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MATHEMATICAL MODELING IN HYDROLOGY

# Topographic Effect for Rossby Waves on Non-Zonal Shear Flow

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**Abstract**—This work continues a series of studies by the authors, where a comparative analysis of the influence of topography, β-effect, and gradient variability of a background flow on the propagation of barotropic topographic Rossby waves is carried out. The novelty of this study lies in the fact that here we consider a non-zonal shear flow and also non-zonal topographic changes. We establish that the non-zonal shear flow enhances the β-effect from the north side of the main flow and reduces from the south side. Contrary, the influence of the joint effect of the Earth rotation and bottom topography enhances the β-effect in the study area and the joint effect of the shear flow and topography reduces it slightly. We exclude the effect of stratification based on the results already obtained that short waves are not observed practically, and also the effect of stratification for Rossby long waves is insignificant. The transverse variability of the shear plane-parallel flow in the WKB approximation is studied. This allows us to obtain a dispersion relation for the flat barotropic topographic Rossby waves with taking into account the above conditions. The estimates of the dispersion equation terms are obtained for the Kuroshio Extension area, where a flow branch turns to the northeast and makes an angle of 55° with latitude. Applied the exponential approximation for bottom topography we obtain that the First approximation by Rhines for the Rossby waves is true for the study area because the parameter of the relief transverse change $l_0$ equals 780 km (exceeds typical Rossby wavelengths). It allows simplifying the corresponding dispersion relation.

**Keywords:** Rossby waves, WKB-approximation, non-zonality, dispersion relation, nonlinear effects, non-zonal jet flow, Kuroshio, GLORYS12v1

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## INTRODUCTION

Rossby ocean waves are large-scale waves that have a low amplitude (some cm), periods from days to months, and lengths from tens to hundreds of km. It may take them months to cross the ocean basin. They receive an impulse from the wind stress in the surface layer of the ocean and are believed to report climate changes due to the variability of the impact caused by both wind and buoyancy. Both barotropic and baroclinic Rossby waves cause fluctuations in sea level height, although the periods and wavelengths made it difficult to detect them before the advent of satellite altimetry. Satellite observations have confirmed the existence of Rossby ocean waves.

Rossby waves propagate to the west, against the global rotation of the Earth. The dispersion equation for Rossby waves derived from the equations of motion and continuity is $\omega = -\frac{k\beta}{k^2 + l^2 + r^{-2}}$ if the ocean depth $H$ is constant. Here $\omega$ is a frequency, $k$ and $l$ are zonal and meridional wave numbers, $r$ is the radius of deformation, $x$ is the coordinate along the parallel (positive direction to the east), $y$ is the coordinate along the meridian (positive direction to the north), $\beta = \frac{df}{dy}$, and $f$ is the Corilis parameter. For the phase speed $c$, the dispersion equation is $c = \frac{\omega}{k} = -\frac{\beta}{k^2 + l^2 + r^{-2}}$ and it is $c = \beta r^2$ in the long-wave approximation. It is shown that the main energy of Rossby waves corresponds to the first baroclinic mode. At almost all middle latitudes, there is good agreement between the empirical speed of the Rosby waves obtained from altimetry maps and the theoretical values of the phase speed [1, 2, 12].

However, this well-known approach applies only to the linear case. It is the obvious fact that it is impossible to explain well the interaction of Rossby waves with ocean currents within the framework of the linear theory because the speed of the currents is much bigger than the wave speed. If consider an interaction of Rossby waves with flows a paradox arises which was formulated by Carl Rossby in 1939. The paradox consists in the fact that the phase velocities of Rossby waves and currents are not comparable in magnitude since the velocities of the current are usually several times (even up to 1–2 order bigger) exceed the phase velocities of Rossby waves. This makes it impossible to interpret the interaction of Rossby waves and flows in terms of simple Doppler effect. The solution to the paradox should be sought in a nonlinear theory [6].

This aspect was considered for the Antarctic Circumpolar Current [7]. The authors, using the non-linear approach, showed that nonlinearity in the long-wave approximation exactly compensates the Doppler shift. This means that the contribution of the component associated with the meridional change in the zonal flow shift $U_{yy}$ can be greater than the influence of the β-effect; $U$ is a zonal component of the velocity, and $V$ is the meridional one. Rossby waves propagating in the waveguide interact nonlinearly with the flow, and the Doppler shift is balanced by nonlinearity. Using altimetry data for the Antarctic Circumpolar Current, the authors demonstrate that in this area, meridional changes in planetary vorticity can be counterbalanced by local meridional changes in relative vorticity. In this case, in the expression for β*, which is "effective" β, the meridional gradient of the zonal velocity shear prevails $U_{yy}$: $\beta^* = \beta - U_{yy}$, and as a result, Rossby waves in the Antarctic Circumpolar Current waveguide can propagate eastward [9].

However, in areas of jet streams, large-scale topographic changes also affect the dynamics of Rossby waves. If the ocean depth $H$ is not constant, then

$$\beta_1 = -\frac{f^2}{H}\frac{\partial H/f}{\partial y} = \frac{\partial f}{\partial y} - \frac{f}{H}\frac{\partial H}{\partial y}. \qquad (1)$$

It follows that both the meridional change of the Coriolis parameter and meridional topographic gradients in the relief influence the Rossby waves propagation [17]. When the fluid elements cross the contours $\frac{f}{H}$, their relative vorticity changes so that it must be compensated by planetary-topographic vorticity according to the law of enstrophy conservation. Consequently, an increase in the planetary wave effects is expected where there is a change of $\frac{f}{H}$. The Eq. (1) follows that in the northern hemisphere ($f > 0$), the decreasing depth to the north leads to a strengthening of β-effect and to its weakening in the opposite case.

Rhines [20] analyzed the joint influence of topography and stratification for Rossby waves. He found a mode when the node of the first baroclinic mode shifts to the bottom with strong bottom relief slopes. Such influence of bottom topography on Rossby waves can be interpreted as a topographic filter. Perhaps, thanks to the topographic filter, short waves are practically not registered in the observations. Thus, bottom topography can also both enhance and weaken the β-effect as well. However, when stratification, topography, and the β-effect are considered together, the problem becomes very complicated because the method of separating variables does not work. This is the reason that the influence of topography and stratification or topography and the β-effect are analyzed separately [18]. Considering these effects separately leads to the formulation of the First and Second approximations of Rhines [14]. The current state of studies for Rossby waves influenced jointly by topography and stratification is introduced by LaCasce [13].

Gnevyshev et al. [7] are the first who provide a joint comparative analysis of the influence of topography, the β-effect, and the gradient of the meridional variability of the background flow on the propagation of barotropic topographic Rossby waves. The authors use the WKB approximation and topography profiles and follow the studies by Gill [5] and Chelton et al. [3, 4], where it is stated that stratification does not affect long waves. Gnevyshev et al. [7] obtained the estimates of components in the dispersion relation for the area of the Antarctic Circumpolar Current with real bottom topography and compared them to each other. The main findings of this study are as follows: (1) the topographic factor in the dispersion equation is the main one; (2) locally, the contribution of the shear flow can overpower the effect of the β-parameter; (3) a new component $\frac{H_y U_y}{H}$ appears in the dispersion relation, and it also affect the β-effect; (4) the comparison to other components reveals that the impact of the last component reflecting the combined effect of topography and meridional variability of the background flow is insignificant. To summarize, Gnevyshev et al. [9] demonstrate that a local contribution of the zonal shear flow may be more that influence of β-effect. Moreover, Gnevyshev et al. [7] show that the contribution of a zonal shear flow locally may exceed the contribution of bottom topography, despite in the whole the impact of the topography in dominating. The result depends on numerical estimates of these components.

However, all these studies only relate to both zonal topography and zonal jet as well. The influence of non-zonal shear flow and non-zonal bottom topography on Rossby waves has not been studied yet. The current research attempts to close this gap. For this goal, we consider an area with a branch of the Kuroshio.

Thus, this study aims to provide a comparative analysis of a rotation (i.e., $f$) and non-zonal topography on the one hand, and velocity gradient of non-zonal barotropic shear flow and topography on the other hand, in the dispersion relation. This formulation of the problem is also new.

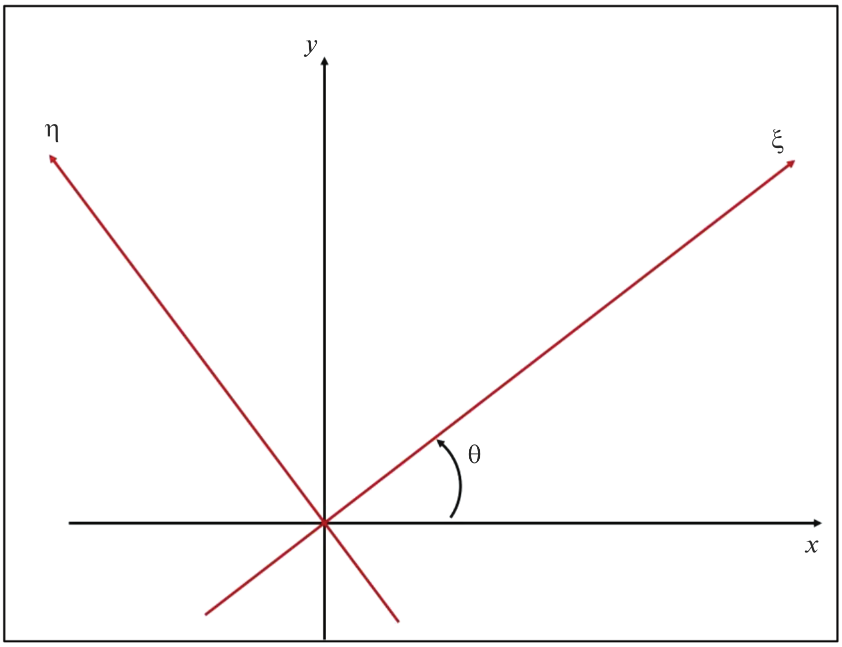


**Fig. 1.** The rotation of a coordinate system $(x, y) \to (\xi, \eta)$.

## METHODS

*The Flow and Isobaths Have the Same Direction Rotated to Latitudes on the Angle* $\theta$

Following to [7], we start from the next basic equations for the mass stream functions:

$$HU = -\Psi_y, \quad HV = \Psi_x, \quad H = H(x,y)$$
$$\zeta = V_x - U_y = \left(\frac{\Psi_x}{H}\right)_x + \left(\frac{\Psi_y}{H}\right)_y,$$
$$\left[\frac{\partial}{\partial t} - \frac{\Psi_y}{H}\frac{\partial}{\partial x} + \frac{\Psi_x}{H}\frac{\partial}{\partial y}\right] \times \left[\frac{1}{H}\left\{\left(\frac{\Psi_x}{H}\right)_x + \left(\frac{\Psi_y}{H}\right)_y + f\right\}\right] = 0. \quad (2)$$

Here $f = f_0 + \beta y$. Following to [14], let's go to a new coordinate system $(\xi, \eta)$ by turning the system $(x, y)$ on the rotation angle $\theta$ (Fig. 1). Now the axis $\xi$ is codirected the flow. The axis $\eta$ is perpendicular to it. We have a coordinate transformation:

$$x = \xi\cos\theta - \eta\sin\theta,$$
$$y = \xi\sin\theta + \eta\cos\theta.$$

According to this section we believe that all variables only depend on the coordinate $\eta$. This means we consider the case of a topographic flow and both flow and topography change in a transversal direction. The conversion formulas have the form:

$$H_\eta = H_x x_\eta + H_y y_\eta = -H_x\sin\theta + H_y\cos\theta,$$
$$U_\eta = -U_x\sin\theta + U_y\cos\theta,$$
$$U_{\eta\eta} = (-U_x\sin\theta + U_y\cos\theta)_\eta$$
$$= \sin^2\theta U_{xx} - 2\sin\theta\cos\theta U_{xy} + \cos^2\theta U_{yy}.$$

The flow and isobaths directions are the same in this case: $\tan\theta = \frac{V}{U}$.

Next, we apply the standard linearization procedure against the background of a stationary zonal flow (see also [7]):

$$\Psi(\xi,\eta,t) = \Psi_s(\eta) + \varepsilon\tilde{\Psi}(\xi,\eta,t), \quad \varepsilon \ll 1, \quad (3)$$

here $U(\eta) = -\frac{(\Psi_s(\eta))_\eta}{H}$ is background current with variable topography, changes of which occur from north to south: $H = H(\eta)$, $\tilde{\Psi}(\xi,\eta,t)$ are perturbations. Note that $U(\eta) = \sqrt{U^2 + V^2}$.

Then, using the invariance of the rotor and Laplace operators with an orthogonal rotation of the Cartesian coordinate system, we obtain the following linear equation for barotropic topographic Rossby waves taking into account the topography and variability of the background flow:

$$(\partial_t + U\partial_\xi)\left[\frac{\tilde{\Psi}_{\xi\xi}}{H^2} + \frac{1}{H}\left(\frac{\tilde{\Psi}_\eta}{H}\right)_\eta\right] + \frac{\tilde{\Psi}_\xi}{H^2}\left(\beta\cos\theta - U_{\eta\eta} - \frac{fH_\eta}{H} + \frac{U_\eta H_\eta}{H}\right) - \frac{\beta\sin\theta}{H^2}\tilde{\Psi}_\eta = 0. \quad (4)$$

For waves $\tilde{\Psi}(\xi,\eta,t) = \tilde{\Psi}(\eta)\exp[i(k\xi - \omega t)]$, we get

$$(\omega - kU)\left[-\frac{k^2\tilde{\Psi}}{H^2} + \frac{1}{H}\left(\frac{\tilde{\Psi}_\eta}{H}\right)_\eta\right] - \frac{k\tilde{\Psi}}{H^2}\left(\beta\cos\theta - U_{\eta\eta} - \frac{fH_\eta}{H} + \frac{U_\eta H_\eta}{H}\right) - i\frac{\beta\sin\theta}{H^2}\tilde{\Psi}_\eta = 0. \quad (5)$$

The first two terms have real coefficients so by a classical substitution eliminating the first derivative (see below), this equation can be reduced to a linear Schrödinger equation with a classical potential. Therefore, this operator is Hermitian and has all the known properties of the spectral problem of Hermitian operators. The last, third term, which is proportional to sin θ, has a complex multiplier if the flow is not zonal ($\theta \neq 0$). Then, the complex conjugate equation, due to the presence of an imaginary additive, will no longer coincide with the original equation and we will leave the space of Hermitian operators and lose all the known results of the mathematical apparatus. As such a loss, we will give a specific example.

It is known that if the flow is strictly zonal (the Hermitian operator), then for Rossby waves, using the classical mathematical apparatus of the calculus of variations, it is possible to construct an analogue of the Howard semicircle theorem, which is a generalization of the classical Howard semicircle theorem and gives a certain general finite restriction on the unstable spectrum of Rossby linear waves for any smooth profile of the background flow velocity. This gives a certain restriction from above on the unstable spectrum. That is, instabilities are fundamentally limited by a certain finite expression, and they are, as it were, under control (see [8, 15, 19]. A fundamentally important point is that the procedure for constructing such a theorem for Hermitian operators has long been known.

However, if we abandon the assumption of zoning of the flow and switch to an arbitrary non-zonal stationary plane-parallel flow, and try to repeat the algorithm, the result will be practically negative. Howard's semicircle theorem does not transfer to a non-zonal flow. And for a non-zonal flow, it is not known whether the instability spectrum is limited from above, or there is no restriction. An attempt at a similar procedure is described by Kamenkovich and Pedlosky [11]. Analyzing concrete examples of background flows, both smooth and piecewise smooth, they come to the conclusion that unstable spectra become more unstable during the transition from a zonal flow to a non-zonal one. But they do not give an answer to the fundamental question: in general, whether there is a certain restriction on the instability increments in the non-zonal case from above. If there is no general theorem that restricts the spectrum from above in principle, the analysis of particular problems, although it has some value, is of extremely little value. This means that all the theorems that are obtained for Rossby waves, and stated, for example, in the book by Pedlosky [18], all this applies only to the case of a strictly zonal flow.

The second important point related to the last term, which is proportional sin θ in (5), is that it, sin θ also appears in a nonlinear problem. If we take the vorticity equation on the β-plane and try to find its stationary solutions in the form of a shear stationary non-zonal flow, we get the following relation [11]:

$$U\beta\sin\theta = F.$$

Here $F$ is the some external force, e. g., the Ekman pumping. If the current is zonal ($\sin\theta = 0$), then the background flow exists independently of the external force. There is a law of conservation of enstrophy (vorticity) and, in fact, the flow does not interact with wave solutions. As a consequence, the linear Hermitian operator and instabilities are bounded from above. If the flow is non-zonal, then constant external pumping is required to maintain it in a stationary state. There is no law of conservation of enstrophy in this case, and perhaps there are no restrictions on the spectral problem either.

Then, using the standard replacement $\tilde{\Psi}(\eta) = \sqrt{H(\eta)}\varphi(t)$, we apply the WKB approximation for the transverse coordinate $\varphi(\eta) = \exp(il\eta)$ and finally obtain the following dispersion relation:

$$(\omega - kU)\left[k^2 + l^2 + \frac{1}{4}\left(\frac{H_\eta}{H}\right)^2 - \frac{1}{2}\left(\frac{H_\eta}{H}\right)_\eta\right] = -\beta^*k - l\beta\sin\theta. \quad (6)$$

Here

$$\beta^* = \beta\cos\theta - U_{\eta\eta} - \frac{fH_\eta}{H} + \frac{U_\eta H_\eta}{H}. \quad (7)$$

Here $k$, $l$ are the lateral and transverse components of the wave number, respectively. Thus, a rotation of the Earth (i.e., $f$) and non-zonal topography on the one hand, and gradient of the non-zonal velocity and topography, on the other hand, compete in the dispersion relation (5). In this case θ is the angle of the shear flow direction and the angle of the isobaths regarding latitudes as well so that the main changes of the all parameters occur in a direction perpendicular to it.

### *The General Case for a Coordinate System Rotated to Latitudes on the Angle θ*

As in the previous section, let's go to the new coordinate system $(x, y) \to (\xi, \eta)$, which is rotated at an angle θ to the parallel. We use the β-flat approximation, $f = f_0 + \beta y$. Then, we omit the index zero so that $f_0 = f$. Further, instead of Eq. (2) we have the next one (see [14]):

$$\left[\frac{\partial}{\partial t} - \frac{\Psi_\eta}{H}\frac{\partial}{\partial \xi} + \frac{\Psi_\xi}{H}\frac{\partial}{\partial \eta}\right] \times \left[\frac{1}{H}\left\{\left(\frac{\Psi_\eta}{H}\right)_\eta + \left(\frac{\Psi_\xi}{H}\right)_\xi + \left(f_0 + \beta[\xi\sin\theta + \eta\cos\theta]\right)\right\}\right] = 0. \quad (8)$$

Since $U = -(\Psi)_\eta/H$, and $V = (\Psi)_\xi/H$, we have:

$$(\partial_t + U\partial_\xi + V\partial_\eta)\left[\frac{1}{H}\left(\frac{\tilde{\Psi}_\xi}{H}\right)_\xi + \frac{1}{H}\left(\frac{\tilde{\Psi}_\eta}{H}\right)_\eta\right] + \frac{\beta^*}{H^2}\tilde{\Psi}_\xi - \frac{\beta_2}{H^2}\tilde{\Psi}_\eta = 0. \quad (9)$$

Here

$$\beta^* = \beta\cos\theta - U_{\eta\eta} - \frac{fH_\eta}{H} + \frac{U_\eta H_\eta}{H}, \quad (10)$$

$$\beta_2 = \beta\sin\theta - U_{\xi\xi} - \frac{fH_\xi}{H} + \frac{U_\xi H_\xi}{H}. \quad (11)$$

In contrast to the previous case, here in the Eqs. (8–10), the direction of isobaths and currents are not tied to each other. If we choose an angle θ in the direction of a large-scale flow, the transverse component of the background flow is eliminated. Then we can linearize Eq. (7) against the background of a large-scale shear flow:

$$\Psi(\xi,\eta,t) = \Psi_s(\eta) + \varepsilon\tilde{\Psi}(\xi,\eta,t), \quad \varepsilon \ll 1.$$

So we get that the velocity has a single component $U(\eta) = -(\Psi_s(\eta))_\eta/H$, and we get the next linear equation:

$$(\partial_t + U\partial_\xi)\left[\frac{\Psi_{\xi\xi}}{H^2} + \frac{1}{H}\left(\frac{\tilde{\Psi}_\eta}{H}\right)_\eta\right] + \frac{\beta^*}{H^2}\tilde{\Psi}_\xi - \frac{\beta_2}{H^2}\tilde{\Psi}_\eta = 0, \quad (12)$$

with

$$\beta_2 = \beta\sin\theta - \frac{fH_\xi}{H}. \quad (13)$$

For wave perturbations of the form $\tilde{\Psi}(\xi,\eta,t) = \tilde{\Psi}(\eta)\exp(i[k\xi - \omega t])$ and with the changes all functions along the transverse coordinate of the form $\sim \exp(il\eta)$, and using the First and Second approximations by Rhines [14] and also the WKB approximation, we get the next dispersion equation:

$$\omega = -\frac{k\beta^* - l\beta_2}{k^2 + l^2} + kU. \quad (14)$$

If we assume that the topography, as well as the background flow, changes only in the transverse direction ($l = 0$), we get the previous case (see section "The Flow and Isobaths Have the Same Direction Rotated to Latitudes on the Angle θ"). Thus, the main finding is as follows: the conclusions for the zonal flow in WKB approximation are transferred to the non-zonal case by a simple adjustment of the β-parameter.

### *Approximation of a Bottom Relief by an Exponential Function*

We apply an exponential approximation of the relief for the selected area to estimate the influence of the topographic factor in the square brackets in Eq. (5). Note, that $\frac{1}{4}\left(\frac{H_\eta}{H}\right)^2 - \frac{1}{2}\left(\frac{H_\eta}{H}\right)_\eta = \frac{3}{4}\frac{(H_\eta)^2}{H^2} - \frac{1}{2}\frac{H_{\eta\eta}}{H}$. According to [14], $H(\eta) = H_0\exp(-\eta/l_0)$ (shallow water to the north). Then $\frac{1}{4}\left(\frac{H_\eta}{H}\right)^2 - \frac{1}{2}\left(\frac{H_\eta}{H}\right)_\eta = 1/(4l_0^2)$, $\beta^* = \beta\cos\theta - U_{\eta\eta} - \frac{fH_\eta}{H} + \frac{U_\eta H_\eta}{H} = \beta\cos\theta - U_{\eta\eta} + (f - U_\eta)/l_0$.

Substituting this expression in Eq. (3), we get:

$$(\omega - kU)\left[k^2 + l^2 + \frac{1}{4l_0^2}\right] = -\left(\beta\cos\theta - U_{\eta\eta} + \frac{f - U_\eta}{l_0}\right)k - l\beta\sin\theta, \quad (15)$$

or for the phase speed:

$$c = \frac{\omega}{k} = -\frac{\left(\beta\cos\theta - U_{\eta\eta} + \frac{f - U_\eta}{l_0}\right) + \frac{l}{k}\beta\sin\theta}{k^2 + l^2 + \frac{1}{4l_0^2}} + U. \quad (16)$$

We apply this approach with exponential approximation of the bottom relief for the study area and estimate $l_0$ using a spatial averaging of bottom topography: $l_0 = 7.8 \times 10^5$ m = 780 km. Such significant magnitude of $l_0$ means that the topography strongly affects the numerator i.e., $\beta^*$, but weakly affects the denominator. Hence, we can use the First approximation by Rhines [14] and consider the dispersion equation in the form:

$$c = \frac{\omega}{k} = -\frac{\beta^* + \frac{l}{k}\beta\sin\theta}{k^2 + l^2} + U. \quad (17)$$

## DATA AND STUDY AREA

### *GLORYS12v1 and GEBCO*

We use the GLORYS12v1 product available on Copernicus Marine Environment Monitoring Service

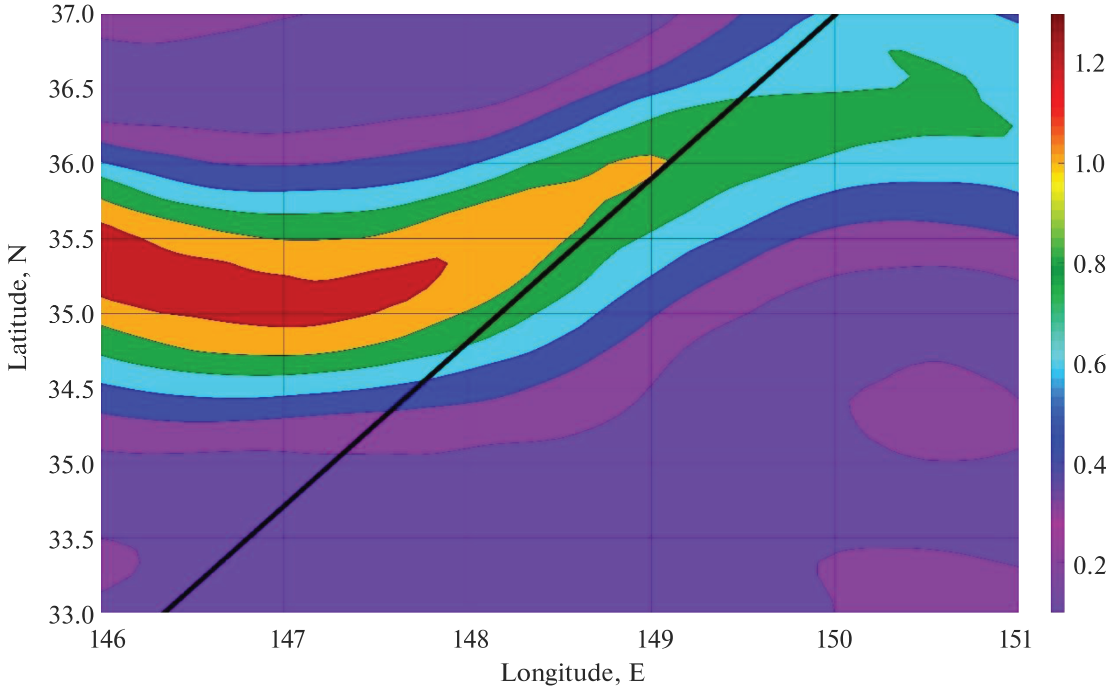


**Fig. 2.** Mean current velocity in the region for the study period (m/s). An averaging for the period 1.01.2018–31.12.2018. The black line shows the flow deviation of 55° regarding latitudes.

(**CMEMS**). It is a vortex-resolving reanalysis based on the NEMO hydrodynamic model of the ocean and the ERA-Interim atmospheric reanalysis, which is used as forcing. The physical (temperature, salinity, current components, sea level anomalies) and ice (thickness, area fraction, velocity) properties are distributed on a regular grid on 50 horizons. The spatial resolution of the data is 1/12°. Observations are assimilated using a reduced-order Kalman filter. In the current study, we use components of the flow velocities $U$ and $V$ for the period 1.01.2018–31.12.2018.

A General Bathymetric Chart of the Oceans (**GEBCO**) is used for the bottom topography. This is the global bathymetrical gridded data set with 15 arc-second (1 arc-second = 1/3600°) special resolution. The chart was created by British Oceanography Data Centre based on various sources from national and international databases, which include regional and global data. Most of the data compilation is multibeam sonar and model data, as well as modernized historical observations. This chart has some gaps in ocean floor data. However, the chart has been improving and updating when new data comes.

### *Topography and Velocities for the Area under Consideration*

Figure 2 shows the Kuroshio branch. The Kuroshio Current separates from the coast of Japan near 35°N, 140°E and veers east, creating the Kuroshio extension [16]. Further east the Kuroshio extension broadens and loses most of its kinetic energy to become the eastward flowing North Pacific Current. The study area in Fig. 2 is part of it. Note, first, the jet runs zonal along 35.5° N so the main axis of the jet corresponds to 35.5° N but then, reaching 147.5° E, it turns northeast with an angle of 55° from a zonal direction. A maximum flow speed of 125 cm/s is achieved between 35.0°–35.5° N with decreasing to 20 cm/s at the periphery of the flow. Figure 2 shows that the flow in the study area is non-zonal.

Although the isobaths of the bottom topography are uneven, we will consider they also tilted on 55° (Fig. 3). The bottom relief is characterized by sharp gradients when in neighboring points of the grid the depth can vary from 4000 to 7000 m. Nevertheless, in averaging the south-eastern part of the area is deeper than the north-western one with the difference between is about 2000 m. The highest point of the area is a mountain located about 34.0°–34.5° N, 148°–148.5° E with the depth of 2000 m. This mountain was excluded from calculations.

### *Comparison of the Components in the Dispersion Relation for the Study Area*

Table 1 shows the influence of the components in $\beta^*$ in Eq. (6) do they reduce or enhance the $\beta$-effect.

Further, we go to a numerical comparison of the components in the dispersion relation. To get the numerical estimates we normalize all components on

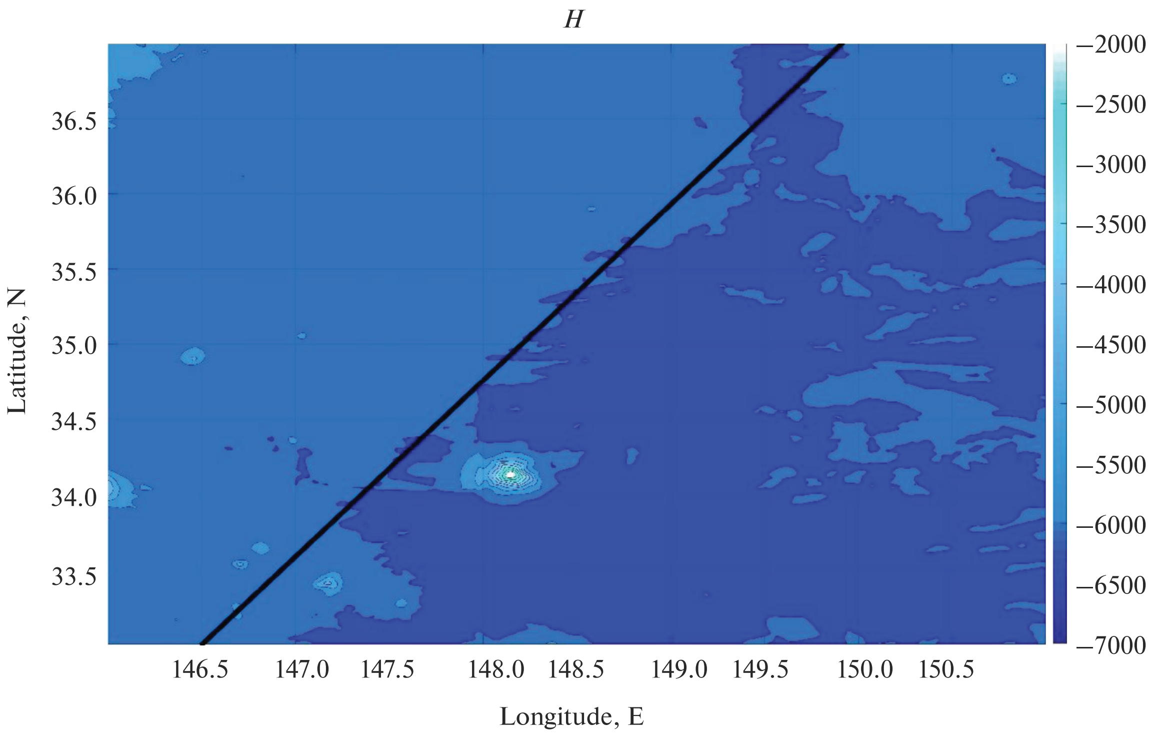


**Fig. 3.** Bottom topography of the study area with various scales of depth $H$ (m).

$\beta$ cos $\theta$. The corresponding graphs are introduced below.

It is clear that the shear flow ($U_{\eta\eta}$) enhances the $\beta$-effect from the north side of the main flow and it reduces from the south side (Fig. 4a). This corresponds well to [14]. There are not such dependencies for other components. In general, the influence of the joint effect of the Earth rotation and bottom topography $\left(\frac{fH_\eta}{H}\right)$ enhances the $\beta$-effect in the whole study area while the joint effect of the shear flow and topography $\left(\frac{U_\eta H_\eta}{H}\right)$ reduces it a bit.

To sum up, the comparison of the graphs in Fig. 4 demonstrates that the most influential component is $\frac{fH_\eta}{H}$ which is responsible of the joint effect of the Earth rotating and topography slope. The numerical values of it are locally bigger than ones of the other terms and on average, the contribution of this component in the area under consideration is 53%. However, $U_{\eta\eta}$ and $\frac{U_\eta H_\eta}{H}$ also affects the wave pattern. Moreover, $U_{\eta\eta}$ can affect significantly at some places and be comparable with the influence of $\frac{fH_\eta}{H}$. On average, the impact of both $U_{\eta\eta}$ and $\beta$-effect is 22% in the area under consideration. The contribution of the term $\frac{U_\eta H_\eta}{H}$ is very insignificant and amounts to 3%.

**Table 1.** The role of the components in the dispersion relation (6) in strengthening or weakening the $\beta$-effect

| Terms | Positive | Negative |
|---|---|---|
| $U_{\eta\eta}$ | Reduces | Enhances |
| $\frac{fH_\eta}{H}$ | Reduces | Enhances |
| $\frac{U_\eta H_\eta}{H}$ | Enhances | Reduces |

## CONCLUSIONS

The problem concerning barotropic Rossby waves in the ocean is developed rather well (see e.g. [14]). However, some subjects need to be studied more. One of these is an interaction of Rossby waves with the flow, and the second one is the influence of topography on propagating of Rossby waves. Transition to non-zonality leads to great difficulties in the main equations for the stream functions because the operator for wave perturbations ceases to be the Hermitian operator. Gnevyshev et al. [7, 9] studied some of these problems, however, their studies related only to zonal cases of shear flow and corresponding bottom topography change.

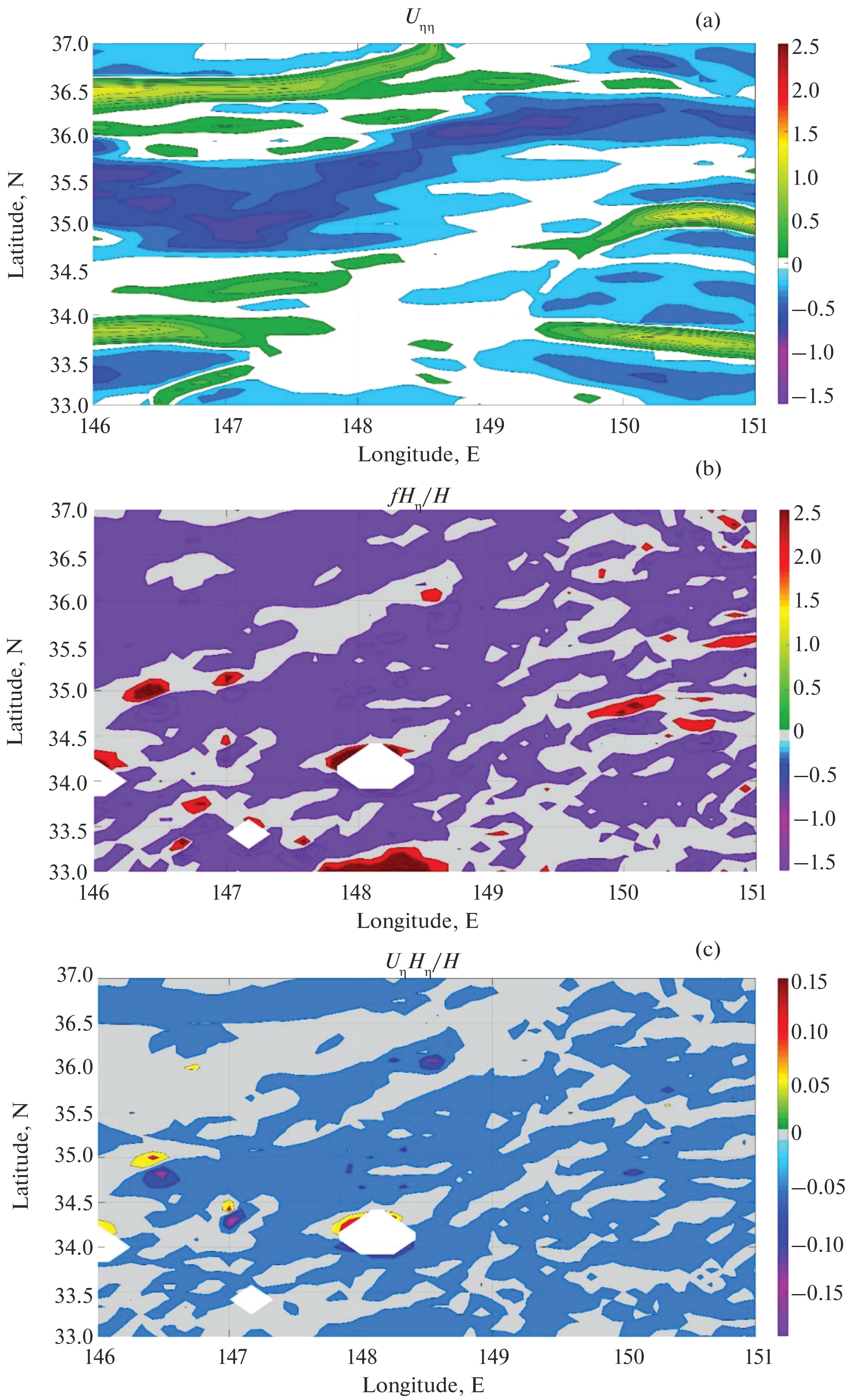


**Fig. 4.** Components normalized to $\beta\cos\theta$: (a) $U_{\eta\eta}$; (b) $\frac{fH_\eta}{H}$; (c) $\frac{U_\eta H_\eta}{H}$.

The current study develops the previous research [7, 9]. It is devoted to non-zonality. It is the joint effect of the influence of non-zonal shear flow and non-zonal bottom topography on Rossby waves propagation. The chosen study area is a branch of Kuroshio bending from a zonal direction on 55°. We estimate terms in dispersion relation compare them with the influence on the β-effect. We reveal that the most influential is the joint effect of the Earth rotation with non-zonal topography, and the second place takes the non-zonal shear flow. The less influential is the joint effect of non-zonal flow with non-zonal topography. We establish that the non-zonal shear flow enhances the β-effect from the north side of the main flow and reduces from the south side. Contrary, the influence of the joint effect of the Earth rotation and bottom topography enhances the β-effect in the study area and the joint effect of the shear flow and topography reduces it slightly. The term $U_{\eta\eta}$ enhances the β-effect from the north side of the main flow and it reduces from the south one. The influence $\frac{U_\eta H_\eta}{H}$ is insignificunt.

Applied the exponential approximation for bottom topography we obtain that the First approximation by Rhines for the Rossby waves is true for the study area because the parameter of the relief transverse change $l_0$ equals 780 km (more than Rossby wave lengths). It allows simplifying the corresponding dispersion relation.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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