Historical Overview of the Speed-gradient Method Applications for Nonlinear Control Problems

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***Abstract*— The present surveyed paper provides a historical overview of the Speed-gradient method and its applications to nonlinear control problems since mid-1970-th, when the method was originated, till the present days. It is demonstrated that it is widely used an efficient and a useful tool for solving various problems in the field of the nonlinear control.**

***Index Terms*— speed-gradient, nonlinear control, mechanical systems, oscillators, distributed parameter systems**

The first publications related to SG-algorithms appeared in 1978. General formulations were proposed simultaneously and independently by Alexander Fradkov and Yuri Neimark in January, 1978 at the 9th All-Soviet school on adaptive systems [1], [2]. Some related formulations for the identification problem were suggested in [3].

First (yet distinct) stability results were published in [4] and in [5]. For the special case affine time-invariant controlled system $\dot{x} = f(x) + g(x)u$ and positive definite goal function $V(x)$ the control algorithm $u = -L_g V(x)$ was proposed in [6]. It is sometimes called "LgV" or "Jurdjevic–Quinn" control. Stability result in [6] is related to the case $\dot{V} \le 0$ and requires some detectability conditions (so called "Jurdjevic-Quinn" conditions).

Non-affine and time-varying case was first studied in [5] for differential form of the SG-algorithms and in [7], [8] for the finite form.

Various types of the speed gradient algorithms were proposed as a set of designing schemes and their applicability conditions by Alexander Fradkov in the framework of the unified *Speed-gradient method* (the SG method). This method was originated in [5] as a universal approach for solving various control problems, originally with a focus on designing the adaptation and identification algorithms, cf. [5], [7], [9]. The basic idea of the method is expressed by [5] as follows: "The paper is concerned with a scheme for design of adaptive control algorithms whereby motion is organized in the space of parameters to be adjusted along the gradient of the speed of change of a evaluative functional". During the subsequent years, the method was further developed by A. Fradkov and his colleagues for elaborating the various schemes of adaptation, non-linear control, identification and synchronization. This method has found application in the works by many researchers worldwide. In [10], the SG method is recognized as as the method, which "enables a transparent trade-off between control performance and design parameters. Furthermore the steps for controller design results are in general simple ... it has become widespread in other multiple successful applications in adaptive control mainly in Physics and Mechanics".

In the present paper the following applications of the SG method to nonlinear control problems are briefly exposed in the historical perspective: design methods for control of generic nonlinear systems; control of mechanical systems and nonlinear oscillators; control and synchronization of chaotic systems; control of spatially-distributed nonlinear systems.

The interested readers may find the detailed description of the method in the dedicated literature, see [5], [9], [11]–[13], for mentioning a few.

Below the most most significant publications of the SG method to nonlinear control problems are listed in the in chronological order.

Works of 1996 – 1999: In [14], a unified framework of the SG method is used for description and comparison of adaptive control algorithms for robot manipulators. The concept of swinging control, meaning achievement of arbitrary large level of the objective function by arbitrary small control level is introduced, and extension of the SG method to oscillating systems with energy-based objective functions is presented in [15]. This approach is applied in [16] to control of oscillations of mechanical systems: physical pendulum and a pendulum on a cart. By these examples, the properties of the SG-based control in the differential and finite forms are compared. The problems of synchronization and control of chaotic systems with uncertain parameters are considered as those of nonlinear adaptive control in [17], where the GS solutions to these problems are found. An example of master-slave synchronization of the pair of forced Duffing-type systems is presented. Book [9] is, possibly, the first monograph, devoted to control of chaotic systems. It gives an exposition of the field of control of oscillatory and chaotic systems, taking in a view numerous applications in mechanics, laser and chemical technologies, communications, biology and medicine, economics, ecology. The proposed control design methods are based on the concepts of Lyapunov functions, the SG method and the Poincare maps. The numerous applications to control of nonlinear (including chaotic) oscillations age presented: coupled pendula; brusselator; Lorenz, Van der Pol, Duffing, Henon and Chua systems; communications;

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growth of thin films; synchronization of chaotic generators based on tunnel diods; stabilization of swings in power systems; increasing predictability of business-cycles, are also presented.

The possibilities of studying nonlinear physical systems by feedback action are discussed in [18]. The feedback resonance phenomenon in nonlinear oscillators and the SG-based method of creating feedback resonance are described and illustrated by the examples of feedback resonance in 1-DOF system (controlled pendulum) and in 2-DOF system (two coupled pendulums).

Works of 2000 – 2004: The investigations on adaptive control of nonlinear chaotic systems in the presence of model uncertainty are continued in [19], where control compensation is incorporated into the SG-based design of the adaptive controller. The authors of [20] consider the problem of the global and local stabilization of invariant sets for general nonlinear controlled systems. New state feedback stabilizing controllers and sufficient conditions of asymptotic stability of a goal set with the specified region of attraction are proposed. The proofs of the obtained results are based on the analysis of closed-loop system with the SG controllers. The results on stabilization of invariant sets for nonlinear systems based on the SG method and the notion of V-detectability are overviewed and extended in [21]. The stabilization problem of the upright position of the spherical pendulum is treated in detail in [22]. It is shown that for any smooth feedback control derived by the SG algorithm with the objective to stabilize Ω_{st}, the closed loop system has a limit cycle Γ, which does not belong to the desired attractor Ω_{st}. The synchronization problem for two 1-DOF pendulums coupled with a weak spring is considered in [23]. The controller is designed with use of the energy-based SG method. Although experimental results showed that the method succeeded in achieving the objective, the mechanism of synchronization was not clear. In this study, the contracted dynamics of the whole system is analyzed and properties of the system are investigated. The energy-based SG method is also employed in [24], [25] to the problem of swinging the Furuta pendulum up. In [26] a framework for system analysis and design is described based on nonlinear system models and nonperiodic signals generated by nonlinear systems. To this end, the notion of the *excitability index* is introduced. The SG algorithms of creating feedback resonance in nonlinear multi-DOF oscillators are described. For strictly dissipative systems bounds of energy and excitability change by feedback are established.

Works of 2005 – 2011: The idea of [26] was implemented in [27] for a special case of numerically estimating the excitability index for a second-order linear oscillator. It is shown that the SG excitation provides an exact solution to the maximum energy problem. In [28] the SG algorithm to control non-linear oscillations of a dynamic system for both regulation and tracking problems by the example of chaotic Colpitts oscillator is presented. A method for rendering passive nonlinear affine-incontrol discrete-time systems based on the discrete-time version of the SG algorithm is proposed in [29], [30]. In [31], a method for control of underactuated nonlinear systems is proposed, based on introducing artificial invariants and using SG algorithms. Its application is illustrated by an example of cart-pendulum oscillations stabilization around the upper equilibrium. In [32] the SG approach is used for dosing system design based on the mathematical model for polycystic ovary syndrome (PCOS). The the SG method is used in [33] for improvement of the time-delayed feedback control by adaptively tuning the controller feedback gain. The Energy-based SG control scheme is used in [34] for designing the swinging-up controller, stabilizing the Reaction Wheel Pendulum about unstable (inverted) position for arbitrary initial conditions. The SG-based inverse optimal control approach for the asymptotic stabilization of discrete-time nonlinear systems is presented in [35].

Works of 2012 – 2015: In [36] the mechanism of entrainment to natural oscillations in a class of (bio)mechanical systems described by linear models is investigated. The SG nonlinear control strategy is analyzed providing the system oscillation in resonance mode with a natural frequency. The possibilities of energy-based SG control implementation for nonlinear oscillations are studied in [37] by the example of controlling the cart-pendulum system. Paper [38] is devoted to networks of delay-coupled Stuart-Landau oscillators. The SG method is used for deriving the adaptive algorithm for an automatic adjustment of the coupling phase with which different states of synchronization, e.g., in-phase oscillation, splay, or various cluster states, can be selected. The adaptive complex scaling factors schemes based on the SG method for the real drive chaotic system and complex response chaotic system are proposed and studied in [39]. Paper [40] presents a SG control strategy for swinging the Furuta pendulum up towards the desired upright position, which uses only directly measured coordinates. The possibility to apply the SG control to elastic structures is analyzed in [41] uses two serial springs for hopping show in simulation the viability of our approach. Proposed here combination of SG control with learning is a novel approach which opens interesting perspectives for further research on passive control. In [42], a control problem for a nonstationary dynamic system with nonfixed termination time and terminal functional in the presence of uncertain parameters is considered. The SG principle is employed, providing a guaranteed value of the quality functional. Paper [43] is concerned with Direct Gradient Descent Control (DGDC) of general nonlinear systems. To improve the convergence, we extend the DGDC by decreasing both the performance function and its time derivative. In [44] the 'nonsmooth' versions of SG-algorithm in differential and finite form are formulated. Conditions for the control goal achievement are derived. In [45] a detailed analysis of the application of the passification approach to the problem of rendering the hyperbolic upright equilibrium of the simplified model of the Furuta pendulum globally attractive. The VSS-like modification of the SG method, stabilizing regulator is suggested and examined.

Works of 2016 – present: In [46] a new distributed SG control algorithm for the sine-Gordon equation is proposed, creating the antikink traveling wave, allowing suppressing defects and obtaining stable propagation of an antikink in the form of the exact traveling wave solution. The boundary energy control problem for the sine-Gordon equation is posed in [47], [48]. The SG control laws with smooth and nonsmooth goal functions in the differential and finite forms are proposed. The conditions ensuring control goal are established. An important feature of the proposed control is the fact that it is continuous along trajectories of the closed-loop system. Theoretical results are illustrated by example of nonsmooth control for a non-affine in control pendulum-like system. In [49] adaptive Complex function projective synchronization (CFPS) schemes and parameters update laws based on the SG method are designed. The convergence factors and pseudogradient condition are added to regulate the convergence speed and increase robustness and the SG method extension from real field to complex field is proposed. The problem of event-triggered sampled-data nonlinear control of Hamiltonian system is considered in [50] by the example of controlling the pendulum's energybased on the SG speed gradient method. In [51] a distributed feedback control algorithm based on the SG method, achieving nonlinear wave localization is developed. This algorithm is extended to coupled nonlinear partial differential equations to obtain consistent localized wave solutions at rather arbitrary initial conditions. An energy control problem is analyzed in [52] setting, where the SG method is developed and justified in PDE setting for nonlinear sine-Gordon equation. The applicability of the Krasovskii-LaSalle principle is established for the resulting sliding-mode closed-loop system. The state feedback control law of this work is numerically studied in [53] for the case of the level quantization and/or time sampling. In [54] the boundary energy control problem for the sine-Gordon and the nonlinear Klein–Gordon equations considered and the SG control laws with smooth and nonsmooth goal functions are proposed. The second control law is proved to steer the system to any required nonzero energy level in finite time. The energy control problem for the nonlinear sine-Gordon model driven by several in-domain actuators is considered in [55]. The SG method is now generalized to the in-domain actuation, aiming to pump/dissipate the energy of the model to a desired level. In [56] the problem of pendulum's energy SG-based control in presence of an irregular input disturbance is considered. The main result is precise estimates for an initial set and a limit set s.t. all the solutions starting in the initial set will enter the limit set in a finite time.

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