Asymptotic Integration of Thin Shell Equations by Means of Computer Algebra Methods

Irina M. [illegible][1] *Andrei L. Smirnov*[2] *Elisa M. [illegible]*[1]

[1]Department of Mechanical Engineering, Concordia University, Montreal, QC, Canada, H3G 1M8
[2]Department of Theoretical and Applied Mechanics, St. Petersburg State University, St. Petersburg, Russia, 198904

The construction of formal asymptotic solutions for linear differential equations describing small vibrations of thin shells of revolution is discussed. An integration method developed by Goldenveizer [1, 2] and his collaborators is employed. A detailed review of this method may be found in [3].

After the separation of variables in circumferential and meridional directions, the equations of vibration for thin shells of revolution have the form

$$y'(s) = A(s, \mu, \lambda, m)y(s), \qquad (1)$$

where $y(s)$ is a vector function of size $n \times 1$, $A(s)$ is a $n \times n$ matrix, s is the length of the arc along the meridian, and μ, λ and m are parameters representing respectively the thickness, the natural frequency and the wave number in circumferential direction. The order n of the system depends on the initial assumptions. For example, for theories of the Kirchhoff-Love type $n = 8$, whereas in Reissner's shell theory $n = 10$.

Equation (1) together with the boundary conditions represents the boundary value problem. For the Kirchhoff-Love theory, employed in the present case, it is most convenient to use the following variables $y = (u, v, w, T_1, S_1, N_1, M_1, \gamma_1)$, where u and v are the tangential components of the displacement, w is the deflection, T_1 and S_{11} are the tangential forces, N_1 is the generalized transverse force, M_1 is the stress-couple and γ_1 is the angle of rotation of the normal [4]. For such variables the boundary conditions have simple forms, for example $u = v = w = \gamma_1 = 0$, for a clamped edge and $T_1 = S_{11} = N_1 = M_1 = 0$, for a free edge.

Sometimes it is more convenient to use one equation of the 8^{th} order with respect to the deflection [6], that is equivalent to system (1):

$$\sum_{i=0}^{8} a_i(s, \mu, \lambda, m)w^{(i)} = 0 \qquad (2)$$

For thin shells the thickness parameter μ is small and it is [illegible] to be the main parameter. The order of the [illegible]

One can construct an asymptotic solution for equation (2) by representing the solution in the form

$$w = \sum_{i=1}^{8} C_i \sum_{k=0}^{\infty} \mu^k w_{ik} \exp\left(\frac{1}{\mu}\int_{s_0}^{s} p_i(s)ds\right) \qquad (3)$$

where C_i are arbitrary constants and [illegible].

To analyze the structure of the asymptotic expansions, a solution of the following form is initially sought [5]:

$$w = w_0 \exp\left(\int_{s_0}^{s} p(s)ds\right) \qquad (4)$$

Substituting (4) into (2), in the first approximation, the characteristic equation for p is obtained as

$$\sum P_i(p, \mu, \lambda, m, s) = \sum_{i=0}^{8} p^i \mu^{\alpha_i} \lambda^{\beta_i} m^{\gamma_i} a_i(s) = 0, \qquad (5)$$

where $a_i \sim 1$.

The case when all roots of equation (5) are simple is considered, i.e. when $p_i(s) \neq p_j(s)$ for all s. Then all solutions are linearly independent and their linear combination provides the general solution of the initial equation.

Substituting the solution $w(s, \mu, \lambda, m)$ into the boundary conditions, the characteristic equation for λ is obtained. The order of the function $p(s)$ in μ is called the index of variation of a solution. The solution is exponentially increasing if $\Re p(s) > 0$, exponentially decreasing if $\Re p(s) < 0$, and oscillating if $\Re p(s) = 0$ and $\Im p(s) \neq 0$. If $p(s) = 0$, a case which can occur for equations (2), the solution is [illegible] slowly varying. If the following conditions are satisfied

$$\Re p(s_1) < 0, \quad \Re\int p(s)ds < 0, \quad s_1 < s \leq s_2$$

the integral is called the integral of the [illegible] [illegible]. This [illegible] from the edge [illegible] all $s > s_1$. In solving the boundary value problem with an error of the order $\epsilon^{-c/\mu}$, where $c > 0$, one may take the value of this integral to be equal to zero at the right end, $s = s_2$. One introduces, in a similar way, the integral of the edge effect at the right end, $s = s_2$. The integral is said to be oscillating if for s, $\Re p(s) = 0$ and $\Im p(s) \neq 0$. There may exist integrals which do not belong to any of the above types.

To study equation (5), traditionally, the Newton polygon method is used [4]. In the present investigation the method of construction of a convex hull for a point set is employed. For example, if $\mu \ll 1$, then the convex hull for the points (l, α_l) is constructed on the plane (l, α). To consider the most important cases when p is large, the new variable $p^* = 1/p$, that is small, is introduced. Each edge of the convex hull determines the vertices that correspond to the main terms of equation (5). This method may be generalized for the case of two parameters (μ, λ), when the convex hull is constructed in 3D.

In the case of 2D the algorithm described by Graham is used, whereas for the 3D case the so-called "gift wrapping" algorithm developed by Chand and Kapur [7] is employed. The code for the algorithm has been written in *Mathematica* 3.0.

As an example, the low frequency axisymmetric vibration of a thin circular cylinder has been analyzed. For this case μ, λ $\ll 1$ and $m = 0$. The results agree well with those obtained in [4].

Since the initial equations may also contain some other small parameters, for example, parameter $1/m$ for vibrations with a large wave number in circumferential direction, or for the rotating shell, the relative angular velocity, the generalization of the above method for the 4D and 5D cases is of practical importance.

ACKNOWLEDGMENT

The authors I.M. Landman and E.M. Haseganu acknowledge the support of the Natural Sc[illegible] Engineering Research Council of Canada.

REFERENCES

[1] A.L. Goldenveizer, "Theory of Thin Elast[illegible] translated from Russian by G. Herrmann, [illegible] Press (1961).

[2] A.L. Goldenveizer, "Asymptotic Methods [illegible] of Shells", *Adv. in Mech.*, 1/2 (5), 137-182 [illegible]

[3] R. Vaillancourt and A.L. Smirnov eds., "A[illegible] Methods in Mechanics", *CRM Proceed[illegible] Lecture Notes*, AMS, Providence, RI (199[illegible]

[4] A.L. Goldenveizer, V.B. Lidsky and P.E. [illegible] "Free Vibrations of Thin Elastic Shells" [illegible] Moscow (1979).

[5] S.M. Bauer, A.L. Smirnov, P.E. Tovstik [illegible] Filippov, "Asymptotic Methods in [illegible] Examples", (in Russian), St. Petersburg [illegible] Press, St. Petersburg (1997).

[6] A.V. Grinkevich and A.L. Smirnov, "The [illegible] of the Transformations of a System [illegible] Differential Equations to the Standard [illegible] *Mechanics Research Communications*, 2[illegible] (1994).

[7] F.P. Preparata and S.J. Hong, "Convex Hul[illegible] Sets of Points in Two and Three Di[illegible] Commun. ASME, 22, 87-93 (1977).