.

Conclusions

- The principal contribution is the analysis of an oligopolistic competition model of river pollution.
- Under cooperation, an upstream firm gets a lower profit than a downstream one, which can be attributed to its support of cooperation despite being located in a less polluted region ($J_i^C < J_i^N$ for $i \leqslant \underline{i}$).

Conclusions

- The principal contribution is the analysis of an oligopolistic competition model of river pollution.
- Under cooperation, an upstream firm gets a lower profit than a downstream one, which can be attributed to its support of cooperation despite being located in a less polluted region ($J_i^C < J_i^N$ for $i \leqslant \underline{i}$).
- A coordinated behavior can be individually rational (Shapley value, ES-value, Nash bargaining solution).

  For a linear network, J_i^N decreases in $i \quad \Rightarrow \quad ES_i$ decreases in i.

Conclusions

- The principal contribution is the analysis of an oligopolistic competition model of river pollution.
- Under cooperation, an upstream firm gets a lower profit than a downstream one, which can be attributed to its support of cooperation despite being located in a less polluted region ($J_i^C < J_i^N$ for $i \leqslant \underline{i}$).
- A coordinated behavior can be individually rational (Shapley value, ES-value, Nash bargaining solution).

  For a linear network, J_i^N decreases in $i \quad \Rightarrow \quad ES_i$ decreases in i.
- PDP/IDP for bargaining/cooperative solutions allow for the implementation of agreed-upon solutions.

Thank you.