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Vibrations and Stability of Non-isotropic Plates with Cutouts

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The research refers to free vibrations and buckling analysis of non-homogeneous non-isotropic elastic plates weakened with holes. The purpose of the study is to examine the effect of shape, area, position, number and proportions of the holes, ratio of the plate sides and boundary conditions on free vibrations and buckling of rectangular plates with different boundary conditions. The plates are considered to be thin enough to apply 2D Kirchhoff-Love theory. Mathematically both stability and vibration problems for plates with cut-outs are reduced to solution of the boundary value problems for nonsimply connected domain, which are solved in the research with asymptotic and/or numerical methods including the Bubnov-Galerkin method and FEM.

The most important and interesting is the effect of the area of a hole on natural frequencies and modes of free vibrations and on critical loadings and buckling modes It appeared that the natural frequencies (critical buckling loadings) may either increase or decrease with the hole since the cut-out affects both the stiffness and the mass of a solid. Special attention is devoted to frequencies those are doubled for homogeneous plates, for example, frequencies of the square plates with similar boundary conditions on all edges. Depending on the wave numbers frequencies may split (or may not) as the hole area increases. For small hole area the asymptotic formula for natural frequencies have been obtained and the asymptotic results have been compared with the results of numerical analysis [3].

Presumably the effect when ''mechanical buckling strengths of the perforated plates, contrary to expectation, increase rather than decrease as the hole sizes grow larger" was firstly reported in [1]. In our research it was found that, for example, for axially compressed rectangular plate for buckling nodes with odd wave numbers the critical loading decreases with the hole area and increases for even wave numbers [2].

The values of the natural frequencies and buckling loading appear to be very sensitive to proportion of the hole of constant area for different boundary conditions on lateral sides and not very sensitive to the shape and position of the hole.

The ratio of the hole plays an important role. If the hole is situated near the top of the wave the frequency increases, if the hole is near the nodal point, the frequency decreases. It may be explained due to energy considerations, in the first case the most important factor is the decreasing of the kinetic energy and in the second the decreasing of the stiffness. For buckling under axial loading for all cases the extension of the hole in the axial direction leads to decreasing of the critical loading and the extension of the hole in the transversal direction makes the critical loading to increase. The change of the ratio may also cause the switch of the buckling or vibration modes.

The buckling behavior of non-isotropic plates has some specific features. As an example we consider the axial buckling of rectangular plate with central square hole with the side d. The plate is made of orthotropic material with Young modules $E_1 = 1/E_2$, Poisson ratios $\nu_1 = 1/\nu_2$ and the bending stiffness G . Is assumed that $E_1 = E_0(1+|\varepsilon|)$, $\nu_1 = \nu_0(1+|\varepsilon|)$, and G=const. Negative values of e means that directions swap. In Figure 1 one can see the effect of orthotropy on the critical buckling loading.

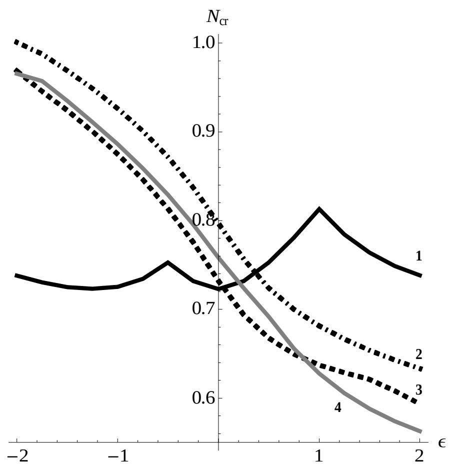

Figure 1: Effect of the non-isotropy on the buckling loading for axially compressed plate with the hole (1 - homogeneous plate, 2 – d=0.1, 3 – d=0.2, 4 - d=0.3)

Curve 1 is for homogeneous plate, curves 2, 3 and 4 are for plates with central square hole with the side length d (the plate width is taken as the characteristic length). Even for relatively small hole the effect of non-isotropy is very significant: if the plate becomes stiffer in the axial direction and softer in transversal the critical buckling loading decreases very speedy with ε and for the plate stiffer in the transverse direction the critical buckling loading increases. One more time it underlines the crucial effect of the initial stresses “carried by the narrow side strips of material along the plate boundaries” [1].

References

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