

# The Galactic Center stars parameters inference

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**1. Introduction & Obs. data**

**2. Modeling**

**3. MCMC Inference**

First of all, it is necessary to introduce the Bayes rule

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)} = \frac{p(\theta)p(X|\theta)}{\int p(\theta)p(X|\theta)d\theta}$$

All of the Bayesian sampling techniques are based on this theorem. It is useful to summarize for a vector of parameters  $\theta$  that we want to estimate and  $X$  represents the observational data. The  $p(\theta)$  is called a prior distribution and the parameter  $\theta$ . It is the probability distribution that represents our beliefs of the parameter  $\theta$  before we get any evidence. The  $p(X|\theta)$  term, which is calculated using the Bayes rule, is called a posterior distribution. It shows us how our beliefs update after taking into account the observational data. The  $p(\theta)$  is called a likelihood function. It is important that the observational data  $X$  is considered to be fixed, while  $\theta$  has varying quantity, which is usually a function of  $\theta$ . The dimensionality of  $X$  is called a  $n$  regardless of likelihood and it should be considered as a secondary constant. In the parameter space  $\theta$ , multidimensional it is a complex standard. To compare the magnitude of likelihood, the Bayes rule can be written as a simplified way

**posterior  $\propto$  prior  $\times$  likelihood**

Since it is difficult to compare the Bayes rule through that, there are other complex and methods to estimate the posterior distribution. MCMC is a series of the class of algorithms that use the Markov chain to obtain a sample of the observed or unobserved, the posterior distribution.

The prior distributions are usually set to be uniform distributions in a given range for each parameter, and the likelihood function is defined as

$$p(\text{data}|\theta) = \exp(-\theta \sum \text{data}_i)$$

**4. Posterior distributions**

**5. The Spacemaker**

**6. Conclusion**

The MCMC parameters are estimated to be roughly equal to 1, which is just not reasonable (another realization of General Relativity, this time in the vicinity of the subatomic compact object).

The Mass of Sag B\* and the Distance to the Galactic Center are also estimated. Though, their estimates are not to be believed from other results because these are the general and theory parameters were also the results values.

AUTHOR INFORMATION | ABSTRACT | REFERENCES | CONTACT AUTHOR | CITE POSTER

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# 1. INTRODUCTION & OBS. DATA

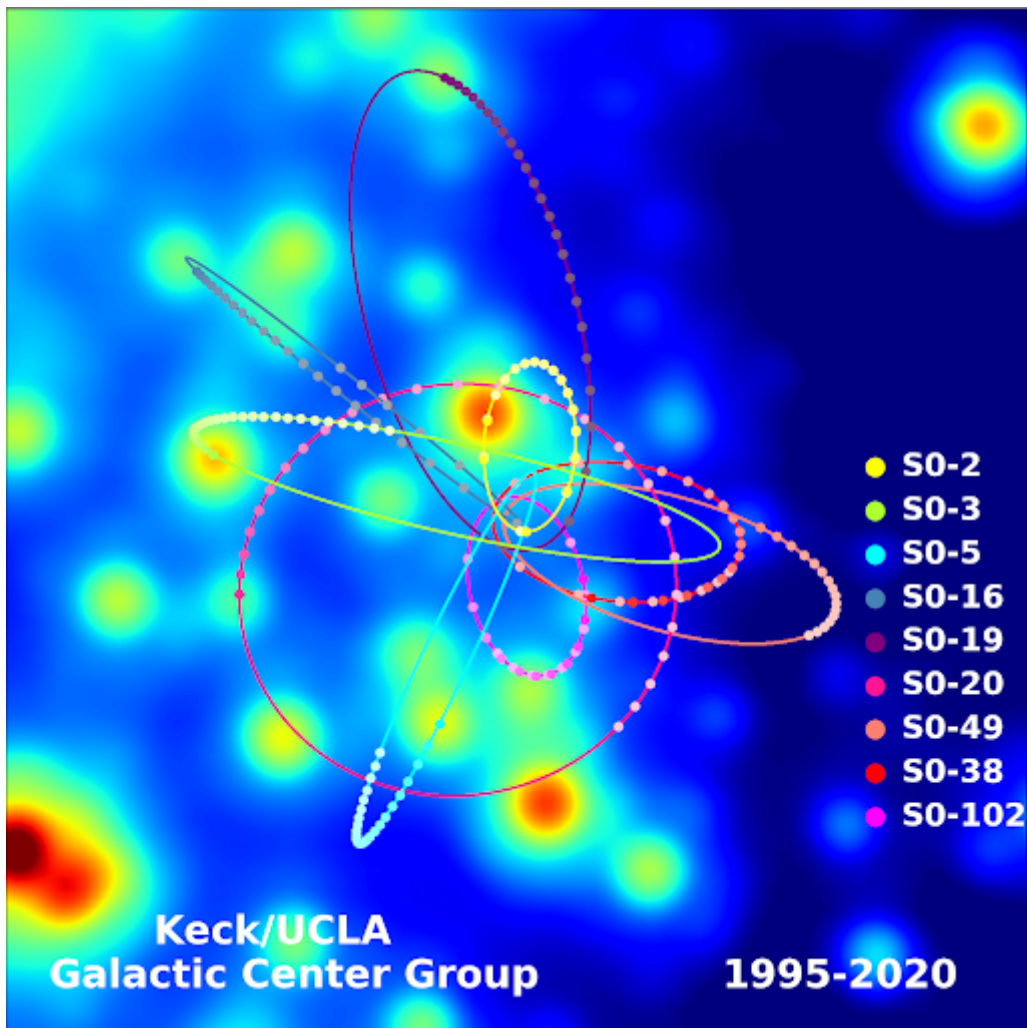


Figure 1. The visual positions and modeled trajectories for several S-stars.

The main subject of this work is the Galactic Center star cluster, also known as the S-stars. The observations of these stars, which began in the early '90s, revealed the existence of the supermassive compact object at the Center of our Galaxy. The discoverers of this phenomenon, Prof. Dr. Reinhard Genzel and Prof. Andrea Ghez, were awarded half of the 2020 Nobel Prize in Physics.

Accurate investigation of the stars' orbital motion can provide the opportunity to test different gravitational theories.

There are many objects in the S-stars cluster. We will point our interest to the stars S2 (S0-2), S38 (S0-38), and S55 (S0-102), as they have relatively close orbits, and the S2 star motion is studied in greatest detail.

The dataset that we use is the same as in works [1] and [2] and consists of the S-stars data published in works [3-6].

## 2. MODELING

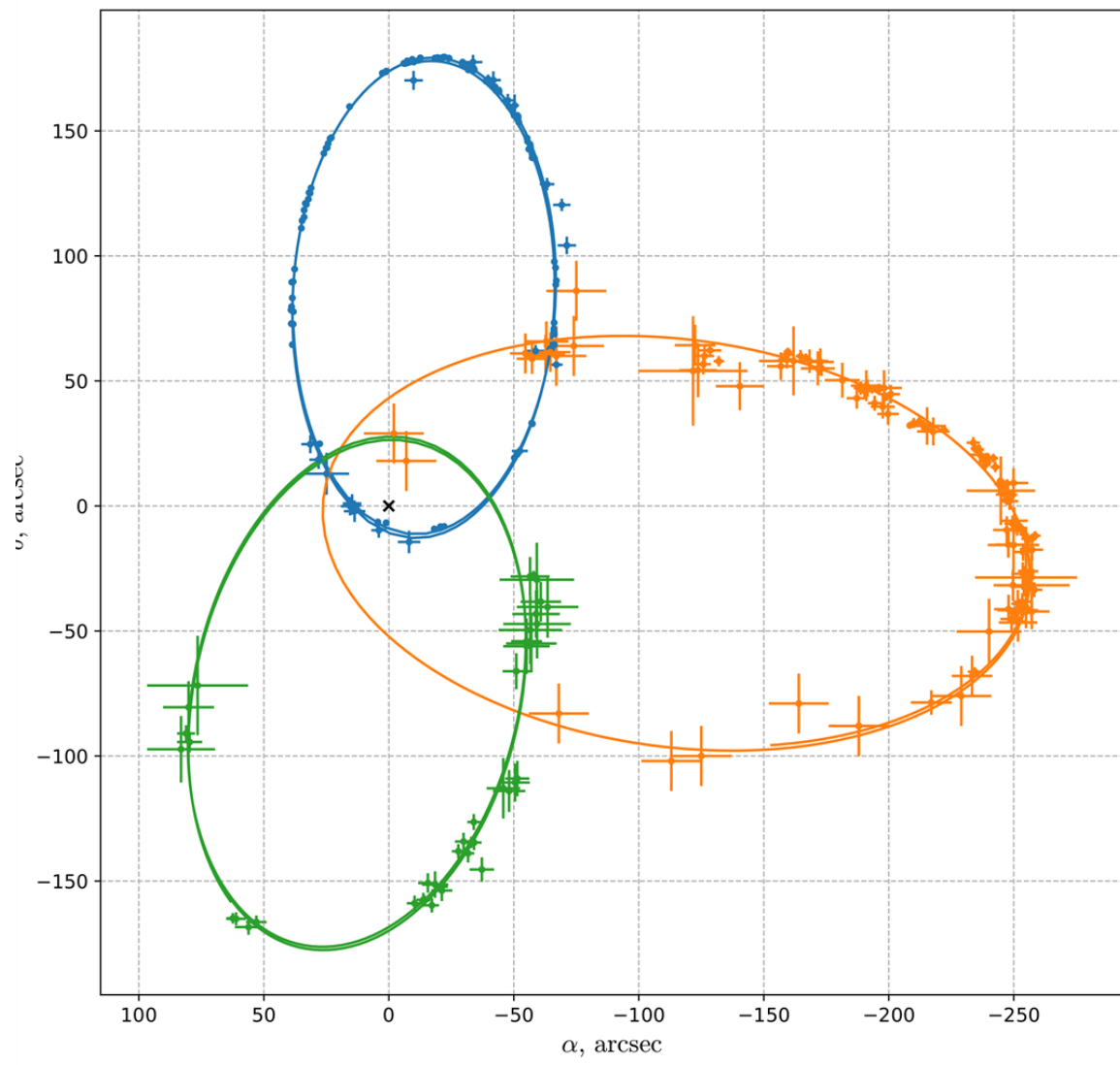


Figure 2. The best-fit trajectories of the S-stars

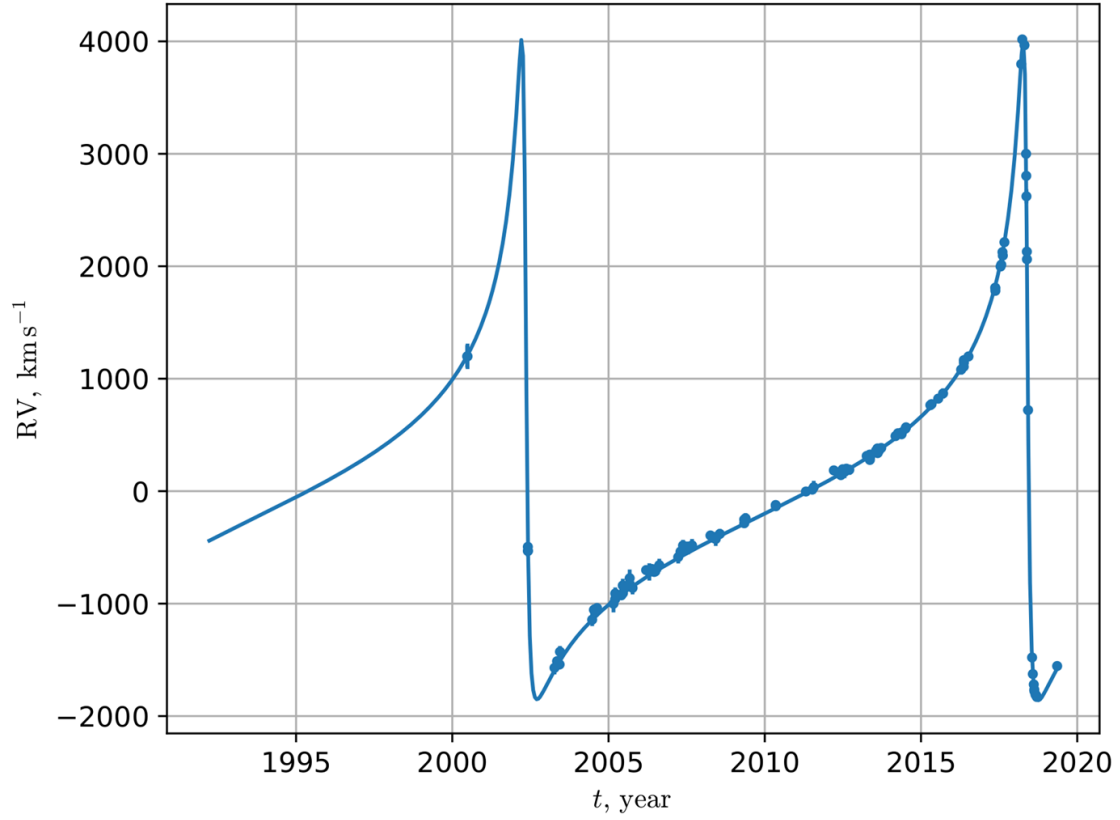


Figure 3. The best-fit RV curve of the S2 star.

Our approach is to define a model depending on several parameters that we could infer using the Bayesian sampling techniques.

The model has these steps:

1. Integration of the PPN Equations of Motion on a plane. This step requires the initial conditions, which is a phase vector of 4 components  $(x_0, y_0, v_{x0}, v_{y0})$ , the mass  $M$  of the supermassive compact object Sgr A\* and the 2 PPN parameters  $\beta$  and  $\gamma$ 

$$\ddot{\mathbf{x}} = -\nabla\varphi_N \left( 1 + 2(\beta + \gamma) \frac{\varphi_N}{c^2} + \gamma \frac{\dot{\mathbf{x}}^2}{c^2} \right) + 2(\gamma + 1) \left( \nabla\varphi_N \cdot \frac{\dot{\mathbf{x}}}{c} \right) \frac{\dot{\mathbf{x}}}{c}$$
2. The next step is to rotate the orbital plane using the given inclination  $i$  and the longitude of the ascending node  $\Omega$ . To define a single star's orbit, we have to use a total of 6 parameters. Then the coordinates must be corrected using the initial position  $(x_{\text{SgrA*}}, y_{\text{SgrA*}})$  and the velocity vector  $(v_x, v_y, v_z)_{\text{SgrA*}}$ . And after that, we have to correct the difference in reference frames using 4 new alignment parameters.
3. After we obtained the 3D trajectories, we should divide the  $x$  and  $y$  coordinates by the distance  $R_0$  to the Galactic Center to obtain the visual RAs and DECs. Also, the Rømer time dilation must be taken into account at this step.
4. The obtained radial velocities must be transformed into observable values taking into account the gravitational redshift and the relativistic Doppler effect using the formula
$$\text{RV}_{\text{obs}}/c = \text{RV}_{\text{model}}/c + \frac{\varphi_N}{c^2} + \frac{v^2}{2c^2}$$

This model is described in more detail in [1].

### 3. MCMC INFERENCE

First of all, it is necessary to introduce the Bayes rule

$$p(\theta|X) = \frac{p(\theta)p(X|\theta)}{p(X)} = \frac{p(\theta)p(X|\theta)}{\int_{\Theta} p(\hat{\theta})p(X|\hat{\theta}) d\hat{\theta}}$$

All of the Bayesian sampling techniques are based on this theorem. Here  $\theta$  is a parameter (or a vector of parameters) that we want to estimate and  $X$  represents the observational data. The  $p(\theta)$  is called a prior distribution of the parameter  $\theta$ . It is the probability distribution that represents our beliefs of the parameter  $\theta$  before we get any evidence. The  $p(\theta|X)$  term, which is calculated using the Bayes rule, is called a posterior distribution. It shows us how our beliefs update after taking into account the observational data. The  $p(X|\theta)$  is called a likelihood function. It is important that the observational data  $X$  is considered to be fixed, while  $\theta$  is a varying quantity, so this is actually a function of  $\theta$ . The denominator  $p(X)$  is called a marginalized likelihood and it should be considered as a normalizing constant. If the parameter space  $\Theta$  is multidimensional it is a complicated task to compute the marginalized likelihood. The Bayes rule can be written in a simplified way

posterior  $\propto$  prior  $\times$  likelihood

Since it is difficult to compute the Bayes rule formula itself, there are other computational methods to estimate the posterior distribution. MCMC is a name of the class of algorithms that use the Markov chains to obtain a sample of the desired (in our case, the posterior) distribution.

The prior distributions are usually set to be the uniform distributions in a given range for each parameter. And the likelihood function is defined as

$$p(\text{data}|\theta) = \exp(-0.5\chi^2(\text{data}, \theta))$$

where

$$\chi^2(\text{data}, \theta) = \sum \left( \frac{y_{\text{data}} - y_{\theta}}{\sigma_{\text{data}}} \right)^2$$

## 4. POSTERIOR DISTRIBUTIONS

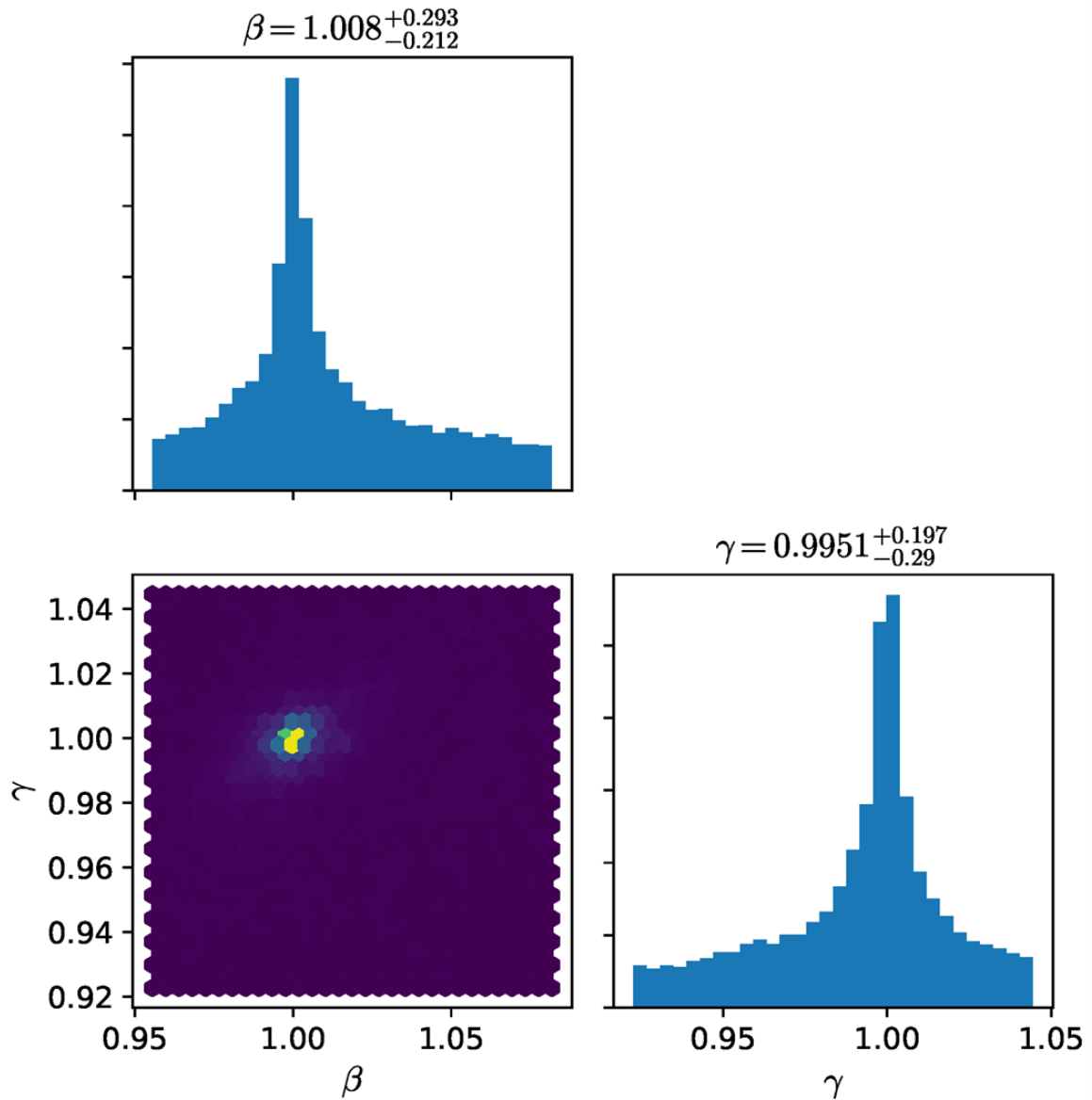


Figure 4. The posterior distributions of the PPN parameters  $\beta$  and  $\gamma$ . The parameters tend to be equal to 1.0, but the accuracy is relatively low.

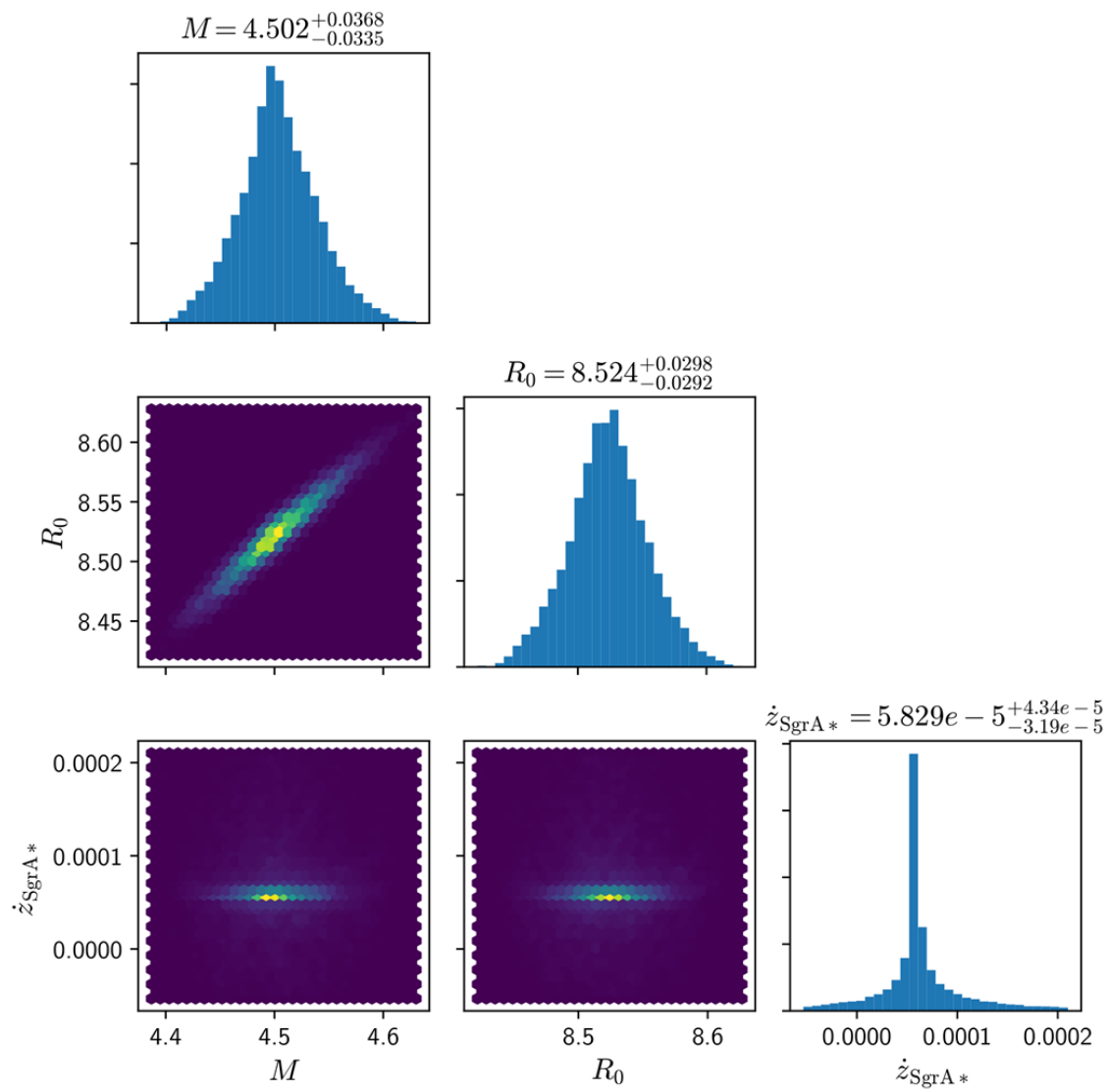


Figure 5. The posterior distribution projections on the planes of parameters  $M$ ,  $R_0$ , and radial velocity of Sgr A\*. We can see that these parameters are highly correlated.

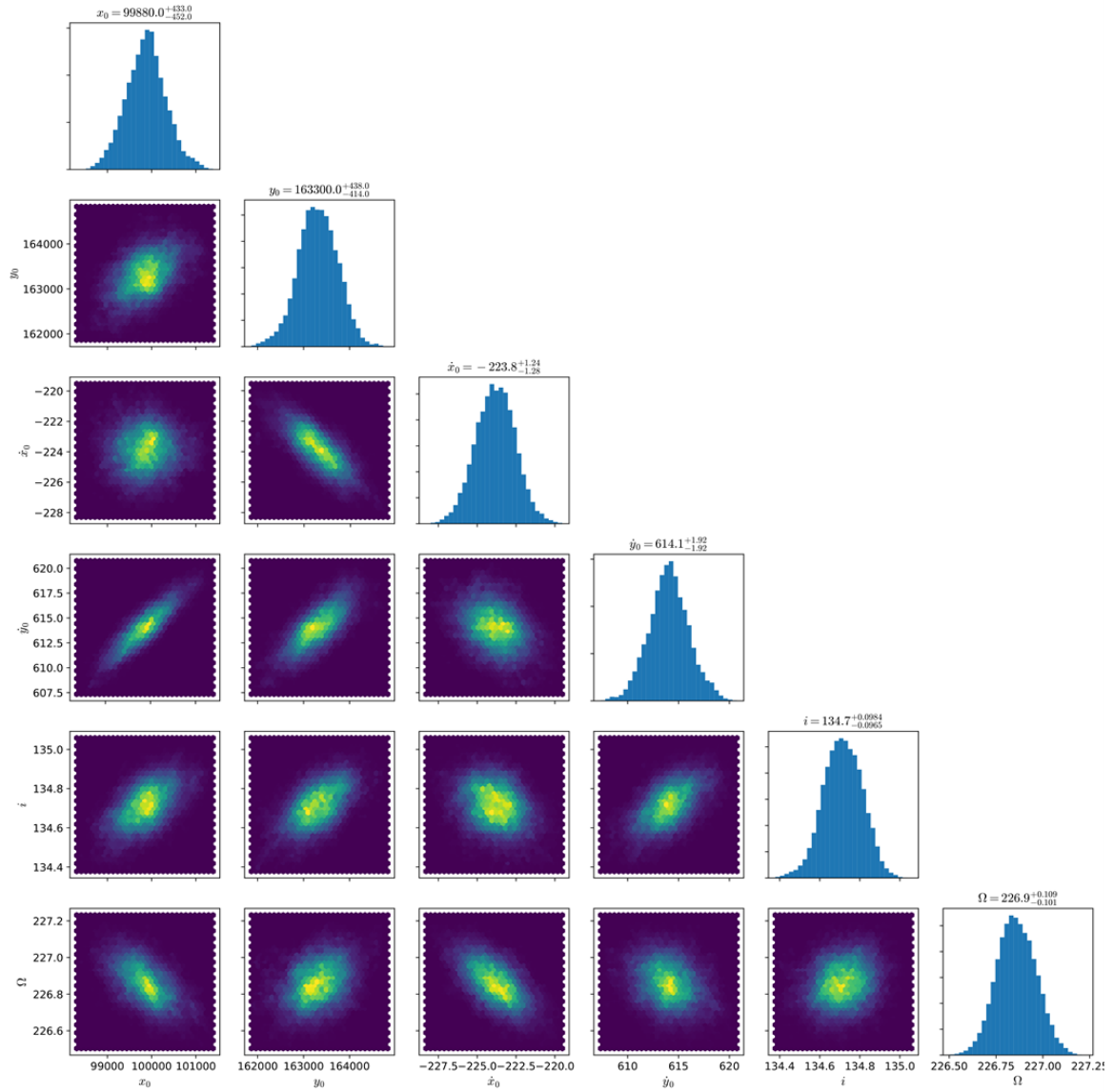


Figure 6. The posterior distributions over the space of the orbital parameters of the S2 star. These parameters are almost independent and they are very well-defined.

We have used the Goodman-Weare MCMC sampler [7] implemented in the `AffineInvariantMCMC.jl` package for Julia language [8]. Our run contained 300 independent chains (so-called 'walkers') and the process was about 2000 iterations.



## 5. THE SPACEWALKER



Figure 7. The example of SW projection of the ranged posterior sample to the leading axes of parameters  $\Omega$  and  $i$  of S2 star animation.

A human's mind is capable to imagine and think of 3D and fewer dimension objects. But when it is necessary to study something more dimensional, it becomes much harder. Since the parameter space in the vast majority of physical problems has a very high dimension, it is difficult to study its objects' properties. That is why the sets of parameter vectors from the high dimensional parameter space are very complicated objects to think about. The usual approach to study such sets is to make a *corner plot*. The examples of such plots are figures in the previous box. But with these corner plots, we can only look at the projections on the 2D spaces of different pairs of parameters. One can also construct a projection on a 3D subspace of a certain three parameters. But it is difficult to think about 4D, 5D projections, or even about the full parameter space (which is 31D in our case). That is the reason why we need cognitive graphics tools.

The Spacewalker software is a tool that allows one to study the multidimensional clouds in the parameter space (unfortunately, only available in Russian).

## 6. CONCLUSION

The PPN parameters are estimated to be roughly equal to 1, which is (yet not so accurate) another confirmation of General Relativity, this time in the vicinity of the relativistic compact object.

The Mass of Sgr A\* and the Distance to the Galactic Center are also measured. Though, their estimate seems to be different from other results because in our case the gravitational theory parameters were also the variable values.

The dynamical visualization of cognitive graphics (the SW software) allows us to study the properties of multidimensional clouds of parameters by rotating the clouds in the multidimensional space and projecting it to certain 2D planes with the leading axes of chosen parameters.

# AUTHOR INFORMATION

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# ABSTRACT

The S-stars, located in the center of our Galaxy, are a truly unique observational phenomenon. They are located close to the relativistic compact object, in a strong enough gravitational field, so there exists an opportunity to test gravitational theories using their observational data. Our approach is to solve the inverse problem and infer several parameters, that include the orbital characteristics of stars, the mass of the central object, and the distance to the Galactic Center, and also the PPN parameters  $\beta$  and  $\gamma$ . Constraining the latter two parameters is a classical test of General Relativity. We suggest the dynamical visualization of cognitive images to help us to study and understand the multidimensional clouds of parameters (the posterior samples).

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