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BUCKLING OF PLATES AND SHELLS WEAKENED WITH CUT-OUTS

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Abstract. *The research concerns behavior of non-homogeneous elastic thin structures weakened with cut-outs, holes and cracks. The purpose of the study is to analyze the effect of shape, area, position and proportions of the rectangular holes on buckling of rectangular plates and cylindrical shells, under external loadings of different types (axial compressing force, hydrostatic pressure etc.). Special attention is devoted to perforated plates and shells.*

The most important and interesting is the effect of the area of a hole on buckling. The results obtained with FEM method by ANSYS package are compared with those obtained with the Rayleigh-Ritz method. The values of the critical buckling loadings appear to be very sensitive to proportion of the hole and not very sensitive to the shape and position of the hole.

It is well known that the critical loadings for shells of revolutions, in particularly, for circular cylindrical shells are asymptotically doubled since the buckling modes are the functions of either sine or cosine of the angle in the circumferential direction. The appearance of a hole or a crack in the shell leads to bifurcation of the values of the critical loadings. The effect of the area of the hole and the ratio of its sides on the critical loading and buckling modes is examined. The most interesting is the effect of the multiple holes (perforation), for which the existence of so-called "resonance" modes is revealed. Indeed, if the wave number in the circumferential direction is divisible by number of holes one of the doubled critical loadings essentially decreases and the other increases. The influence of the shape of the hole is also analyzed [4].

1 INTRODUCTION

The research concerns the buckling of thin-walled isotropic structures, such as plates and shells, weakened with the holes or cut-outs under the compressive external loadings. The purpose of the study is to analyze the effect of shape, area, position and proportions of the rectangular holes on buckling of rectangular plates and cylindrical shells, under external loadings of different types (axial compressing force, hydrostatic pressure etc.) for different boundary conditions. The results obtained with FEM method by ANSYS package and those obtained with the Rayleigh-Ritz method[1] and analytical results for plates without holes [2, 3] are compared.

2 THE STATEMENT OF THE PROBLEM

In this paper we restrict ourselves to analysis of thin isotropic plate under axial compressive load q directed along the axis OX (see Fig. 1)

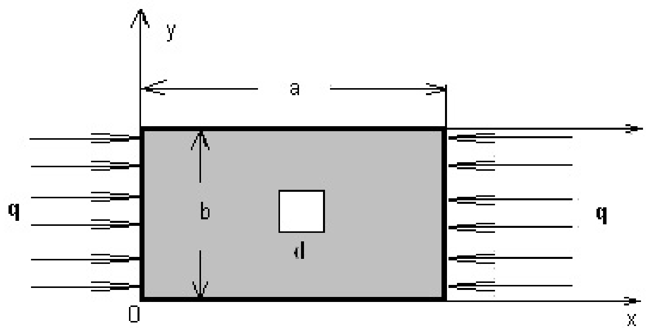

Figure 1: Rectangular plate under axial compression.

The lateral faces of the plate are of the length a, the side ends have the length b and $a \geqslant b$, the sides of the central square hole are parallel to the plate sides and have the length d.

Equation for evaluation of the critical load for the homogeneous plate under axial compression has the form [2]:

$$D\nabla^2\nabla^2 w + q\frac{d^2w}{dx^2} = 0, \qquad D = \frac{Eh^3}{12(1-\nu^2)}, \tag{1}$$

where w is the deflection, D is the bending stiffness, E is the Young modulus, ν is the Poisson ratio, h is the plate thickness. Here we use the non-dimensional variables, which relate to dimensional (with $\sim$) as:

$$D = \tilde{D}/(Eb^3), q = \tilde{q}/(Eb), \{a, d, h, w\} = \{\tilde{a}, \tilde{d}, \tilde{h}, \tilde{w}\}/b \tag{2}$$

Later only symmetric boundary conditions of clamped, free and simply supported types are considered.

For homogeneous ($h =$ const) simply supported long plate equation (1) may be reduced to more simple equation [2]:

$$D\frac{d^4w}{dx^4} + q\frac{d^2w}{dx^2} = 0, \qquad w|_{x=0,a} = 0, w''|_{x=0,a} = 0, \tag{3}$$

from which the critical load may be evaluated as

$$q_{cr} = \frac{\pi^2 D}{a^2}, \tag{4}$$

and the buckling mode has the form of cylindrical bending $w = A \sin \frac{\pi n x}{a}$.

For numerical analysis of the problem the FEM package ANSYS was used. As the elements SHELL63 were chosen. The convergence of the method was examined for buckling of a steel ($E = 2.0 \cdot 10^{11}$N/m^2, $\nu = 0.3$) homogeneous plate of thickness $h = 0.01$ with free lateral sides and simply supported end sides ($a = 2$, $b = 1$). For the square elements of the relative sizes $t = 0.025$ the relative difference between the numerical results and those obtained by (4) is about 7%. The problem is in formula (4), which is accurate only for long enough plates or plates with small Poisson's ratio. Solution of equation (1) with proper boundary conditions leads to formula

$$q_{cr} = 0.928 \frac{\pi^2 D}{a^2}, \tag{5}$$

which gives rather better agreement with the numerical results (error about 0.1%).

3 EFFECT OF THE HOLE AREA

3.1 Buckling of the plate with free lateral edges under axial compression

Now we consider the plate with the central square hole with the sides of the length d parallel to the sides of the plate. The effect of the hole area may be roughly estimated, if we introduce the averaged plate thickness as $\bar{h} = h(1 - d^2/ab)$. For such thickness the homogeneous plate has same volume as the plate with the hole. Substituting in (4) we obtain

$$q_{cr} = \frac{\pi^2 D}{a^2}(1 - d^2/ab)^3. \tag{6}$$

In Fig. 2 the effect of the hole area on the critical loading is shown.

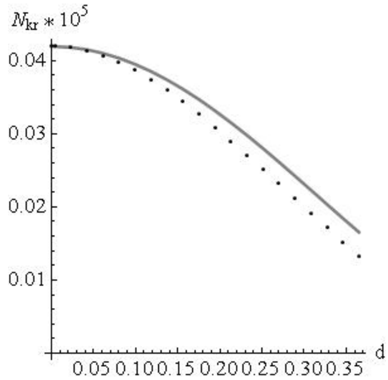

Figure 2: The effect of the hole area on the critical loading for the plate with free lateral sides and simply supported end sides.

The numerical results are plotted with the dotted line and the solid line corresponds to the results due to (6). Therefore for the plate with free lateral sides the critical loading decreases with the hole area.

3.2 Buckling of the plate with simply supported edges under axial compression

Consider here same plate as in 3.1 with simply supported boundary conditions at each edge. For the homogeneous plate the critical buckling load and buckling modes may be obtained from (1). The deflection w has the form

$$w = A \sin \frac{\pi n x}{a} \sin \frac{\pi m y}{b}, \quad (7)$$

and the buckling load is the following [2]

$$q_{cr} = \min_{m,n} \left[((mr)^2 + n^2) \frac{\pi^2 D}{a^2 n^2} \right]; r = \frac{a}{b} \quad (8)$$

For homogeneous simply supported plate the results of buckling analysis by means of ANSYS well agree with the analytical results with the relative error less than 1%.

For the plate with the central square hole the interaction of the hole and the buckling mode plays significant role and affects the critical loading. In Fig. 3 the effect of the plate length on the critical loading is shown for the plate with the hole of constant area (dotted line) and for the homogeneous plate (solid line). The critical loading is the lower envelope for the curves. For the homogeneous plate the minimum attains at $r = n$, where n is the wave number in the axial direction.

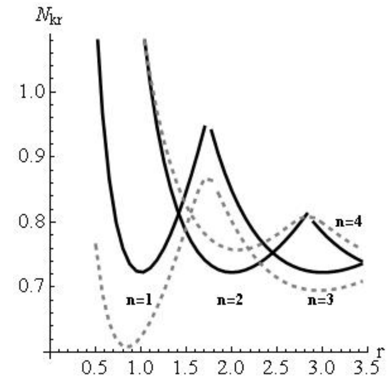

Figure 3: Effect of the relative length of the plate (r) on the critical loading for the plate with the hole (dotted line) and for the homogeneous plate (solid line)

One can see in Fig. 3 that for buckling modes with odd wave number n the critical loading decreases as the hole area increases, and for even n the critical loading goes up with the hole area. This problem was considered in [1] by means of Rayleigh-Ritz method. For odd modes the results of that work well agrees with our numerical results, but for even modes the results differ drastically. It seems that the representation of deflection with only one term of the series selected by the authors of [1] does not sufficiently well describe the shape of the buckling modes.

In Fig. 4 we compare our results (dotted lines) and results of [1] (solid lines) for the critical loadings for plates of relative length $r = 2$ and $r = 3$.

It seems that support of the lateral sides under some conditions leads to the growth of the initial stresses in the transverse direction, that in its turn provides the growth of the critical loadings. The fall of the critical loading for the large hole area in Fig. 4a is explained with the local buckling of the thin stripes between the hole and the lateral plate edges.

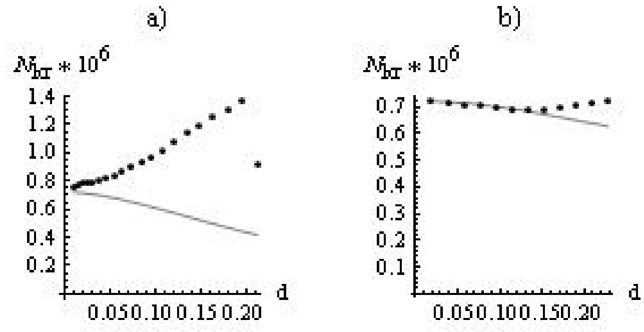

Figure 4: Effect of the hole area on critical loadings for the plates of relative thickness a) $r = 2$ and b) $r = 3$

3.3 Buckling of the plate with clamped lateral edges under axial compression

Let the lateral edges of the plate with the hole be clamped and the end edges be simply supported. The numerical results for the critical loadings are given in Fig. 5 for different values of the hole sides (d).

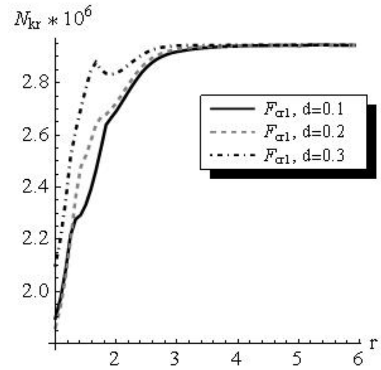

Figure 5: Effect of the parameter r on the critical buckling loading of the laterally clamped plate.

As one can see the critical buckling loading increases with the hole area regardless of the buckling mode. For the sufficiently long plate ($r > 3$) the critical buckling loading does not depend on the hole area, if the hole is not too large. At the points where the curves lose smoothness the wave number of the critical mode changes.

4 EFFECT OF THE RATIO OF THE HOLE SIDE LENGTHS

The next parameter that plays an important role in plate buckling is the ratio of the sides of the hole. Here we consider the plate with the central rectangular hole and the with lengths of its sides $a_1 = d(\xi + 1)$ and $a_2 = d/(\xi + 1)$ respectively. The hole area is constant d^2 for any ξ. If $\xi = 0$ it is a square hole, if $\xi > 0$ the hole is extended in axial direction and $\xi < 0$ it is extended in the transversal direction.

In Fig. 6 we plot the curves for the buckling critical loading vs. the ratio of the sides of the hole for different lengths of the plate ($r = 1, 2, 3$). The horizontal line is the critical buckling loading for the homogeneous plate.

For all cases the extension of the hole in the axial direction leads to the decreasing of the

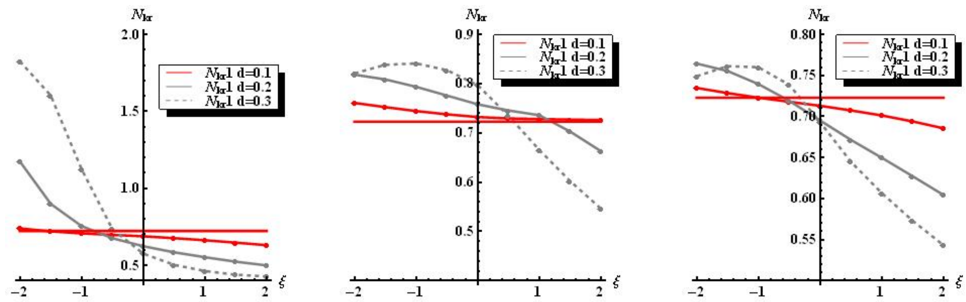

Figure 6: Effect of the hole side ration of the buckling loading for $r = 1, 2, 3$

critical loading and the extension of the hole in the transversal direction makes the critical loading to increase. The change of the ratio may also cause the switch of the buckling modes. The unsmooth behavior of the curve is caused with that fact. For example, it happens when ξ is close to 1 for $r = 2$.

5 EFFECT OF THE PERFORATION ON THE CRITICAL BUCKLING LOADING

Finally consider the effect of perforation on buckling. Let the plate be simply supported and weakened with multiple holes placed regularly along the plate middle lines in the axial or in the transversal directions. The number of the holes changes whereas the total area of the holes is constant. As an example, we consider the plate with the ratio of the sides $r = 2$. The plate has n holes, the total area of which is equal to 0.04 and 0.09 (see Fig. 7). We plot curves for critical load vs. hole number (n), for perforation in axial direction (H) and transversal direction (V).

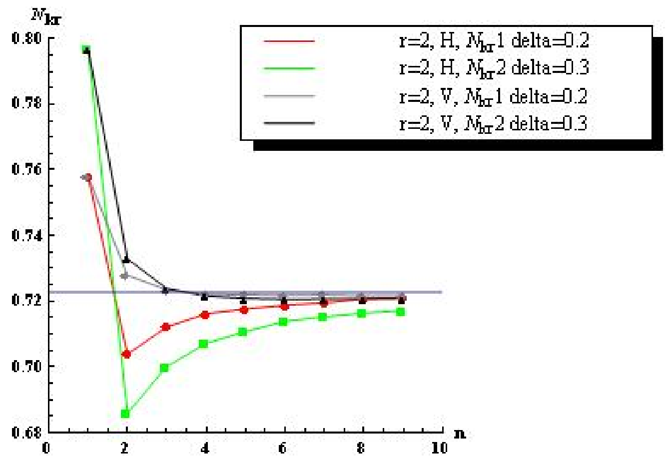

Figure 7: Effect of perforation of the critical buckling loadings: axial perforation, $d = 0.2$ (red line), axial perforation, $d = 0.3$ (green line), transversal perforation, $d = 0.2$ (grey line), transversal perforation, $d = 0.3$ (black line).

One can see that the critical loading converges to the value of the critical loading for the homogeneous plate as the number of holes increases.

6 CONCLUSION

The presence of the hole or cut-outs may lead to either increasing or decreasing of the critical buckling load depending on the boundary conditions and geometric parameters of the plate and the hole. The more rigid support of the lateral edges produces larger initial stresses that raises the critical buckling loading.

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