ASYMPTOTIC INTEGRATION OF FREE VIBRATION EQUATIONS OF CYLINDRICAL SHELLS

Andrei L. Smirnov

Dept. of Theoretical and Applied Mechanics, St. Petersburg State University,
2 Bibliotechnaya Sq., St. Petersburg, 198904, Russia,
Email: smirnov@bals.usr.pu.ru

The asymptotic method of solution of the equations describing the small nonaxisymmetric vibrations of a closed thin cylindrical shell is discussed in this report. The linear shell theory of Kirchhoff-Love type is applied. After separating the nondimensional variables in the equations for shell vibrations the singularly perturbed boundary value problem is obtained

$$y'(s) = A(s; h, \lambda, m)y(s),$$

where $y(s)$ is a vector-function of size, $A(s)$ is matrix, s is the length of arc of the meridian ($0 \leq s \leq s_1$), λ is the parameters of natural frequency and m is the wave number in the circumferential direction. The relative shell thickness, h, is considered to be the main small parameter and the rest of parameters are expressed through h. The algorithm for solving this boundary value problem is based on the method developed in [1] and is realized with the computer algebra package Mathematica 3.0. For axisymmetric vibrations of a cylindrical shell such an algorithm has been reported in [2].

First, the asymptotic solution is constructed. Each solution is represented in the exponential form

$$y_j(s) = \sum_{i=1}^{8} C_i \sum_{k=0}^{\infty} h^k y_{j,ik} \exp \int_0^{s_1} p_i \, ds,$$

where the values of p_i are determined from the characteristic equation

$$P_8(p; h, \lambda, m) = \sum_i a_i p^i h^{\alpha_i} \lambda^{\beta_i} m^{\gamma_i}$$

To find the roots p_i for different values of parameters the generalization of the Newton polygon method is used. The representative points $(i, \alpha_i, \beta_i, \gamma_i)$ are plotted in the diagram and then the convex hull is constructed. Each facet of the convex hull determines the relations between the parameters (the separative points) where the asymptotic character of solutions changes. For example, the facets of the convex hull plotted for $m \sim 1$ determine the critical points $\lambda \sim 1$, $\lambda \sim h^4$, $\lambda \sim h^{-4}$. All these cases together with the intermediate zones have been considered and the corresponding solutions have been constructed for each case. For example, for the low frequency vibrations $\lambda \sim h^{\varkappa}$ ($4 > \varkappa > 0$) four roots have the order h^{-1} and four the order of $h^{\varkappa/4}$.

Then, after obtaining the values of p_i, the solution is substituted into the boundary conditions to form the characteristic equation for the frequency λ. In several cases for some types of boundary conditions the determinant obtained may be simplified as $h \to 0$, to get the analytical formulas for λ [1,2]. For other cases λ should be found numerically from the 8th order equation.

References: [1] Vaillancourt R. and Smirnov A.L. eds., 1993, Asymptotic Methods in Mechanics, CRM Proceedings and Lecture Notes, AMS, Providence, RI. [2] Landman, I.M., Smirnov, A.L., Haseganu, E.M., 1999, Asymptotic Integration of Thin Shell Equations by Means of Computer Algebra Methods, Proceedings of the 17th Canadian Congress of Applied Mechanics, McMaster University, Hamilton, ON, 37-38.