THIN PRESSURIZED SPHERE EXPOSED TO INSIDE GENERAL CORROSION AND NONUNIFORM HEATING

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In this paper, the problem of the internal corrosion of a thin-walled elastic spherical vessel subjected to internal and external pressures of media with different temperatures is considered. The linear dependence of the corrosion rate on the mechanical stress is used. A new analytical solution of the problem is derived. The new solution allows to estimate the lifetime of the vessel taking into account the hydrostatic pressure applied both inside and outside.

Introduction. Thin-walled elastic spheres are widely used in engineering, for example, as a part of high-pressure vessels. Pressure vessels are often exploited in conditions of both mechanical loads, heating and chemical attack. This may lead to mechanochemical corrosion process, i.e. uniform corrosion which is accelerated by mechanical stresses and steady-state heat flow. In such situations, the thickness of the shell decreases due to corrosion process, shell thinning causes mechanical stress increase, which, in turn, accelerates the corrosion rate.

Herein, we derive a solution for a thin spherical shell subjected to inside uniform corrosion and loaded with internal and external pressures of media with different temperatures. Being based on Laplace's law, previous solutions of other authors (e.g., [3, 5]) do not reflect the effect of pressure values themselves, but only their difference. However, as it was shown in [7–9], the hydrostatic pressure may significantly affect the lifetime of a vessel. Unlike the previous solutions based on Laplace's law, we present the solution reflecting the effect of hydrostatic pressure on the lifetime of the vessel.

Problem Statement. Consider a linearly elastic thin-walled closed spherical shell loaded with internal p_r and external p_R pressures with temperatures T_r on the inner surface and T_R on the outer. The outer radius is constant through time and denoted by R_0 . The shell is subjected to inside uniform corrosion with the rate v_r, so the inner radius $r = r(t)$ of the sphere increases with time t, while its thickness $h(t) = R_0 - r(t)$ decreases uniformly. Let the inner radius of the sphere at the initial time $t_0 = 0$ be denoted by $r(0) = r_0$.

It is known that the corrosion rate depends on temperature and the stress state of the solid surface [1, 2, 6]. Experimental data shows that for some steels, corrosion rate is often exponentially dependent on temperature and linearly dependent on mechanical stress [1, 6]:

$$v_r = \frac{dr}{dt} = [a_r + m_r s(r)] \exp\left(b_r [T_r - T_r^{th}]\right),$$

where a_r, m_r and b_r, T_r^{th} are empirically determined constants; $s(r)$ is the maximum principal stress on the inside surface, which is the sum of the maximum principal elastic stress caused by the mechanical load and the maximum principal thermal stress caused by the steady-state heat flow (defined in [4]).

It is required to derive a solution which allows finding the stress value in the shell, its thickness for $t > 0$, and to assess the lifetime of the shell. The purpose of the present paper is to receive a solution that will be more accurate than the solutions based on Laplace's law (i.e. will reflect the effect of the hydrostatic pressure $p = \min\{p_r, p_R\}$), but retain a simple form like the solutions based on Laplace's law.

Problem Solution. The problem can be reduced to the differential equation

$$\frac{dh}{dt} = -A_r - M_r\left(\frac{(p_r - p_R)r_c}{2h} + \frac{T_k h}{3r_c}\right),$$

Where $M_r = m_r \exp\left(b_r\left[T_r - T^{th}{}_r\right]\right)$, $A_r = \left[a_r + \frac{m_r}{2}\left(T_k - \frac{p_r + 3p_R}{2}\right)\right]\exp\left(b_r\left[T_r - T^{th}{}_r\right]\right)$,

$r_c = (R_0 + r_0)/2$, $T_k = \frac{Ek(T_R - T_r)}{1-u}$; E is the Young module, u is the Poisson coefficient, k is thermal expansion coefficient of a material.

The initial condition to be satisfied is $h(0) = h_0 = R_0 - r_0$.

Solution of this equation is

$$t = \int \frac{6r_c h dh}{2T_k M_r h^2 + 6rA_r h + 3M_r(p_r - p_R)r^2{}_c}\Bigg|_{h_0}^{h}.$$

This expression provides corresponding to each other values of thickness h and time t. Since h and t are found, the stress can also be found for any t. This solution allows finding critical time t^*, corresponding to the critical stress value s^* (for example, a strength limit taking into account safety factors, or any other critical stress). The present solution reflects the effect of hydrostatic pressure $p = \min\{p_r, p_R\}$ on the lifetime of the vessel.

Conclusion. The solution obtained in this paper allows to estimate the lifetime of a thin-walled elastic spherical shell subjected to simultaneous action of internal and external pressures, nonuniform heating (in the case of steady-state heat flow) and inside uniform corrosion as well as to find the stress value in the shell and its thickness for any time. The values of internal and external pressures are taken into account, but not only their difference.

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