Impact of ULF oscillations in solar wind dynamic pressure on the outer radiation belt electrons

A. Y. Ukhorskiy,[1] B. J. Anderson,[1] K. Takahashi,[1] and N. A. Tsyganenko[2]

Received 11 August 2005; revised 22 November 2005; accepted 24 February 2006; published 31 March 2006.

[1] This work addresses radial transport of outer radiation belt electrons due to ULF disturbances of geomagnetic field. A new approach to calculating inductive electric field is developed and implemented using a dynamical model of the storm-time geomagnetic field. The approach is used to analyze the effects associated with solar wind dynamic pressure (P_{dyn}). It is found that P_{dyn} produces large-scale electric fields with maximum intensity at noon and midnight local time. Derived fields are used in test particle calculations of relativistic electron dynamics. The results show that even moderate oscillations of P_{dyn} typical for quiet-time magnetosphere can result in rapid electron scattering across the drift shells, which identifies P_{dyn} as one of the primary mechanisms of radial transport in the belt. Calculations show that electron motion is inconsistent with radial diffusion, and hence a more detailed description is required for accurate predictions of electron fluxes in the belt. **Citation:** Ukhorskiy, A. Y., B. J. Anderson, K. Takahashi, and N. A. Tsyganenko (2006), Impact of ULF oscillations in solar wind dynamic pressure on the outer radiation belt electrons, *Geophys. Res. Lett.*, *33*, L06111, doi:10.1029/2005GL024380.

1. Introduction

[2] Earth's outer radiation belt is populated by electrons of energies $\gtrsim$1 MeV which are referred to as relativistic electrons due to non-negligent values of relativistic corrections in their equation of motion. The outer belt electrons respond nonlinearly to geomagnetic activity. In the course of a storm electron fluxes exhibit erratic variations over several orders in magnitude which may lead to an increase as well as a decrease of the final flux levels [*Reeves et al.*, 2003]. One of the main control mechanisms of large-scale variability of electron fluxes in the belt is radial transport. While radial transport is generally recognized to be important to both electron acceleration and loss, there is little consensus on what its main drivers and the corresponding transport rates are. One reason for the lack of closure is sensitive dependence of relativistic electron motion on dynamic variations in the inner magnetospheric fields which are not treated realistically in most of contemporary radiation belt models. In this paper we develop a consistent framework for modeling dynamics of relativistic electrons in time-dependent disturbed geomagnetic field and then use it in the analysis of radial transport in the belt.

[1]Johns Hopkins University Applied Physics Laboratory, Laurel, Maryland, USA.

[2]Universities Space Research Association, NASA Goddard Space Flight Center, Greenbelt, Maryland, USA.

0094-8276/06/2005GL024380$05.00

[3] Radial transport of charged particles trapped in the geomagnetic field requires violation of the third adiabatic invariant associated with particle's drift around the Earth due to magnetic field gradient and curvature. The third invariant of the outer belt electrons can be violated in the process of resonant interaction of the electron drift motion with geomagnetic field fluctuations in the ULF frequency range. Since the electron drift period greatly exceeds periods of the gyro and the bounce motions the first and the second adiabatic invariants usually remain constant. In this case variation in electron energy is directly related to the change in radial position. Outward motion yields energy loss while inward motion is accompanied by acceleration. While it is generally believed that radial transport of the outer belt electrons is a quasi-linear diffusion process and therefore can be described by the Fokker-Plank equation, the comparison between a test particle and radial diffusion simulations [*Riley and Wolf*, 1992] shows only mediocre agreement for particular storm events.

[4] In the inner magnetosphere electric and magnetic field fluctuations in the ULF frequency range have different source mechanisms and exhibit diverse spatial and temporal structure. They can be broadly classified as either waves or as quasi-periodic oscillations, directly induced by ULF variations in solar wind parameters (e.g., magnetopause compressions). Both ULF waves and induced oscillations can have a substantial impact on radiation belt electrons. There has been a considerable progress in quantifying radial transport rates due to ULF waves based on various observational [*Brautigam et al.*, 2005] and theoretical [*Ukhorskiy et al.*, 2005] techniques. In this paper we study the implications of large-scale induced ULF fields. This requires a continuous global representation of the magnetic and electric fields that is realistic in the inner magnetosphere. For this purpose we develop a new technique of calculating the self-consistent inductive electric field corresponding to time evolution of geomagnetic field models and implement it for *Tsyganenko and Sitnov* [2005, hereinafter referred to as TS05] storm-time magnetic field model.

[5] There are other models such as global MHD models, that naturally yield self-consistent electromagnetic field and are currently used to model electron transport in the outer belt (Y. F. Fei et al., Radial diffusion and MHD-particle simulations of relativistic electron transport by ULF waves in the September 1998 storm, submitted to *Journal of Geophysical Research*, 2005). There are several reasons why TS05 is more suitable for this study. First, TS05 is specifically designed to represent the realistic magnetic field during storms including the strong influence of the ring current. Since in the inner magnetosphere the drift motion of relativistic electrons is mainly controlled by magnetic field

terms it is crucial to have as realistic a representation of the field as possible and the MHD codes do not yet have a ring current module so are not yet suitable for this analysis. Second, TS05 with the added inductive electric field module is computationally efficient allowing a comprehensive study of the effects on the electron dynamics. Finally, TS05 allows independent variation of different physical sources of the magnetic field in the inner magnetosphere, so it is possible to study the effects of different causes of field fluctuations in the model to determine their relative roles in radial transport.

[6] In this paper we focus on the effects due to solar wind dynamic pressure (P_{dyn}). We show that changes in P_{dyn} can induce strong inductive electric fields in the inner magnetosphere. The field amplitude is asymmetric in magnetic local time. It exhibits two distinct maxima one on the dayside and the other on the nightside. To evaluate the response of radiation belt electrons to time varying magnetic and inductive electric fields a test particle approach is used. Particle dynamics are calculated numerically in the guiding center approximation. The results show that P_{dyn} produces a large impact on the outer belt electrons. Drift resonance of P_{dyn}-induced electric fields with radiation belt electrons yields electron radial motion across the drift shells. Large values of transport rates indicate that global magnetospheric compression due to P_{dyn} variations is one of the predominant mechanisms of radial transport in the outer belt. It is also shown, that although in the course of radial motion the electrons from a given drift shell are spread over a range of shells, radial transport is not a diffusion process in the sense that it is not described by the Fokker-Plank equation.

2. Inductive E-Field

[7] In a steady-state magnetosphere the motion of trapped relativistic electrons is mainly defined by the gradient-curvature drift while the $\mathbf{E} \times \mathbf{B}$ force due to convection electric field is generally small. In the time varying case, however, global variations in the geomagnetic field produce large-scale electric fields which may exhibit resonance with a quasi-periodic electron motion. Thus, in the analysis of radial transport in the belt it is important to quantify ULF oscillations in global electric fields, while it is possible to neglect electric fields which can be considered static on the time scales of electron drift motion ($T_D \sim 10$ min for a 1.5 MeV electron at $L = 6$).

[8] In general, electric fields in magnetospheric plasmas have both potential and inductive components. However, according to global MHD simulations [*Slinker et al.*, 1995], the time required for magnetospheric convection to acquire a new steady state (characterized by potential electric field) after an impulsive change in solar wind conditions exceeds 1 hr. On the other hand, spacecraft observations [*Aggson et al.*, 1983] show that electric field oscillations are inductive on minute time scales due to rapid magnetic field changes at substorm onset. Thus, we can assume that ULF variations of electric field induced by global perturbations of geomagnetic field can be estimated using the inductive component of the field.

[9] Previous models of electric fields induced by magnetospheric disturbances [*Birmingham and Jones*, 1968; *Fok and Moore*, 1997] were based on the assumption that magnetic field lines are frozen in a stationary ionosphere. Under this assumption magnetic field lines are identified by their stationary foot points in the ionosphere. In this case electric field at any given point in the magnetosphere can be computed from field line disablement, calculated by tracing the field line from its foot point to the point of interest before and after the disturbance. In the real magnetosphere, however, this assumption is not valid due to presence of ionospheric convection which moves foot points of magnetic field lines. Neglecting the foot point motion in the ionosphere may lead to large errors in field line displacement and associated electric field values at the equator. To avoid this uncertainty we suggest an alternative approach which does not rely on field line tracing. Inductive electric field is estimated directly from Faraday's law with $\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r}, t)$ being an output of a global time-dependent model of geomagnetic field. Since for inductive part of the field $\nabla\cdot\mathbf{E} = 0$, we can introduce a vector potential $\mathbf{\Lambda}$ of electric field, such that $\mathbf{E} = \nabla \times \mathbf{\Lambda}\ (\mathbf{r}, t)$. As in the case of magnetic field, $\mathbf{\Lambda}$ is not uniquely defined. To calculate $\mathbf{\Lambda}$ we impose an additional condition $\nabla \cdot \mathbf{\Lambda} = 0$, which is equivalent to the Coulomb gauge for magnetic field vector potential. In this gauge Faraday's law can be written as the Poisson equation:

$$\nabla^2 \mathbf{\Lambda}(\mathbf{r}, t) = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r}, t). \tag{1}$$

The solution of (1) is used to obtain the expression for electric field in the form of the Biot-Savart integral:

$$\mathbf{E}(\mathbf{r}, t) = -\frac{1}{4\pi c}\frac{\partial}{\partial t}\int_V d^3r' \frac{\mathbf{B}(\mathbf{r}', t) \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}, \tag{2}$$

where the integration is carried out over the domain of the geomagnetic field model. It has to be noted, that for any given model of geomagnetic field the consistency of solution (2) has to be verified separately. Indeed, since model domains are finite, boundary terms in the expression for $\mathbf{\Lambda}$, which vanish in the case of an infinite space, may become significant and violate the imposed gauge $\nabla\cdot\mathbf{\Lambda} = 0$.

[10] To study the impact of global perturbations in the geomagnetic field on the outer belt electrons this approach was implemented for the TS05 model. TS05 can be used as a dynamical model, since its ten control parameters depend on the current value and the history of solar wind conditions (for details see *Tsyganenko and Sitnov* [2005]). Its output is calculated from contributions of seven major magnetospheric current systems driven by different input parameters and varying on different time scales. The most abrupt changes of the field are attributed to variations in P_{dyn} which directly effects the magnetopause surface current and cross-tail current systems. Since the electric field (2) is proportional to the time derivative of the integrated magnetic field, it can be expected that P_{dyn} also produces large inductive electric fields.

[11] TS05 implies spatial coherence of magnetic field variations over the whole domain of the model and therefore does not describe wave phenomena. Thus, the analysis has to be restricted to phenomena that are longer than the characteristic wave propagation time. Since the drift period of relativistic electrons is much longer than the time, 1–3 min, for the fast mode wave to travel through the

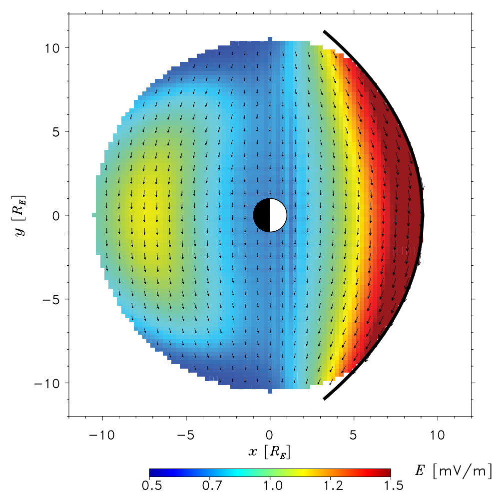

Figure 1. Equatorial electric field calculated from the TS05 magnetic field model. The global compression of magnetic field was simulated by an increase in the solar wind dynamic pressure from 2 to 4 nPa over the period of 4 min.

inner magnetosphere, TS05 is suitable for the analysis of radial transport in the outer belt.

[12] To calculate inductive electric field due to magnetopause compressions we varied only P_{dyn}, while the other nine input parameters were held constant. Integral (2) was computed on a cubical grid with the resolution of 0.25 R_E. To verify whether the obtained estimate is consistent with the imposed gauge, the ratio $\epsilon = \nabla \cdot \mathbf{\Lambda} / |\nabla \times \mathbf{\Lambda}|$ was calculated on the grid for various values of input parameters. It was found that $\epsilon < 10^{-2}$, which validates the assumption $\nabla \cdot \mathbf{\Lambda} = 0$.

[13] A typical pattern of global inductive electric fields generated by P_{dyn} is shown in Figure 1. It shows the distribution of equatorial electric field due to a P_{dyn} increase from 2 to 4 nPa over the time period of 4 min. The field amplitude is shown in color while its direction is indicated by the overplotted arrows. Electric fields exceeding 1 mV/m are found in two extended regions of the inner magnetosphere. The electric field on the day side is associated with the growth of the magnetopause currents while the night side field is driven by the tail current increase. P_{dyn} exhibits fluctuations in the ULF frequency range [e.g., *Sibeck et al.*, 1989] which results in variations of the whole electric field pattern at the time scale of relativistic electron drift.

3. Radial Transport Due to P_{dyn}

[14] To analyze the dynamics of relativistic electrons due to ULF variations of P_{dyn} we used a test particle approach in the guiding center approximation [*Northrop*, 1963]. To simplify calculations and save computational time, the consideration was restricted to the case of equatorial (90° pitch angle) electrons. Such a simplified 2D model is expected to capture key features of the full 3D motion, since ULF oscillations do not violate the first and the second adiabatic invariants of relativistic electrons. In a steady state, equatorial electrons follow contours of constant magnetic field intensity. To specify their drift trajectories we use the parameter $\mathcal{L} = (B_0/B)^{1/3}$, where $B_0 = 0.311G$. In the case of equatorial electrons and quiet-time geomagnetic conditions $\mathcal{L}$ is analogous to the generalized L^* [*Elkington et al.*, 2003], which allows representing electron phase space density (PSD) as: $f = f(\mathcal{L}, \varphi, t)$, where φ is the azimuthal angle.

[15] To isolate the effects due to P_{dyn} we selected a 13-hour quiet-time interval on November 4–5, 2000 with $D_{st} \sim 0$ and $B_z \geq 0$. The time series of P_{dyn} were calculated from 5-min solar wind measurements at the ACE spacecraft (see Figure 2c) and then used to derive time varying magnetic and electric fields according to the approach discussed in previous section. Other input parameters of the field model were put to zero.

[16] The obtained fields were used in simulation of dynamics of 10^4 1 MeV electrons initially located at $\mathcal{L}$ = 5.8 and evenly distributed in local time. To visualize radial transport in the system particle motion was converted into a radial distribution function: $F(\mathcal{L}, t) = \langle f(\mathcal{L}, \varphi, t) \rangle$, where $\langle \ldots \rangle = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \ldots$ is the phase average. Calculation

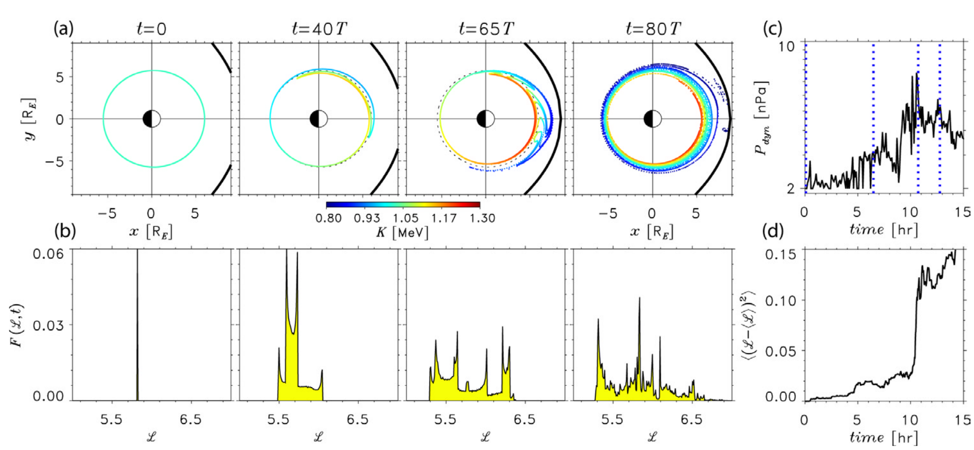

Figure 2. Test-particle simulation of radial transport in the outer belt driven by quiet-time variations in P_{dyn} on November 4–5, 2000. (a) Snapshots of 10^4 electrons in x-y plane at times indicated by vertical dotted lines in Figure 2c; electron energy is shown with color. (b) Electron radial distribution functions $F(\mathcal{L}, t)$ for snapshots in Figure 2a. (c) Time series of P_{dyn} (log scale). (d) The second moment of $F(\mathcal{L}, t)$.

results are illustrated in Figure 2. Figure 2a shows four snapshots of electron dynamics at times indicated by vertical lines in Figure 2c. Electron position is plotted in *x-y* plane, while energy is indicated with color. For each snapshot, $F(\mathcal{L}, t)$ are shown in Figure 2b. Initial conditions are shown in the left most snapshot. Next snapshot corresponds to $t = 40T$ ($\sim$6.5 hr, where T is the electron drift period). At this point electrons form a ring with a single fold corresponding to a four-peak F (the second plot in Figure 2b), which is typical for a motion in the vicinity of a nonlinear resonance. This indicates that at a given $\mathcal{L}$-shell electron dynamics are controlled by the drift-resonance with P_{dyn}-induced electric field oscillations.

[17] In time, while the first fold stretches in local time new folds emerge and stretch in a self-similar manner spreading electrons in radial direction and mixing their drift phases (see the third and the forth snapshots). Thus, in 80 drift periods ($\sim$13 hr) electrons spread over $\Delta\mathcal{L} \simeq 1.2$. $F(\mathcal{L}, t)$ exhibits a complex structure with multiple peaks which move and mix with time (the third and the forth plots in Figure 2b). A multi-peak radial profile may be a result of strong nonlinearity in the system; diffusion in chaotic systems (e.g., standard map) often produce fractal structures in coarse-grained PSD [*Zaslavsky*, 2002]. However, if electron radial transport is indeed a diffusion, evolution of $F(\mathcal{L}, t)$ is described with the Fokker-Plank equation and its second moment must exhibit linear growth with time. The second moment $\langle(\mathcal{L}-\langle\mathcal{L}\rangle)^2\rangle$ is shown in Figure 2d. Its complex time profile and evident dependence on P_{dyn} (Figure 2c) indicate that electron radial motion is not a diffusion process. Thus, the description of electron motion due to global ULF perturbations in geomagnetic field cannot be reduced to the Fokker-Plank equation, evolution of the full phase space density must be considered.

4. Conclusions

[18] In this paper we developed a new method for calculating inductive electric field due to geomagnetic field disturbances and implemented it for the time-dependent TS05 magnetic field model. Calculated fields were used in a test particle simulation of Earth's outer radiation belt. In particular, we examined the role of solar wind P_{dyn} in radial transport of relativistic electrons in the belt. It was shown, that ULF variations in P_{dyn} induce large-scale electric fields which can exhibit drift resonance with relativistic electrons and drive their radial motion. Even moderate fluctuations of P_{dyn} typical for quiet solar wind conditions can efficiently scatter particles across the drift shells, which identifies P_{dyn} as one of the primary drivers of radial transport in the belt.

[19] Current modeling efforts of global storm-time dynamics of the outer belt heavily rely on the assumption of radial diffusion. However, our analysis suggests that even though radial motion might be stochastic, it cannot be described by a diffusion equation. A fully kinetic treatment of electron motion is required for accurate predictions of storm-time evolution of the belt.

[20] Although these results demonstrate the relevance of global ULF disturbances in geomagnetic field to radial transport in the outer belt, there are many fundamental questions which are yet to be addressed. In particular, it is not clear whether electron radial transport is attributed to a random character of solar wind drivers or to a strong nonlinearity of electron interaction with the oscillating fields. Another issue is the role of storm-time magnetospheric current systems, which control the magnetic field in the inner magnetosphere and therefore may impact radial motion of radiation belt electrons. It is also important to analyze the discrepancy of electron transport and radial diffusion, to determine whether a diffusion approach can still be used as a description of statistically averaged transport properties even though the fundamental physical process is not a diffusion.

[21] **Acknowledgment.** The research is supported by NSF grants ATM-0318556, ATM-0540121 and NASA grant NAG5-13509.

References

Aggson, T. L., J. P. Heppner, and N. C. Maynard (1983), Observations of large magnetospheric electric fields during the onset phase of a substorm, *J. Geophys. Res.*, *88*, 1981.
Birmingham, T. J., and F. C. Jones (1968), Identification of moving magnetic field lines, *J. Geophys. Res.*, *73*, 5505.
Brautigam, D. H., G. P. Ginet, J. M. Albert, J. R. Wygant, D. E. Rowland, A. Ling, and J. Bass (2005), CRRES electric field power spectra and radial diffusion coefficents, *J. Geophys. Res.*, *110*, A02214, doi:10.1029/2004JA010612.
Elkington, S. R., M. K. Hudson, and A. A. Chan (2003), Resonant acceleration and diffusion of outer zone electrons in an asummetric geomagnetic field, *J. Geophys. Res.*, *108*(A3), 1116, doi:10.1029/2001JA009202.
Fok, M. C., and T. E. Moore (1997), Ring current modeling in a realistic magnetic field configuration, *Geophys. Res. Lett.*, *24*, 1775.
Northrop, T. G. (1963), *The Adiabatic Motion of Charged Particles*, Wiley-Interscience, Hoboken, N. J.
Reeves, G. D., K. L. McAdams, R. H. W. Friedel, and T. P. O'Brien (2003), Acceleration and loss of relativistic electrons during geomagnetic storms, *Geophys. Res. Lett.*, *30*(10), 1529, doi:10.1029/2002GL016513.
Riley, P., and R. A. Wolf (1992), Comparison of diffusion and particle drift description of radial transport in the Earth's inner magnetosphere, *J. Geophys. Res.*, *97*, 16,865.
Sibeck, D. G., et al. (1989), The magnetospheric response to 8-minute period strong-amplitude upstream pressure variations, *J. Geophys. Res.*, *94*, 2505.
Slinker, S. P., J. A. Fedder, and J. G. Lyon (1995), Plasmoid formation and evolution in a numerical simulation of a substorm, *Geophys. Res. Lett.*, *22*, 859.
Tsyganenko, N. A., and M. I. Sitnov (2005), Modeling the dynamics of the inner magnetosphere during strong geomagnetic storms, *J. Geophys. Res.*, *110*, A03208, doi:10.1029/2004JA010798.
Ukhorskiy, A. Y., K. Takahashi, B. J. Anderson, and H. Korth (2005), The impact of toroidal ULF waves on the outer radiation belt electrons, *J. Geophys. Res.*, *110*, A10202, doi:10.1029/2005JA011017.
Zaslavsky, G. M. (2002), Chaos, fractional kinetics, and anomalous transport, *Phys. Rep.*, *371*, 461.

B. J. Anderson, K. Takahashi, and A. Y. Ukhorskiy, Johns Hopkins University Applied Physics Laboratory, Johns Hopkins Road, Laurel, MD 20723, USA. (aleksandr.ukhorskiy@jhuapl.edu)
N. A. Tsyganenko, Universities Space Research Association, NASA GSFC, Greenbelt, MD 20771, USA.