ELSEVIER

Discrete Mathematics 135 (1994) 395–397

DISCRETE MATHEMATICS

Research problems

The purpose of the research problems section is the presentation of unsolved problems in discrete mathematics. Older problems are acceptable if they are not as widely known as they deserve. Problems should be submitted using the format as they appear in the journal and sent to

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Readers wishing to make comments dealing with technical matters about a problem that has appeared should write to the correspondent for that particular problem. Comments of a general nature about previous problems should be sent to Professor Alspach.

Problem 213. Posed by O.V. Borodin

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A *pseudo-plane* P^n is obtained from the euclidean plane P by pairwise identification of $2n$ points. There are three types of possible embeddings of graphs into P^n:

(a) the edges of a graph cannot pass through any pair of identified points of P^n [4];

(b) there are no restrictions on a graph [3]; and

(c) the vertices of a graph do not lie on any pair of identified points of P^n [2,6].

Let $\mathrm{chr_D}(P^n)$, $\mathrm{chr}(P^n)$ and $\mathrm{chr_K}(P^n)$ be the chromatic numbers of the pseudo-plane corresponding to the three possible types of embeddings, respectively. It is known that

0012-365X/94/$07.00
SSDI 0012-365X(93)E0154-V

$$\mathrm{chr}_{\mathrm{D}}(P^n)=\begin{cases} n+4 & 0\leqslant n\leqslant 4,\\ 8 & n=5,\\ \left\lfloor\frac{7+\sqrt{1+24n}}{2}\right\rfloor & 5<n\leqslant 12,\\ 12 & n\geqslant 12,\end{cases}$$

$$\mathrm{chr}(P^n)=\begin{cases}\mathrm{chr}_{\mathrm{D}}(P^n) & n\leqslant 12,\\ \left\lfloor\frac{11+\sqrt{73+8n}}{2}\right\rfloor & n\geqslant 12,\end{cases}$$

$$\mathrm{chr}_{\mathrm{K}}(P^n)=\left\lfloor\frac{7+\sqrt{1+8n}}{2}\right\rfloor \qquad n>0.$$

The first of these formulas was obtained in [3] and [5], the second in [3], and the third in [6].

In 1965 Ringel introduced [7] the concept of the 1-embedding of a graph into a surface. Accordingly, a graph is 1-embeddable if any edge intersects at most one other edge. Let $\mathrm{chr}^+(S)$ be the 1-chromatic number of a surface S, that is, the maximum chromatic number of all graphs which are 1-embeddable into a surface S. Ringel conjectured [7] that $\mathrm{chr}^+(P)=6$ for the plane P and this conjecture was proved in [1]. In addition, it is shown in [2] that

$$\mathrm{chr}_{\mathrm{K}}^+(P^n)=\begin{cases}\left\lfloor\frac{9+\sqrt{17+16n}}{2}\right\rfloor & n\geqslant 0 \text{ and } n\neq 4,\\ 8 & n=4.\end{cases}$$

The present problem is to find $\mathrm{chr}^+(P^n)$ and $\mathrm{chr}_{\mathrm{D}}^+(P^n)$ for all $n>0$.

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Problem 214. Posed by V.D. Noghin.

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A poset is said to be *m-irreducible* if it has dimension m and the removal of any element results in a poset of smaller dimension. What is the maximum value $f(n,m)$ of the number of maximal elements of an m-irreducible poset on n elements with $m \geqslant 4$?