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GALACTIC KINEMATICS DERIVED FROM DATA IN THE RAVE5, UCAC4, PPMXL, AND GAIA TGAS CATALOGS

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Abstract—The spatial velocities of the stars with high-precision positions, proper motions and parallaxes from the Gaia TGAS catalog and line-of-sight velocities from the RAVE5 catalog are considered. From the sample of 92395 stars with the age estimates we have obtained the following kinematic parameters: $(U, V, W) = (9.42, 20.34, 7.21) \pm (0.12, 0.10, 0.09)$ km s^{-1}, $\Omega = 26.29 \pm 0.39$ km s^{-1} kpc^{-1} and $\Omega' = -3.89 \pm 0.08$ km s^{-1} kpc^{-2}, where $V_0 = 210 \pm 6$ km s^{-1} (for adopted $R_0 = 8.0 \pm 0.2$ kpc), and the Oort constants $A = 15.57 \pm 0.31$ km s^{-1} kpc^{-1} and $B = -10.72 \pm 0.50$ km s^{-1} kpc^{-1}. It is shown that the parameters Ω and Ω' are stable to the star age. A comparative analysis of the Bottlinger model parameters obtained separately from the RAVE5 catalog line-of-sight velocities and the Gaia TGAS, UCAC4 and PPMXL catalogs proper motion has been made. It is shown that these parameters are in good agreement with each other when derived from the proper motions of both the terrestrial catalogs and catalog Gaia TGAS. At the same time, it was established that the values of the Bottlinger model parameters obtained from the line-of-sight velocities can differ from the corresponding parameters obtained from the proper motions. The reduction of the line-of-sight velocities from the RAVE5 catalog is proposed for eliminating these differences.

Keywords: radial velocities of stars: proper motions: RAVE5: Gaia DR1: Galactic kinematics

Introduction

At the threshold of the completion of the Gaia space mission [1], the RAVE (Radial Velocity Experiment [2]) experiment devoted to mass determination of the radial (line-of-sight) velocities of faint stars is under successful development. Observations in the southern hemisphere with the 1.2-m Schmidt telescope system at the Anglo-Australian Observatory began in 2003. Since then five issues of this catalog have been published. The mean error in determining the radial velocity is about 3 km s^{-1}. Combining the high precision proper motions and trigonometric parallaxes of the stars in the Gaia project with radial velocities of the stars in the RAVE project has made it possible to analyze the three-dimensional motion of stars in the Galaxy.

Data from RAVE1 [2], RAVE2 [3], and RAVE3 [4] have been used to obtain a number of important results in stellar astronomy. For example, there has been a search for new star flows and groups [5–7], the characteristics of the velocity ellipsoids for different samples

of stars in the vicinity the Sun have been refined [8,9], a new value has been obtained for the Sun's peculiar velocity relative to the local standard of rest [10], the kinematic characteristics of stars in the thin and thick disks have been refined [11,12], the properties of the spiral density wave in the Sun's region have been studied [13], and the asymmetric drift parameters have been refined [14]. These things have been done by drawing on different catalogs of the proper motions of stars and using estimates of their age obtained by different methods from spectral data in the RAVE program [15–18].

The RAVE4 catalog [19] is already an extensive data base which includes data on the proper motions of stars from several catalogs, as well as the measured radial velocities of the stars, along with data on infrared photometry in several bands, photometric distances, ages, effective temperatures, values of the acceleration of gravity, and several other estimates. The RAVE4 catalog has been used [20] in a redetermination of the average radial velocities of a large sample of diffuse star clusters. Data from this catalog have been used to estimate a new value for the rate at which stars escape the Galaxy [21], as well a new value for the virial mass of the Galaxy. New estimates have been obtained for the rotation parameters of the Galaxy [22]. This used the radial velocities from RAVE4, along with the proper motions and photometric distances from the UCAC4 catalog [23]. It was found that stars from a circumsolar vicinity of radius 3 kpc yield fairly good values for the peculiar velocity U, V, W of the Sun, and the angular rotation velocity Ω of the Galaxy and its first derivative Ω', but the determination of the second derivative Ω'' was poor. The RAVE5 catalog [24] contains data on 457588 stars. The most interesting feature of this catalog is the more than 200000 stars in common with the Gaia catalog with high precision proper motions and trigonometric parallaxes.

With the publication in 2016 of the first data from the Gaia satellite experiment, some new possibilities arose for studying the structure and kinematics of the Galaxy. The proper motions of the stars were determined by comparing the positions obtained by the Gaia satellite as second epochs with the positions of these stars measured in the Tycho experiment [25] as the first epochs, with an average difference in the epochs of about 26 years. This version of the catalog is referred to as TGAS (Tycho-Gaia Astrometric Solution [26,27]) and contains the trigonometric parallaxes and proper motions for about 2 million stars. The proper motions of the stars from Gaia DR1 are of the greatest value. For roughly 90000 stars in common with the HIPPARCOS catalog, the average random error in their proper motions is about 0.06 milliarcseconds per year (mas yr^{-1}), and for the remaining stars this error is about 1 mas yr^{-1} [26]. The measured radial velocities will be published later with low accuracy and an expected average random error of about 15 km s^{-1}.

Here we point out some important kinematic results obtained on the basis of data from the Gaia DR1 catalog. New values of the rotation parameters of the Galaxy using the proper motions from Gaia DR1 for about 250 classical Cepheids from a wide vicinity of the Sun (<3–4 kpc) have been found by Bobylev [28]. Roughly 230 OB-stars with three different distance scales and proper motions from the Gaia DR1 catalog (<3–4 kpc) have been used [29] for the same purpose. The proper motions of roughly 300000 near (< 250 kpc) stars in the Main Sequence from the Gaia DR1 catalog have been used by Bovy [30] to estimate the Oort constants A, B, C, and K for the local kinematics. The kinematic parameters of a number of nearby diffuse star clusters have been refined [31]

using the proper motions of the stars from the Gaia DR1 catalog.

This article is a continuation of studies begun by Bobylev and Bajkova [22]. Two problems are solved here. The first involves refining the rotation parameters of the Galaxy with combined use of the radial velocities of the stars from the RAVE5 catalog and the proper motions and parallaxes of the stars from the Gaia DR1 catalog. The second problem, on the other hand, is a kinematic study of the velocity field of the stars based on separate solutions of the kinematic equations for the radial velocities and the proper motions of the stars. Since the radial velocities and proper motions of the stars are determined in fundamentally different ways, the separate solutions for the basic kinematic equations can be used to study the consistency of these data with one another from the standpoint of kinematics.

Combined analysis of the radial velocities and proper motions of stars

From observations we know three components of the star velocity: the line-of-sight velocity V_r and the two projections of the tangential velocity $V_l = 4.74 r \mu_l \cos b$ and $V_b = 4.74 r \mu_b$, directed along the Galactic longitude l and latitude b respectively and expressed in km s^{-1}. Here the coefficient 4.74 is the ratio of the number of kilometers in astronomical unit by the number of seconds in a tropical year, and r is a heliocentric distance of the star in kpc. The components of a proper motion of $\mu_l \cos b$ and μ_b are expressed in the mas yr^{-1}.

To determine the parameters of the Galactic rotation curve, we use the equation derived from Bottlinger's formulas in which the angular velocity Ω was expanded in a series to terms of the second order of smallness in r/R_0:

$$V_r = -U \cos b \cos l - V \cos b \sin l - W_\odot \sin b+$$
$$+R_0(R - R_0) \sin l \cos b \Omega' + 0.5 R_0 (R - R_0)^2 \sin l \cos b \Omega'' + Kr \cos^2 b, \tag{1}$$

$$V_l = U \sin l - V_\odot \cos l - r\Omega \cos b+$$
$$+(R - R_0)(R_0 \cos l - r \cos b)\Omega' + 0.5(R - R_0)^2(R_0 \cos l - r \cos b)\Omega'', \tag{2}$$

$$V_b = U \cos l \sin b + V \sin l \sin b - W_\odot \cos b-$$
$$-R_0(R - R_0) \sin l \sin b \Omega' - 0.5 R_0 (R - R_0)^2 \sin l \sin b \Omega'' + Kr \cos b \sin b, \tag{3}$$

where R is the distance from the star to the Galactic rotation axis,

$$R^2 = r^2 \cos^2 b - 2R_0 r \cos b \cos l + R_0^2. \tag{4}$$

Ω is the angular velocity of Galactic rotation at the solar distance R_0, the parameters Ω' and Ω'' are the corresponding derivatives of the angular velocity, $V_0 = |R_0 \Omega|$, and K is one of the Oort constants describing the expansion/compression effect for the star system, while the two constants can be found using the formulas

$$A = -0.5\Omega' R_0, \quad B = -\Omega + A, \tag{5}$$

In this paper we take $R_0 = 8.0 \pm 0.2$ kpc, found by Vallée to be the most probable value [32].

3

Data

The sample is made up of stars for which estimates of their trigonometric parallaxes and proper motions are available from the Gaia DR1 catalog, their radial velocities, from RAVE5, and their ages, from version RAVE4. The method for determining the individual ages of the stars is described elsewhere [17,33]. These estimates were obtained by comparison with suitable isochrones on the Hertzsprung-Russell diagram, for stars in the Main Sequence as well as for red giants.

It turns out that in the RAVE5 catalog, the radial velocity has been measured several times for a fairly large fraction ($\approx$10%) of the stars. Thus, we have set up a sample in which each star is represented once. When the radial velocity has been measured several times for a star, we did not take an average, but chose the measurement with the smallest error in the measured radial velocity. A total of about 200000 stars are in the sample.

In the RAVE5 catalog there are stars with very large $|V_r| > 600$ km/s. These values are usually obtained from low quality spectra with low signal/noise ratios. Thus, we do not use stars with these velocities. We also do not use stars with large random errors σ_{V_r} in the radial velocity. Ultimately, to select candidates without substantial random observational errors we have chosen the stars that satisfy the following criteria:

$$|V_r| < 600 \text{ km/s}, \qquad \sigma_{V_r} < 5 \text{ km/s},$$
$$|\mu_\alpha \cos\delta| < 400 \text{ mas/yr}, \qquad |\mu_\delta| < 400 \text{ mas/yr}, \qquad (6)$$
$$|z| < 0.3 \text{ kpc}, \qquad \sqrt{U^2 + V^2 + W^2} < 300 \text{ km/s},$$

where the velocities U, V, and W are freed of the galactic differential rotation, i.e., they are residuals. For this procedure, any of the known galactic rotation curves are suitable, e.g., from [29]. The limit on the z coordinate is used to eliminate the influence of stars in the halo during searches for the rotation parameters of the Galaxy.

Results and discussion

For analyzing the sample of stars with estimates of their age, a system of nominal equations of the form of Eqs. (1)–(3) was solved by a least squares method with size unknowns: $U, V, W, \Omega, \Omega', \Omega''$, i.e., without the K-term. Partition into small areas with equal cross sections was not used, so that each star provided three equations. Table 1 lists the values of the kinematic parameters found from the stars with intrinsic motions and trigonometric parallaxes from the Gaia DR1 catalog; shown there is the error per unit weight, σ_0 obtained by solving the system of the form (1)–(3) by a least squares method. The parameters in this table were calculated for different values of the relative error in the parallaxes. These results are of great interest because for small relative errors in determining the trigonometric parallaxes ($\sigma_\pi/\pi : 10\% - 15\%$) the influence of the Lutz-Kelker effect is negligible [34]. For larger σ_π/π, this effect must be taken into account [35].

Table 1 shows that for different limits on the error σ_π/π ranging from 10–25% there is good agreement in the values of all these parameters, except for the second derivative of the angular rotation speed, Ω'', which is always determined with large errors.

Table 2 lists the kinematic parameters found by using the photometric distances from the RAVE5 catalog, as well as by using the trigonometric parallaxes from the Gaia TGAS

Table 1: Parameters for the Galactic Rotation Based on Stars with Proper Motions and Trigonometric Parallaxes from the Gaia TGAS Catalog for Different Limits on the Relative Error in the Parallaxes

Parameters	$\sigma_\pi/\pi < 10\%$	$\sigma_\pi/\pi < 15\%$	$\sigma_\pi/\pi < 20\%$	$\sigma_\pi/\pi < 25\%$
U, km/s	9.10 ± 0.19	9.24 ± 0.15	9.44 ± 0.13	9.49 ± 0.12
V, km/s	20.55 ± 0.17	20.44 ± 0.13	20.40 ± 0.12	20.33 ± 0.11
W, km/s	7.79 ± 0.13	7.68 ± 0.11	7.54 ± 0.10	7.35 ± 0.09
Ω, km/s/kpc	26.81 ± 1.53	26.67 ± 0.89	25.88 ± 0.60	26.03 ± 0.46
Ω', km/s/kpc^2	-3.76 ± 0.25	-3.90 ± 0.15	-3.81 ± 0.11	-3.87 ± 0.09
Ω'', km/s/kpc^3	2.77 ± 1.84	1.09 ± 0.83	0.68 ± 0.51	0.35 ± 0.35
σ_0, km/s	27.68	27.73	27.70	27.65
$N_\star$	43813	63926	76966	86060
A, km/s/kpc	15.05 ± 0.99	15.59 ± 0.61	15.23 ± 0.45	15.49 ± 0.36
B, km/s/kpc	-11.76 ± 1.83	-11.08 ± 1.08	-10.65 ± 0.75	-10.55 ± 0.59

catalog. Here the limitation $\sigma_r/r < 30\%$ was assumed when using the photometric distances and $\sigma_\pi/\pi < 30\%$ when using the trigonometric parallaxes. It is clear from this table that there are roughly 2.5 times fewer stars with photometric distances than with trigonometric parallaxes. With increasing age of the stars, the dispersions in their velocities increase. We have attempted a partition into age groups in a way such that the random errors in the found parameters are ultimately comparable for each group. Here it is clear from Table 2 that the oldest stars have the largest random errors, even though there are large numbers of them. Almost all the kinematic parameters in the top and bottom parts of the table are in good agreement with one another. In addition, the trigonometric parallaxes are more reliable from an ideological standpoint. These remarks indicate that using the trigonometric parallaxes is preferable. We note the result in the bottom part of Table 2 which was obtained using the largest number of stars:

$$(U, V, W) = (9.42, 20.34, 7.21) \pm (0.12, 0.10, 0.09) \text{ km/s},$$
$$\Omega = 26.29 \pm 0.39 \text{ km/s/kpc}, \tag{7}$$
$$\Omega' = -3.89 \pm 0.08 \text{ km/s/kpc}^2.$$

Here the linear rotation speed of the Sun around the center of the Galaxy is $V_0 = 210 \pm 6$ km/s (for adopted $R_0 = 8.0 \pm 0.2$ kpc).

It is interesting to compare the parameters for the galactic rotation obtained here with, for example, the estimates of Rastorguev, et al. [36] derived from data on 136 masers with measured trigonometric parallaxes covering a wide range of distances R : 0–16 kpc. As an example, for model C1 (model of a constant radial dispersion in the velocities), the components of the Sun's velocity were $(U, V, W) = (10.98, 19.62, 8.93) \pm (1.40, 1.15, 1.05)$ km/s, and following components of angular velocities of Galactic rotation: $\Omega = 28.35 \pm 0.45$ km/s/kpc, $\Omega' = -3.83 \pm 0.08$ km/s/kpc^2 and $\Omega'' = 1.17 \pm 0.05$ km/s/kpc^3, $V_0 = 235 \pm 7$ km/s (for the found value of $R_0 = 8.27 \pm 0.13$ kpc). An earlier analysis of masers [37] yielded an estimate of $V_0 = 238 \pm 14$ km/s for the Sun's velocity (for the found $R_0 = 8.05 \pm 0.45$ km/s), and Reid, et al. [38] obtained a velocity of $V_0 = 240 \pm 8$ km/s

Table 2: Parameters for the Galactic Rotation

Parameters	All stars	$\lg t < 9.5$	$\lg t$: 9.5–9.7	$9.7 < \lg t$
U, km/s	9.34 ± 0.16	9.68 ± 0.27	9.63 ± 0.33	8.99 ± 0.25
V, km/s	17.81 ± 0.15	14.20 ± 0.25	17.02 ± 0.29	20.26 ± 0.24
W, km/s	7.73 ± 0.13	7.42 ± 0.23	7.56 ± 0.25	8.05 ± 0.22
Ω, km/s/kpc	26.71 ± 0.48	25.07 ± 0.94	26.61 ± 0.69	27.86 ± 1.16
Ω', km/s/kpc^2	-4.09 ± 0.10	-3.37 ± 0.19	-3.95 ± 0.15	-4.28 ± 0.24
Ω'', km/s/kpc^3	0.65 ± 0.26	3.00 ± 0.40	-1.80 ± 0.46	0.06 ± 0.51
σ_0, km/s	25.87	22.92	26.44	27.15
$N_\star$	36858	10256	11125	15477
A, km/s/kpc	16.38 ± 0.41	13.49 ± 0.75	15.78 ± 0.58	17.13 ± 0.94
B, km/s/kpc	-10.33 ± 0.63	-11.58 ± 1.20	-10.83 ± 0.90	-10.73 ± 1.50
U, km/s	9.42 ± 0.12	9.85 ± 0.21	9.88 ± 0.27	9.09 ± 0.17
V, km/s	20.34 ± 0.10	14.75 ± 0.18	20.14 ± 0.21	22.66 ± 0.16
W, km/s	7.21 ± 0.09	6.85 ± 0.15	6.39 ± 0.18	7.82 ± 0.14
Ω, km/s/kpc	26.29 ± 0.39	25.55 ± 0.58	25.86 ± 0.65	27.00 ± 0.98
Ω', km/s/kpc^2	-3.89 ± 0.08	-3.68 ± 0.11	-3.87 ± 0.12	-3.99 ± 0.18
Ω'', km/s/kpc^3	0.39 ± 0.27	-0.33 ± 0.38	-1.70 ± 0.40	-3.29 ± 0.80
σ_0, km/s	27.66	22.93	27.24	29.97
$N_\star$	92395	24286	22960	45149
A, km/s/kpc	15.57 ± 0.31	14.70 ± 0.45	15.48 ± 0.50	15.95 ± 0.71
B, km/s/kpc	-10.72 ± 0.50	-10.85 ± 0.73	-10.38 ± 0.82	-11.05 ± 1.21

The top part of the table lists the parameters found for $\sigma_\pi/\pi < 30\%$ based on stars with proper motions from the Gaia TGAS catalog and photometric distances from the RAVE5 catalog and the bottom, the parameters calculated using the trigonometric parallaxes from the Gaia TGAS catalog.

(for the found $R_0 = 8.34 \pm 0.16$ kpc).

Based on the velocities of 260 Cepheids with proper motions from the Gaia DR1 catalog, it was found [28] that $(U, V, W) = (7.90, 11.73, 7.39) \pm (0.65, 0.77, 0.62)$ km/s, along with the following values for the parameters of the Galactic rotation curve: $\Omega = 28.84 \pm 0.33$ km/s/kpc, $\Omega' = -4.05 \pm 0.10$ km/s/kpc^2, and $\Omega'' = 0.805 \pm 0.067$ km/s/kpc^3, (for $R_0 = 8.0 \pm 0.2$ kpc), $V_0 = 231 \pm 6$ km/s, $A = 16.20 \pm 0.38$ km/s/kpc, and $B = -12.64 \pm 0.51$ km/s/kpc.

238 OB-stars with proper motions from the TGAS catalog have been used [29] to find the following: $(U, V, W) = (8.19, 9.28, 8.79) \pm (0.74, 0.92, 0.74)$ km/s, $\Omega = 31.53 \pm 0.54$ km/s/kpc, $\Omega' = -4.44 \pm 0.12$ km/s/kpc^2, $\Omega'' = 0.706 \pm 0.100$ km/s/kpc^3, $A = 17.77 \pm 0.46$ km/s/kpc, and $B = -13.76 \pm 0.71$ km/s/kpc, with a linear circular velocity of the Sun of $V_0 = 252 \pm 8$ km/s (for the assumed distance $R_0 = 8.0 \pm 0.2$ kpc).

Stars have been analyzed [22] using radial velocities from the RAVE4 catalog and proper motions from the UCAC4 catalog. A sample of 145000 stars yielded the following parameters: $(U, V, W) = (9.12, 20.80, 7.66) \pm (0.10, 0.10, 0.08)$ km/s, $\Omega = 28.71 \pm 0.63$ km/s/kpc, and $\Omega' = -4.28 \pm 0.11$ km/s/kpc^2, where $V_0 = 230 \pm 12$ km/s (for $R_0 = 8.0 \pm 0.4$ kpc), as well as the Oort parameters $A = 17.12 \pm 0.45$ km/s/kpc and
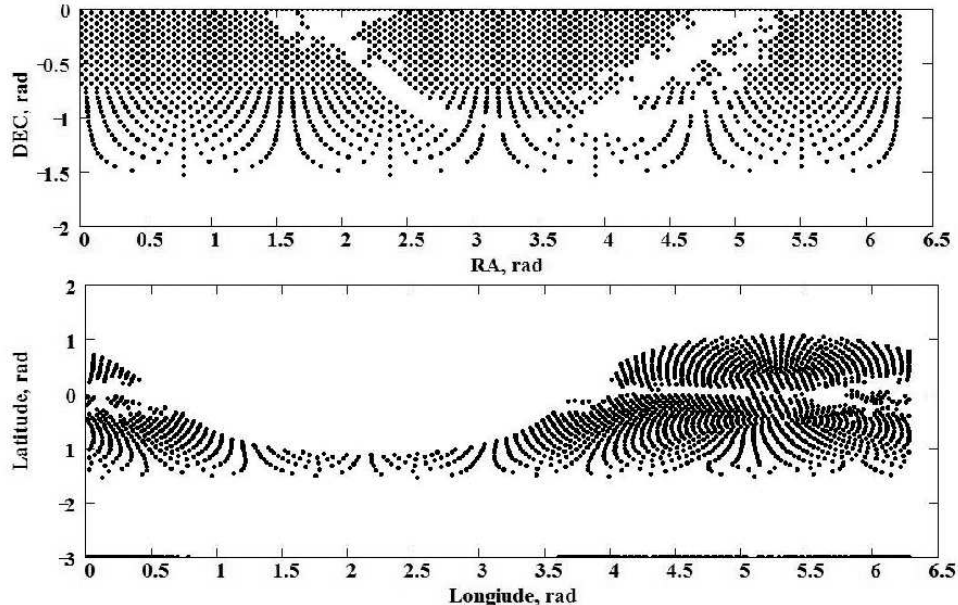
Figure 1: The filling of Healpix-areas ($N_{side} = 20$) by stars from the RAVE5 catalog for distances in the range of $100 < r < 300$ pc. The equatorial coordinate system (top) and galactic coordinate system (bottom).

$B = -11.60 \pm 0.77$ km/s/kpc. When samples of stars with different ages were analyzed (Table 2 of [22]), fairly low values of the angular rotation speed, $\Omega \sim 24$ km/s/kpc, were found. We may conclude that using the proper motions of the stars from the Gaia TGAS catalog yields estimates of the kinematic parameters for our model that are in good agreement with the results of an analysis of independent data.

Separate analysis of the radial velocities and proper motions of stars

The second part of our paper is devoted to a comparative analysis of the velocity fields obtained separately using the radial velocities and proper motions of stars. As noted above, the radial velocities and proper motions are found in fundamentally different ways, so that our separate solutions of the basic kinematic equations using the radial velocities and proper motions of the stars make it possible to study the consistency of these data relative to one another from the standpoint of kinematics. A comparison of the results obtained from the proper motions of stars from different catalogs is one way of comparing the systems of these catalogs. Comparing the results obtained from the radial velocities and proper motions of stars from the same catalogs can, in turn, be regarded as a comparison of the system of radial velocities and proper motions with respect to the chosen kinematic model. The detection of differences in this case can indicate systematic errors in the observational data or incompleteness of the kinematic model that is being used.

As opposed to the problem solved above, it makes sense to examine the agreement of

Table 3: Conditional Numbers and Correlation Coefficients for Combined Solutions of the Nominal Equations. Sample from RAVE5 with $100 < r < 300$ pc

Model	Solution based on proper motions and radial velocities
OM	cond= 488, corr$(V, M_{22}) = 0.803$
Bottlinger	cond=1349, corr$(\Omega, \Omega') = 0.732$

the results derived from the radial velocities and proper motions of the stars for samples of stars at different distances from the sun. We shall make a kinematical analysis of the radial velocities and proper motions of stars in the RAVE5 catalog using the Bottlinger formulas (1)–(3) reduced to a form in which the left hand sides of Eqs. (2) and (3) contain $4.74\mu_l \cos b$ and $4.74\mu_b$, instead of $V_l = 4.74r\mu_l \cos b$ and $V_b = 4.74r\mu_b$.

This requirement makes it necessary to set up a sample of stars belonging to narrow spherical shells. The distribution of the stars over the sky in the RAVE5 catalog has two distinctive features. First, the bulk of these stars lie in the southern equatorial hemisphere; second, this distribution is not uniform, since the regions through which the Milky Way passes do not contain stars (Fig. 1).

Direct solution of the Ogorodnikov-Milne and Bottlinger equations for such a set of stars belonging to spherical shells in which the width of a shell is considerably smaller than its average radius makes it necessary to solve a ill-conditioned system of equations with strong correlations in the estimates of the unknown parameters. For example, Tables 3 and 4 list the condition numbers (calculated using an infinite norm for the matrices) and the largest correlation coefficients for a sample of 52640 stars at distances of $100 < r < 300$ pc. The values of the proper motions and radial velocities are averaged on a HealPix grid with $N_{side} = 20$.

In order to avoid dealing with a situation of this sort, we use a two-step procedure based on an initial representation of the radial velocities and proper motions with a system of orthogonal spherical functions with subsequent determination of the parameters of the model being used. We now describe the main steps in this method.

Initial data:

i. A sample of stars from RAVE5 that contains average values for the radial velocities $V(h) = V(\alpha_h, \delta_h)$ and proper motions $\mu_\alpha^*(h) = \mu_\alpha^*(\alpha_h, \delta_h)$, $\mu_\delta(h) = \mu_\delta(\alpha_h, \delta_h)$, of the stars relative to the centers (α_h, δ_h) of the HealPix areas with numbers $h = 0, 1, ..., N - 1$. In our case a grid with parameter $N_{side} = 20$ and 4800 areas (pixels) was constructed. For the areas with negative declinations, $h = 2440, ..., 4779$. In the following, we indicate that an index h belongs to this set by $h \in H$.

ii. The pixel weights w_h. The weight of a pixel with index h is equal to unity if at least one star falls into this pixel; otherwise, the weight of the pixel is assumed to be zero. It is clear that the number of filled areas is equal to $\sum_h w_h$.

iii. The galactic coordinates l_h, b_h of the centers of all the areas.

iv. The galactic proper motions $\mu_\alpha^*(h) = \mu_l^*(l_h, b_h)$, $\mu_b(h) = \mu_b(l_h, b_h)$, derived from the initial equatorial components:

1. Initially we represent the radial velocities by an expansion in the system of vector

Table 4: Conditional Numbers and Correlation Coefficients for Separate Solutions of the Nominal Equations. Sample from RAVE5 with $100 < r < 300$ pc

Model	Solution based on proper motions	Solution based on radial velocities
OM	cond=1185, $\text{corr}(W, \Omega') = 0.784$	cond=5038, $\text{corr}(V, M_{23}) = 0.972$
Bottlinger	cond=1383, $\text{corr}(\Omega, \Omega') = 0.734$	cond=2243, $\text{corr}(\Omega, \Omega'') = 0.856$

spherical functions that are orthogonal in the southern equatorial hemisphere. For the pixel h we have

$$V_h = \sum_{nkp} v_{nkp} K_{nkp}(\hat{x}_h, \alpha_h), \tag{8}$$

where v_{nkp} are the coefficients of the expansion in the spherical functions $K_{nkp}(\hat{x}, \alpha)$, which are orthogonal for declinations $\delta_1 \leq \delta \leq \delta_2$. The explicit form and equations for calculating these functions are given by Vityazev and Tsvetkov [39].

Introducing a continuous enumeration $i = (nkp)$, for all the triplets, we rewrite Eq. (8) as

$$V_h = \sum_{i=0}^{I_v} v_i f_i(\hat{x}_h, \alpha_h) = \bar{v}\,\bar{f}, \tag{9}$$

where the vector $\bar{v}$ and the vector functional $\bar{f}$ have the following components:

$$\bar{v} = (v_0, v_1, ..., v_{I_v}), \tag{10}$$

$$\bar{f} = (f_0, f_1, ..., v_{I_v}) = (K_0, K_1, ..., K_{I_v}). \tag{11}$$

The unknown vector $\bar{v}$ is determined by a least squares method:

$$\bar{v} = z^{-1}\bar{a}, \tag{12}$$

where the components of the matrix z and the vector $\bar{a}$ are given by

$$z_{ij} = \sum_{h \in H} f_i(\hat{x}_h, \alpha_h) f_j(\hat{x}_h, \alpha_h) w_h, \tag{13}$$

$$a_j = \sum_{h \in H} V_h f_j(\hat{x}_h, \alpha_h) w_h. \tag{14}$$

2. Expansion of the proper motions of the stars in terms of the vector spherical functions. Using the unit vectors $\bar{e}_l$ and $\bar{e}_b$ along the directions of variation in the galactic longitudes and latitudes, we form the vector of the galactic proper motions of the stars:

$$\bar{\mu} = \mu_l^* \bar{e}_l + \mu_b \bar{e}_b. \tag{15}$$

We write this vector as an expansion in the vector spherical functions,

$$\bar{\mu} = \sum_{i=0}^{I_m} m_i \bar{F}_i(\hat{\delta}_h, \alpha_h) = \bar{m}\,\bar{F}, \tag{16}$$

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where

$$\bar{m} = (s_1, s_2, ..., s_n; t_1, t_2, ..., t_n), \tag{17}$$

$$\bar{F} = (\bar{S}_1, \bar{S}_2, ..., \bar{S}_n; \bar{T}_1, \bar{T}_2, ..., \bar{T}_n). \tag{18}$$

Here $\bar{S}_i$ and $\bar{T}_i$ are spheroidal and toroidal vector functions, respectively, which are orthogonal in the southern hemisphere of the equatorial coordinate system and are described in [39].

The unknown vector $\bar{m}$ is found by the method of least squares:

$$\bar{m} = Z^{-1}\bar{A}, \tag{19}$$

where the components of the matrix Z and vector $\bar{A}$ are given by

$$Z_{ij} = \sum_{h \in H} \bar{F}_i(\hat{\delta}_h, \alpha_h)\bar{F}_j(\hat{\delta}_h, \alpha_h)w_h, \quad i, j = 1, 2, ...I_m, \tag{20}$$

$$A_j = \sum_{h \in H} \bar{\mu}_h \bar{F}_j(\hat{\delta}_h, \alpha_h)w_h, \quad j = 1, 2, ..., I_m. \tag{21}$$

3. Determination of the parameters of the physical models for the radial velocities and proper motions of the stars. The models (9) and (16) are formal, so the significance of the coefficients v_i and m_i can be clarified only in terms of concrete physical models for the radial velocities and proper motions of the stars. We specify physical models in the form of linear combinations of some functions $\varphi_j(r, l, b)$ and $\psi_j(r, l, b)$:

$$V(r, l, b) = \sum_{j=1}^{J_v} p_j \varphi_j(r, l, b), \tag{22}$$

$$\bar{\mu}(r, l, b) = \sum_{j=1}^{J_m} q_j \bar{\psi}_j(r, l, b). \tag{23}$$

It is obvious that many models have this form and are used in kinematical analysis of the velocity field of stars, such as the Ogorodnikov-Milne model, the Lindblad-Oort model, the generalized Oort model, etc. In our case (the Bottlinger model), the significance of the coefficients p_j, q_j, and the form of the functions $\varphi_j(r, l, b)$ and $\psi_j(r, l, b)$ are easily established by comparing Eqs. (22)–(23) with Eqs. (1)–(3).

To determine the parameters p_j in terms of the coefficients v_i, we use Eqs. (22) and (9):

$$\sum_{j=1}^{J_v} p_j \varphi_j(r, l, b) = \sum_{j=0}^{I_v} v_i \, f_i(\bar{x}, \alpha). \tag{24}$$

Solving this equation by the method of least squares yields

$$z\bar{v} = \Phi\,\bar{p}, \tag{25}$$

where the matrix z has the components (20), and the components of the matrix Φ are given by

$$\sum_{h \in H} \varphi_j(r, l_h, b_h) \, f_i(\bar{x}, \alpha_h) \, w_h, \quad i = 0, 1, ..., I_v; \;\; j = 1, 2, ..., J_v. \tag{26}$$

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This yields the important relation

$$\bar{v} = z^{-1}\Phi\bar{p}. \tag{27}$$

This equation relates the physical parameters p_j to the formal parameters v_i and makes it possible to interpret the physics of the phenomena in terms of the coefficients of the formal representation v_i of the radial velocities.

To solve the inverse problem, i.e., determine the physical parameters p_j in terms of the coefficients v_i, it is necessary to select from the matrix $z^{-1}\Phi$ a square matrix N_v of size $J_v \times J_v$. In this case the unknown parameters are found in the following way:

$$\bar{p} = N_v^{-1}\bar{v}. \tag{28}$$

Here the vector $\bar{v}$ consists of J_v elements with indices that match the numbers of the rows in the matrix $z^{-1}\Phi$ selected to form the matrix N_v. It is evident that several square matrices can be chosen in this way. In choosing them it is necessary to be guided by the criterion of minimizing the condition number. A few square matrices (usually two, in practice) can be used to test the model with the aid of the principal solution and an alternative one [39].

Following similar arguments, we write the equation

$$\sum_{j=1}^{J_m} q_j \bar{\psi}_j(r, l, b) = \sum_{j=0}^{I_m} m_i \bar{F}_i(\bar{\delta}, \alpha). \tag{29}$$

The solution of this equation by the method of least squares has the form

$$Z\bar{m} = \Psi\,\bar{q}, \tag{30}$$

where the coefficients of the matrix Ψ are calculated using the formula

$$\Psi_{ij} = \sum_{h \in H} \psi_j(r, l_h, b_h)\,\bar{F}_i(\bar{\delta}, \alpha_h\,w_h, \quad i = 0, 1, ..., I_m; \quad j = 1, 2, ..., J_m. \tag{31}$$

Thus,

$$\bar{m} = (Z^{-1}\,\Psi)\,\bar{q}. \tag{32}$$

This equation can be used to interpret the coefficients of m_i in the expansion of the proper motions of the stars in terms of the parameters of the physical model.

To determine the coefficients $q_1, q_2, ...q_{I_m}$ it is necessary to select, from the matrix $Z^{-1}\,\Psi$, a square matrix N_μ of size $J_m \times J_m$. By analogy with the case of the radial velocities, the choice of the matrix N_μ is not unique. Here it is necessary to choose a matrix with a minimum conditionality number. The solution has the form

$$\bar{q} = N_\mu^{-1}\,\bar{m}. \tag{33}$$

On constructing any two of these matrices it is possible to check the adequacy of the model for the observational data.
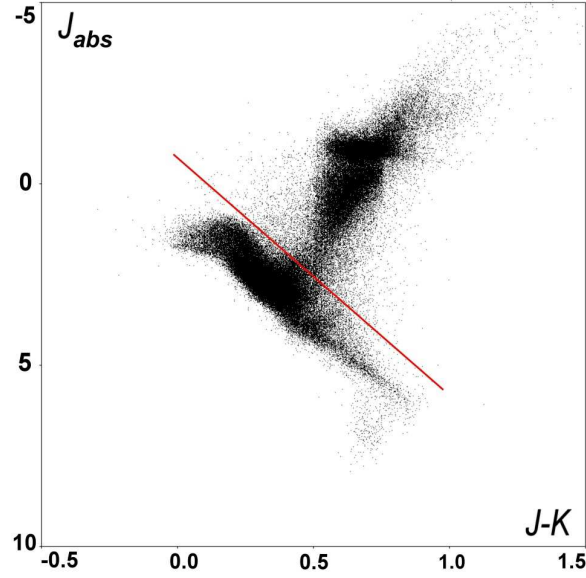
Some comments on the method:

Figure 2: The Main Sequence and the red giant branch on a Hertzsprung-Russell diagram. The line of separation is $J_{abs} = 7(J - K) - 1$.

1. Equations (27) and (32) are overdetermined systems of equations. Thus, they can be solved by a least squares method over the entire range of significant coefficients v_i and m_i. We rewrite them in the form

$$\bar{v} = X\bar{p}, \quad X = z^{-1}\Phi, \tag{34}$$

$$\bar{m} = Y\bar{p}, \quad Y = Z^{-1}\Psi, \tag{35}$$

and we then obtain

$$\bar{p} = (X^T X)^{-1} (X^T \bar{v}), \tag{36}$$

$$\bar{q} = (Y^T Y)^{-1}) (Y^T \bar{m}). \tag{37}$$

But this approach again leads to the solution of ill-conditioned systems of equations.

2. The matrices $z^{-1}\Phi$ and $Z^{-1}\Psi$ can be calculated once if all the cells are filled. Individual nonuniformities in the filling of the pixels, which are typical of any sample, as well as variations during the filtration process when processing the data, make it necessary to calculate these matrices for each specific sample run. In this way the results are protected from distortions owing to unfilled areas.

3. The method described here can be used to study the kinematics of stars separately in the northern and southern hemispheres. In this case we know the galactic coordinates of the pixels in both hemispheres and the average radial velocities, as well as the galactic proper motions of the stars relative to the centers of the pixels. At present we have no need to use the equatorial coordinate system, so in all the formulas the coordinates $(\bar{x}, \alpha)$ and $(\bar{\delta}, \alpha)$ should be replaced formally by their analogs in the galactic coordinate system.

Table 5: Characteristics of the Samples of Stars in the Main Sequence

Sample boundary, pc	Average distance, pc	Number of stars	Average of relative error in parallaxes
100–200	155	21737	0.061
200–300	249	26541	0.099
300–400	346	19161	0.136
400–500	444	10137	0.168
500–700	574	6914	0.213
700–900	776	1243	0.296

4. The clear advantage of this method over the standard method of least squares is the absence of strong correlations of the unknown parameters and the fact that well-conditioned systems of equations are to be solved.

5. The shortcomings of our method relative to the method of least squares are: it is impossible to obtain a joint solution of the kinematic equations with respect to the radial velocities and proper motions of the stars, and solutions have to be obtained for stars located within relatively narrow distance ranges. This is done in order to reduce the influence of different distances on the results for the kinematic parameters. Numerical simulations show that a distance interval of up to 200 pc introduces distortions in the values for the unknown parameters of no more than 5%.

Numerical results and discussion

This method was used for Main Sequence stars with the boundary shown in the Hertzsprung-Russell (HR) diagram of Fig. 2. The HR diagram was constructed using photometry from the 2MASS catalog and the distance estimates were taken from the TGAS catalog. The characteristics of the samples are listed in Table 5. Since the average relative error in the parallax does not exceed 0.15 for 95% of the stars in our samples, we have neglected the Lutz-Kelker effect in the kinematical analysis.

These observations (radial velocities and proper motions) were averaged using Healpix areas with a parameter $N_{side} = 20$. Besides solutions for the radial velocities, solutions have been obtained for the proper motions of the stars from the TGAS, UCAC4, and PPMXL catalogs. The data were approximated using spherical functions twice. In the first step, the error in unit weight σ_0 was obtained, after which the data were filtered, i.e., the trapezoids whose content exceeded a threshold of $3\sigma_0$ were excluded from further processing. The results were values of the parameters for the Bottlinger model as functions of the average distance to the stars in the sample. These curves were obtained by smoothing over three points. Then the data obtained from the terrestrial catalogs UCAC4 and PPMXL were averaged and a separate curve was plotted from them which we shall denote by UCPP. In the following, we refer to the curves derived from the proper motions of the stars as "μ–curves." Correspondingly, for the curve derived from the radial velocities, we use the term "V_r–curves."

The averages of the parameters for the Bottlinger model derived from the radial velocities in the RAVE5 catalog and from the proper motions in the TGAS and UCPP catalogs
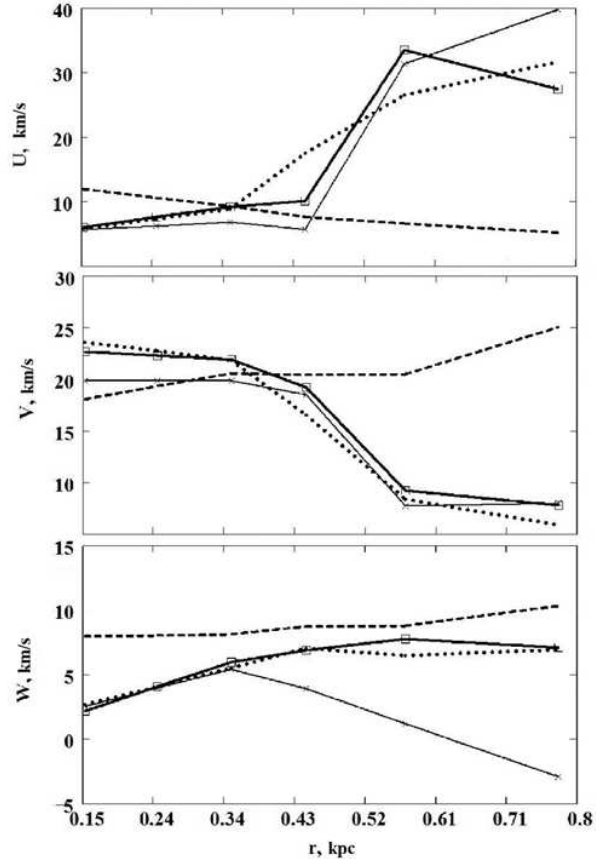
Figure 3: Distance dependences of the parameters U, V, and W derived from the radial velocities (dashed curve) and proper motions of Main Sequence stars in the TGAS (squares) and UCPP (crosses) catalogs. The dotted curves are the radial velocities reduced to the system of proper motions of the stars in the TGAS catalog.

are shown in Figs. 3 and 4. Analysis of these results yields a very important conclusion: excepting the parameter W for $r > 0.35$ kpc, the values of all the other parameters agree very well with the values derived from the proper motions in the TGAS space catalog and from the proper motions in the UCPP terrestrial catalogs. At the same time, the values determined from the radial velocities in the RAVE5 catalog are not always close to the results derived from the proper motions of the stars. Since we use the proper motions of stars from terrestrial catalogs and the independent TGAS space catalog, when there is good agreement in the variation of all the μ–curves and in the difference of the V_r–curves from that variation, we should prefer the results derived from the proper motions of the stars. In addition, since the terrestrial catalogs confirm the results from TGAS, it should be recognized that the values obtained in the TGAS system are more reliable than those derived from an analysis of the radial velocities of the stars.

For quantitative estimates of the agreement between the μ- and V_r–curves we calculated the variation with distance in the differences of the same parameters of the Bottlinger model obtained from the radial velocities and proper motions of stars in the TGAS cata-
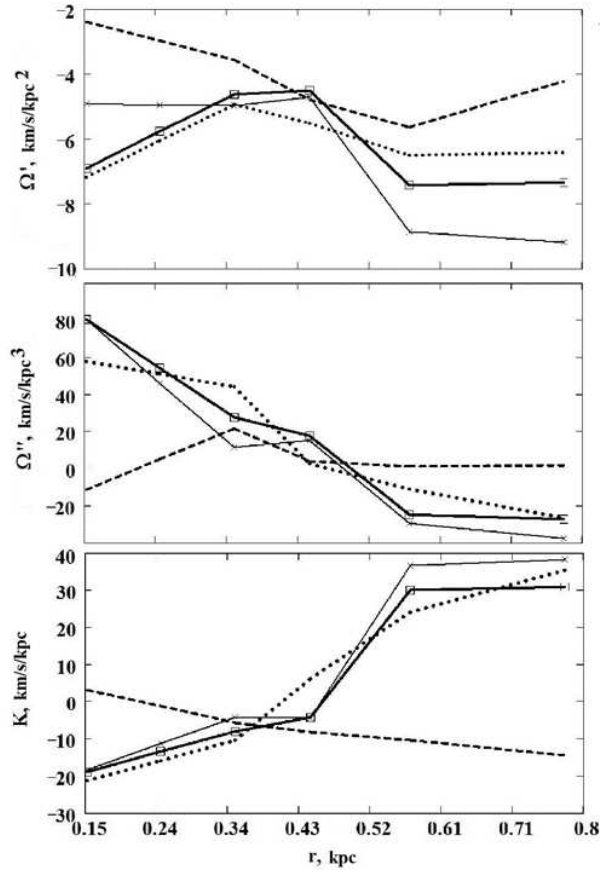
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Figure 4: Distance dependences of the parameters Ω', Ω'', and K derived from the radial velocities (dashed curve) and proper motions of Main Sequence stars in the TGAS (squares) and UCPP (crosses) catalogs. The dotted curves are the radial velocities reduced to the system of proper motions of the stars in the TGAS catalog.

log. For each curve the ranges of the heliocentric distances were found within which the modules of these differences did not exceed certain prespecified tolerances. These tolerances, the ranges of distances that were found, and the values of the kinematic parameters derived from the radial velocities and the proper motions from TGAS and UCPP in the zones of agreement are shown in Table 6. A study of Figs. 3–4 and Table 6 yields the following conclusions:

- The parameters U, V, and W. The results based on the radial velocities and proper motions agree to within 2–3 km/s for distances ranging from 0.2–0.5 kpc.

- The parameter Ω. It is determined only from the proper motions of the stars. The values based on UCPP and TGAS are in good agreement (at a level of 5 km/s/kpc) for distances ranging from 0.3 to 1.1 kpc.

- The parameter Ω'. The results based on radial distances and proper motions differ by no more than 3 km/s/kpc^2 for distances ranging from $0.2 \div 0.7$ kpc. This cir-

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Table 6: Parameters of the Bottlinger Model. Main Sequence

Parameter	Tolerance	Range for averaging, kpc	Radial velocities	UCPP	TGAS
U, km/s	$\|\Delta U\| < 2$	0.3–0.5	8.48 ± 0.11	6.27 ± 0.07	9.68 ± 0.11
V, km/s	$\|\Delta V\| < 2$	0.2–0.5	20.09 ± 0.06	19.38 ± 0.03	21.12 ± 0.05
W, km/s	$\|\Delta W\| < 3$	0.3–0.4	8.10 ± 0.03	5.38 ± 0.14	5.96 ± 0.22
Ω, km/s/kpc		0.7–0.8	—	34.88 ± 2.58	34.66 ± 4.46
Ω', km/s/kpc^2	$\|\Delta\Omega'\| < 2$	0.3–0.4	-3.56 ± 0.14	-4.98 ± 0.11	-4.63 ± 0.16
Ω'', km/s/kpc^3	$\|\Delta\Omega''\| > 4$	0.1–0.9	—	—	—
K, km/s/kpc	$\|\Delta K\| < 2$	0.3–0.5	-6.86 ± 0.29	-4.36 ± 0.44	-6.14 ± 0.40
A, km/s/kpc			14.24 ± 0.56	19.92 ± 0.44	21.44 ± 0.64
B, km/s/kpc			—	-14.96 ± 2.62	-13.22 ± 4.51

cumstance is of great interest from the standpoint of a method for monitoring the distance scale used here. This method is based on the fact that the errors in the radial velocities (see the left hand side of Eq. (1)) are independent of the errors in the distances, while the errors in the tangential components (see the left hand sides of Eqs. (2) and (3)) are dependent. Thus, a comparison of the differences between the various ways of finding Ω' (the most sensitive to the assumed distance scale) makes it possible to find the distance scale coefficient p [40,29]. We can calculate p in two ways using the data of Table 6: $p_1 = (-3.56)/(-4.98) = 0.72$ and $p_2 = (-3.56)/(-4.63) = 0.77$. Both values of p were found for nearby distances $(0.2 \div 0.7 \text{ kpc})$ and differ significantly from the value 0.96 obtained in those papers for distant stars. We note that an analysis of the radial velocities and proper motions of red giants in the RAVE5 catalog using our method also yielded a distance scale coefficient of 0.96 for distances of $0.7 \div 1.5$ kpc.

- The parameter Ω''. Estimates of this parameter based on the radial velocities and proper motions of stars do not agree, even at a level of 4 km/s/kpc^3 any where in the $0.1 \div 1.1$ kpc range. This makes it impossible to find concordant values of this parameter based on the radial velocities and proper motions.

- The parameter K. At a level of 2 km/s/kpc the results based on the radial velocities and proper motions for the Main Sequence are in agreement within $0.3 \div 0.5$ kpc.

Reduction of radial velocities to the system of proper motions of stars in the TGAS catalog

We now estimate the distance over which the accuracies of the tangential V_t and radial V_r velocities are equal. Using an upper bound for the estimated accuracy of the tangential component from the equation $\sigma_{V_r} = 4.74r\sqrt{\sigma^2_{\mu\alpha\cos\delta} + \sigma^2_{\mu_\delta}}$, we find the critical distance within which the tangential velocities are more exact than the radial velocities. Since $\sigma_{V_r} = 3$ km/s and $\sigma_{\mu\alpha\cos\delta,\mu_\delta} = 1$ mas/year for stars in TGAS ($\sigma_{\mu\alpha\cos\delta,\mu_\delta} = 0.1$ mas/year

Table 7: Coefficients in Eq. (38) for Main Sequence Stars

r	ΔU	ΔV	ΔW	$\Delta \Omega'$	$\Delta \Omega''$	ΔK
kpc	km/s	km/s	km/s	km/s/kpc^2	km/s/kpc^3	km/s/kpc
0.15	5.9 ± 0.4	-4.6 ± 0.3	5.8 ± 0.5	4.5 ± 0.8	-92.1 ± 6.8	22.1 ± 2.2
0.25	3.0 ± 0.2	-2.9 ± 0.1	4.0 ± 0.2	2.8 ± 0.4	-49.1 ± 3.3	12.2 ± 1.1
0.35	0.1 ± 0.2	-1.3 ± 0.1	2.1 ± 0.2	1.1 ± 0.2	-6.1 ± 1.6	2.4 ± 0.7
0.44	-2.5 ± 0.3	1.2 ± 0.2	1.9 ± 0.3	-0.3 ± 0.2	-13.7 ± 1.2	-3.9 ± 0.6
0.57	-26.8 ± 2.2	11.3 ± 1.2	1.0 ± 0.9	1.8 ± 0.3	25.6 ± 3.4	-40.4 ± 3.2
0.78	-22.3 ± 6.1	17.2 ± 1.9	3.2 ± 2.2	3.1 ± 0.9	28.8 ± 4.2	-45.5 ± 5.2
0.98	-17.8 ± 12.3	23.2 ± 3.0	5.4 ± 4.5	4.5 ± 1.8	32.0 ± 9.0	-50.5 ± 10.8

for subset of HIPPARCOS stars), we obtain $r = 0.45$ kpc for all the TGAS stars and $r = 1.4$ kpc for the subset of HIPPARCOS stars. Given that our samples are within these distance limits, it should be recognized that, in terms of the Bottlinger kinematic model, the radial velocities of the Main Sequence in the RAVE5 catalog may also have systematic biases relative to the proper motions of the same stars in the TGAS catalog. These biases can be eliminated using the corrected radial velocities given by

$$
\begin{aligned}
V_r^{corr} = V_r &+ \Delta U \, \cos l \cos b + \Delta V \, \sin l \cos b + \Delta W \, \sin b - \\
&- \Delta \Omega' \, R_0 (R - R_0) \sin l \cos b - \\
-0.5 \Delta \Omega'' \, R_0 (R - R_0)^2 &\sin l \cos b - \Delta K r \cos^2 b.
\end{aligned}
\tag{38}
$$

The coefficients $\Delta U, \Delta V, ...,$ in this formula have the are in the sense of the form "$V_r - \mu_{TGAS}$". Their numerical values are given in Table 7.

In Figs. 3 and 4 the dotted curves show the parameters of the Bottlinger model calculated using the corrected radial velocities (38). We now see that these parameters agree much better with the parameters derived from TGAS. Evidently, at the distances for which the accuracy of the tangential velocities is lower than that of the radial velocities it is necessary to use the opposite procedure, i.e., to reduce the proper motions to the system of their radial velocities. The general causes of significant differences in the V_r-and μ-curves require special study.

Conclusion

We have examined the three-dimensional velocities of stars with highly accurate positions, proper motions, and parallaxes from the Gaia TGAS catalog and radial velocities from the RAVE5 catalog. Based on a sample of 92395 stars with estimated ages, the following parameters have been determined: $(U, V, W) = (9.42, 20.34, 7.21) \pm (0.12, 0.10, 0.09)$ km/s, $\Omega = 26.29 \pm 0.39$ km/s/kpc, and $\Omega' = -3.89 \pm 0.08$ km/s/kpc^2, where $V_0 = 210 \pm 6$ km/s (for the assumed $R_0 = 8.0 \pm 0.2$ kpc), as well as the Oort constants $A = 15.57 \pm 0.31$ km/s/kpc and $B = -10.72 \pm 0.50$ km/s/kpc. We have found that the values of the parameters Ω and Ω' have good stability depending on the age of the stars.

Separate solutions have been obtained for the basic kinematic equations based on the radial velocities from the RAVE5 catalog and on the proper motions from the Gaia

TGAS, UCAC4, and PPMXL catalogs. This has made it possible to trace the mutual inconsistency of the data from a kinematic standpoint. The proper motions of the stars from three catalogs, Gaia TGAS, UCAC4, and PPMXL have been used. This yielded the following results:

- zone scalar and vector spherical functions have been used to construct a method for solving the Bottlinger equations based on stars in the southern equatorial hemisphere, so that it was possible to avoid strong correlations among the unknown parameters without needing to solve ill-conditioned systems of normal equations;

- the dependences of the parameters of the Bottlinger model on the average distance to the stars in the sample have been calculated separately using the radial velocities and proper motions of stars in the Main Sequence;

- the distance ranges have been obtained within which the Bottlinger model parameters are in good mutual agreement when derived from the radial velocities in the RAVE5 catalog and from the proper motions of the stars in the UCAC4, PPMXL, and Gaia TGAS catalogs;

- estimates of the parameters of the Bottlinger model for Main Sequence stars have been obtained within the conformity ranges;

- it has been concluded that when there are inconsistencies among the estimates of the kinematic parameters derived from the proper motions and radial velocities of stars, the results based on the proper motions are to be preferred, since the proper motions of the stars in the terrestrial UCAC4 and PPMXL catalogs and the Gaia TGAS catalog, which give consistent results, are fully independent;

- the radial velocities have been reduced to the system of proper motions of stars in the Gaia TGAS catalog, 482 thereby eliminating the differences in the values of the Bottlinger model parameters found by analyzing the radial velocities and proper motions of the stars.

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