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Bifurcation of a Cylindrical Panel Made of Elastomers under Uniform Pressure

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Abstract.
The loss of stability of a cylindrical panel under the action of uniform pressure is considered. As a model, the nonlinear theory of shells from elastomers of K.F. Chernykh is used. The cylindrical bending of the panel is investigated. The problem reduces to solving a nonlinear one-dimensional boundary value problem for a system of nonlinear ordinary differential equations and the associated nonlinear system of algebraic equations. The solution of the nonlinear boundary value problem is based on method the solution continuation on the parameter and Newton-Kantorovich's linearization. The corresponding linear boundary value problems are solved using the Godunov's orthogonal sweep method. The numerical solution of the problem is realized in the MatLab environment. The results are presented in the form of "load-displacement" diagrams characterizing the process of both symmetric loss of stability and asymmetric bifurcation from a symmetric state. Deformed shell shapes are given at various points in the loading diagrams.

INTRODUCTION

Currently, rubber-like structures are widely used, so the calculation of stability losses in the nonlinear theory of elastic elastomer shells is of sufficient interest. Many papers are devoted to sustainability issues [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], etc. The main difficulty of stability research is related to the non-linearity of the equations describing the behavior of elastic systems. In this paper, we consider bifurcation loss of stability, i.e. the transition to adjacent forms of equilibrium

The search for bifurcation branches of the equilibrium state diagram is demonstrated by the example of cylindrical bending of a cylindrical panel under the action of uniform external pressure. This study uses the method of finding bifurcation branches, based on the idea of "de-idealization" of elastic system [6, 9]

PROBLEM FORMULATION

A cylindrical bend of a panel under the action of distributed pressure q_n is considered (Fig.1). The panel is made of elastomeric material and has a constant thickness of h_0. For the analysis the equations of the one-dimensional model of the nonlinear theory of elastic shells of elastomers are used [9, 11]. The purpose of the analysis is to search for asymmetric equilibrium forms and bifurcation critical load.

The panel's middle surface in Cartesian coordinate system is given by parametric equations (the values of the undeformed configuration is supplied with index zero):

$$x_0(s_0) = R\sin(\varphi_0),\quad y_0(s_0) = const,\quad z_0(s_0) = R\cos(\varphi_0),\quad \varphi_0 = \frac{s_0}{R}.$$

Here s_0 is the length of the arc before deformation, $s_0/R \in [-\pi/2, \pi/2]$, φ_0 - the angle between the normal to the middle surface of the panel and the z-axis.

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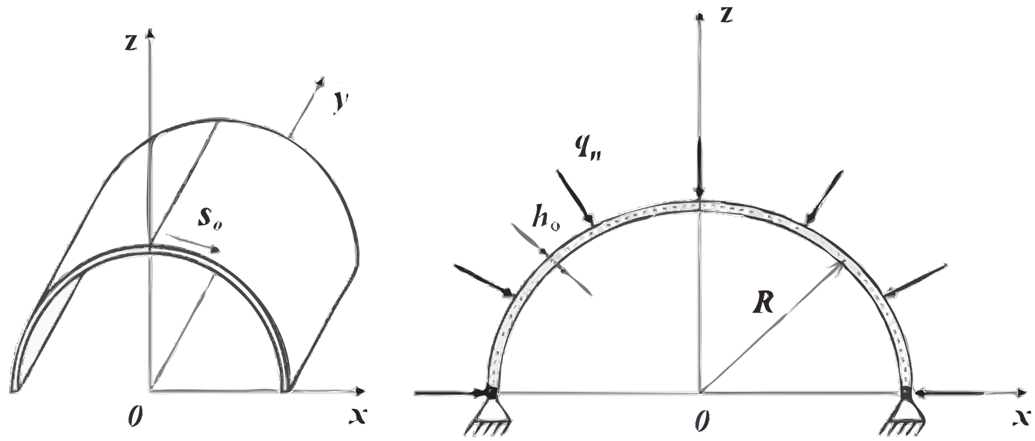

FIGURE 1. The cylindrical panel with hinged edge support.

The deformed configuration is defined by the ratio:

$$x' = \lambda_s \cos\varphi, \quad y' = 0, \quad z' = -\lambda_s \sin\varphi, \quad \varphi' = \lambda_s^2(\varkappa + \varphi_0') \tag{1}$$

Apostrophe means the differentiation by s_0, $\lambda_s = ds/ds_0$ - the multiplicity of elongation of the arc, $\varkappa$ – component bending deformation(the change of curvature).

The Equilibrium equations of a cylindrical panel take the form:

$$T_s' + \varphi' T_n = 0, \quad T_n' - \varphi' T_s + \lambda_s q_n = 0, \quad M' - \lambda_s T_n = 0. \tag{2}$$

where T_s, T_n is a projection of resultant tension to a tangent and a normal of an arch, M - the stress couple, q_n - external pressure.

The elasticity relations for the neo-Hooke potential take the form [9]

$$T_s = \mu h_0(\lambda_s - \lambda_s^{-3}), \quad M = \frac{\mu h_0^3}{3}\lambda_s^{-4}\varkappa \tag{3}$$

The boundary conditions are set depending on a way of fixing the edges. For hinge support at $s_0 = \pm\pi R/2$:

$$(x - x0) = 0, \quad (z - z_0) = 0, \quad M = 0 \tag{4}$$

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PROBLEM SOLUTION

The solution of the nonlinear boundary value problem (1-4) is based on the method of discrete continuation of the solution by parameter and Newton-Kantorovich linearization. The subsequent linear boundary value problems is solved by the Godunov's orthogonal sweep method. To implement the parameter continuation method, the system (2) is supplemented by the equation

$$q_n' = 0 \tag{5}$$

and the boundary condition (4) at $s_0 = -\pi R/2$ is supplemented with one of four equalities :

$$\varphi - \varphi_0 = \theta^*, \quad T_n = T_n^*, \quad T_s = T_s^*, \quad q_n = q_n^*. \tag{6}$$

The process of discrete continuation for a parameter from the set (6) is performed as follows. The parameter that received the maximum relative increment in the previous step is selected as the continuation parameter in the next step. The size of the next step is determined depending on the convergence rate at the previous step. If this step fails to get a solution in an acceptable number of iterations, the step value is reduced by 2 times. This algorithm allows you to overcome any limit points of the equilibrium state diagram. Finally, we obtain the curve of equilibrium States in the 4-dimensional space of variables (6).

The result of using the implemented methods is shown in the ”load-displacement” diagram (Fig. 2). The curves show the dependence of the dimensionless pressure $\hat{q}_n = q_n R^3/(\mu h^3)$ on the dimensionless displacement $\hat{u} = (z(0) - z_0(0))/R$ of the shell pole for the case of symmetric deformation relative to the z axis (curve 1). The corresponding deformed shapes are shown in Fig. 3(left).

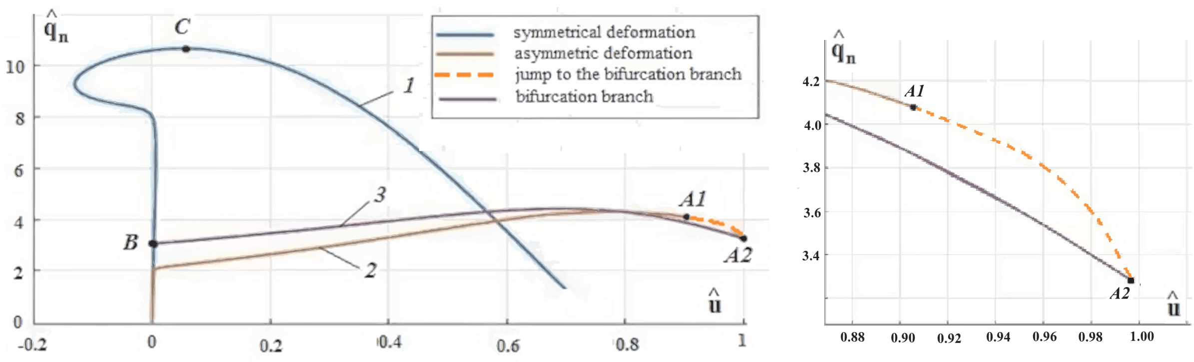

FIGURE 2. The load-displacement diagram of the deformation process of a cylindrical panel with hinged fastening of the ends.

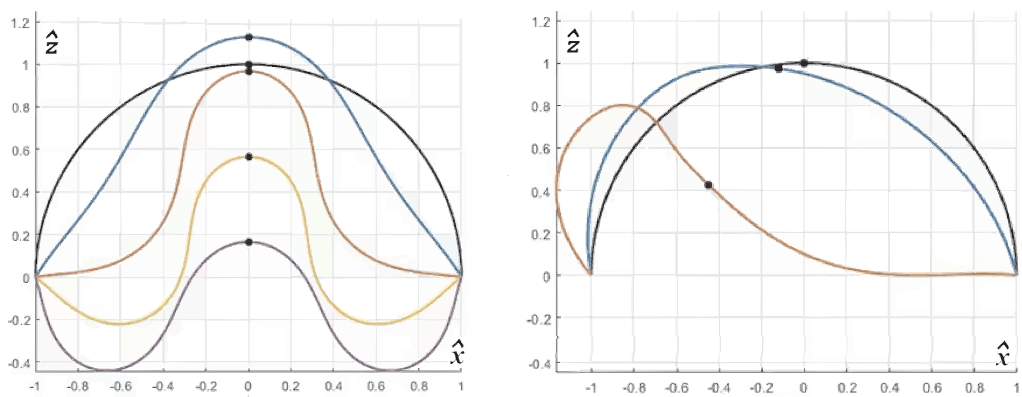

FIGURE 3. Deformed panel configurations corresponding to curve 1 and 2 in Fig.1.

When numerically constructing the equilibrium state diagram, we always get the curve 1. The load corresponding to point C is a critical load at which the loss of stability occurs, such as a limit point. However, there is a suspicion that at lower loads, stability loss may occur in an asymmetric form - the so-called bifurcation (branching) of the solution. To search for the bifurcation branches of the equilibrium state diagram, the imperfection method of the elastic system is used [6, 8, 9]: transition to a system with rejected from idealized conditions. The deviation from symmetric conditions is set by reducing the thickness of the panel on the left side of the arc according to the formula $h^* = h_0(1 + \alpha \sin(2s_0/R))$, with $s_0 \in [-\pi R/2, 0]$, where α is a parameter, $\alpha \in [0, 0.3]$. The process of deformation of the panel with a modified thickness is shown in fig. 2, curve 2, and forms of the panel in Fig. 3(right).

Having the loaded panel to a significant asymmetrical state (this state in this case corresponds to point A1) (Fig. 2), the value of the continuation parameter (4) is fixed and the parameter α changes from a certain step from 0.3 to 0. Thus, the transition from the solution for the panel with asymmetrical thickness to the bifurcation solution for the panels with an initial constant thickness. The transition is presented in Fig. 2, by dashed line. Thus, the point A2 at which we arrive at $\alpha = 0$ is the point of the bifurcation curve. Next, we move along the bifurcation curve to the left,

”jump” over the bifurcation point B and get the 2nd part of the bifurcation curve, shown in three-dimensional space in Fig.4 (left) The numbers represent the points on the diagram for which the corresponding shell profiles are shown (on the right).

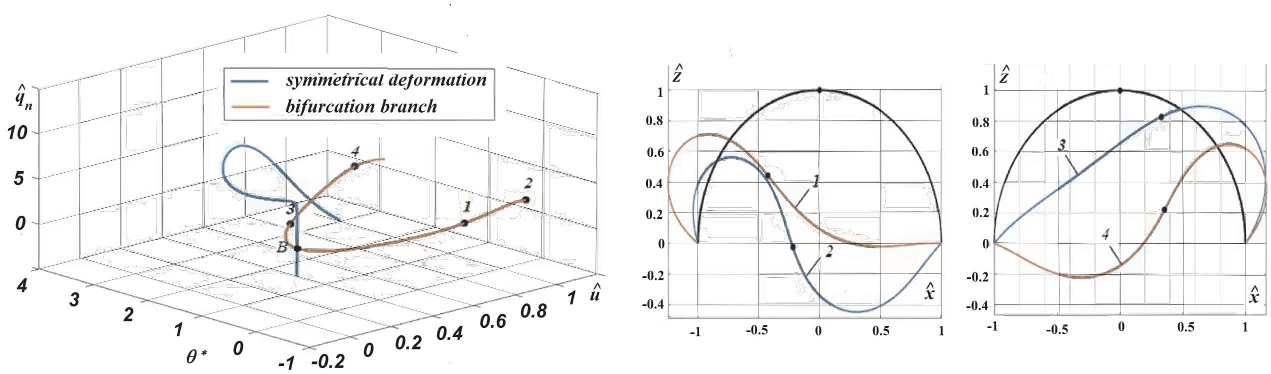

FIGURE 4. Diagram ”load displacement” in 3-dimensional space. The deformed configuration of the panel.

CONCLUSION

The process of branching solutions in the transition from a symmetric deformed configuration to an asymmetric one is studied. An algorithm for constructing bifurcation branches based on the idea of imperfection of an elastic system is developed. Critical loads that cause loss of stability of cylindrical panels are established.

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