Kari Astala (Aalto University)

Non-linear Beltrami equation

The non-linear Beltrami equation parametrises all planar elliptic systems of PDE's, and arises as a key method for wide range of topics, from calculus of variations to geometry of scaling limits of random tilings. In this talk I discuss these equations from a general point of view, and in particular, establish a new and unexpected regularity result for their solutions. The talk is based on joint work with Albert Clop, Daniel Faraco, Jarmo Jääskeläinen and Aleksis Koski.

Anton Baranov (Saint Petersburg State University)

Spectral synthesis for systems of exponentials and reproducing kernels

Let x_n be a complete and minimal system of vectors in a Hilbert space H. We say that this system is hereditarily complete or admits spectral synthesis if any vector in H can be approximated by linear combinations of partial sums of the Fourier series with respect to x_n. It was a long-standing problem whether any complete and minimal system of exponentials in $L^2(-a,a)$ admits spectral synthesis. Several years ago we gave a negative answer to this question. At the same time we showed that any such system admits the synthesis up to a one- dimensional defect. In the talk we will also discuss related problems for systems of reproducing kernels in Hilbert spaces of entire functions (such as Paley-Wiener, de Branges, Fock).

Yuri Belov (Saint Petersburg State University)

The Newman-Shapiro problem

In 1966 D. Newman and H. Shapiro posed the following approximation problem. Let G be a function from Fock space $\mathcal{F}$ and such that $e^{wz}G(z) \in \mathcal{F}$ for any $w \in \mathcal{F}$. Is it true that

$$span\{e^{wz}G(z)\}_{w\in\mathbb{C}} = \{FG : FG \in \mathcal{F}\}?$$

Recently the author (joint with A. Borichev) has constructed a counterexample to this conjecture. On the other hand, we are able to show that if G satisfies some regularity or growth conditions, then conjecture holds. These results are closely connected to some spectral synthesis problems in Fock space.

Linxiao Chen (University of Helsinki)

Phase transition in the Ising model on random triangulations of the disk

The Ising model is one of the first statistical mechanics models known to have a non-trivial phase transition in two dimensions. On regular lattices, this phase

transition has been extensively studied. In this talk I will present an annealed Ising model on a random two-dimensional lattice, introduced first in the Physics literature as a discretization of the Liouville quantum gravity in 2D. We will see that the partition function of this model is exactly solvable. We show that this model has a phase transition at a unique temperature by examining its free energy, its critical exponents, and the scaling limit of its interface length. In particular, the result confirms the physical intuition that a random lattice coupled to a non-critical Ising model has a geometry similar to a uniform random lattice with an instance of Bernoulli percolation on it. I will also discuss how the phase transition takes place in the near-critical window. Based on a joint work in progress with Joonas Turunen (University of Helsinki).

Erik Duse (KTH Stockholm)

On some conditions for minimality for non-smooth functionals arising from dimer models

In this talk I will discuss certain sufficient conditions for minimality for non-smooth functionals arising from dimer models. In particular I will discuss how this can be turned into a geometric question regarding tangents to a certain class of algebraic curves. This is based on joint work with Kari Astala, István Prause and Xiao Zhong.

Alex Karrila (IHES)

On multiple SLE type scaling limits

Schramm-Loewner evolution (SLE) curves were introduced in 2000 as the natural candidate for scaling limits of random interfaces in critical planar lattice models. Since then, celebrated proofs have established SLE convergence for several prominent lattice models. In this talk, I will discuss various aspects related to multiple SLE curve families and convergence proofs realizing them as scaling limits of lattice models. The talk is mainly based on [arXiv :1903.10354] and [arXiv :1810.05608].

Sergei Kislyakov (PDMI RAS)

A property of ideals in a uniform algebra

Let I and J be to closed ideals in a uniform algebra A on a compact space S. It will be shown that if the complex conjugate $\overline{I \cap J}$ of their intersection is not included in some of them, then the sum $I + \bar{J}$ is not closed in $C(S)$.
The question arose during the joint work of the author and I. Zlotnikov on interpolation properties of coinvariant subspaces of the shift operator. The answer indicated above may be viewed as a far-reaching generalization of the universally known fact that $C_A + \overline{C_A} \neq C(\mathbb{T})$, where C_A is the disk-algebra,

$$C_A = \{f \in C(\mathbb{T})\colon \hat{f}(n) = 0 \text{ for } n < 0\}.$$

An equivalent way of expressing the same property of the disk-algebra is simply to say that the Riesz projection

$$f \mapsto \sum_{n \geq 0} \hat{f}(n) z^n$$

is unbounded on $C(\mathbb{T})$. Around 1961, I. Glicksberg conjectured that no proper uniform algebra on a compact space S admits a bounded linear projection from $C(S)$ onto it. A far-reachng generalization of this conjecture was proved by the author around 1987. The above statement about nonclosed sums of ideals is proved by using similar technique, which is based on the presence of certain very slight traces of analytic structure on an arbitrary proper uniform algebra.

Tuomo Kuusi (University of Helsinki)

Higher-order linearization and regularity in nonlinear homogenization

The analysis of higher-order linearized equations lets us develop an incisive large-scale higher regularity theory for solutions of nonlinear elliptic equations in the context of homogenization. We proceed in analogy to the role of the Schauder theory in resolving Hilbert's 19th problem on the regularity of solutions to nonlinear equations with smooth coefficients.

Eveliina Peltola (University of Bonn)

Conformal invariance of crossing probabilities in critical 2D lattice models

For a number of lattice models in 2D statistical physics, it has been proven that the scaling limit of an interface at criticality (with suitable boundary conditions) is a Schramm-Loewner evolution (SLE). Similarly, collections of several interfaces converge to families of interacting random curves, multiple SLEs. Connection probabilities of these interfaces encode crossing probabilities in the lattice models, which should also be related to correlation functions of appropriate fields in the corresponding conformal field theory (CFT).
In this talk, I discuss results towards conformal invariance of critical lattice models in terms of interfaces and crossing probabilities. In general, there are no explicit formulas for the crossing probabilities, but their scaling limits can still be completely understood in terms of so-called pure partition functions of multiple SLEs. In particular, all of the expected CFT properties : conformal invariance, null-field equations, and fusion rules, are satisfied.

Fedor Petrov (Saint Petersburg State University)

Selberg integral and polynomial identities

Selberg-type integrals that can be turned into constant term identities for Laurent polynomials arise naturally in conjunction with random matrix models in statistical mechanics. We discuss a general method based on the multi-dimensional polynomial interpolation identity related to the so called Combinatorial Nullstellensatz of Alon, that is powerful enough to establish many such identities, both known before and new, in a simple manner. Based on joint works with R. Karasev, G. Károlyi, Z. Nagy and V. Volkov.

Olga Postnova (PDMI RAS)

Large tensor product multiplicities and asymptotics of character distributions of representations of simple Lie algebras

Let $\mathfrak{g}$ be a simple Lie algebra and V_i, $i = 1, \cdots, m$ be finite dimensional representations of $\mathfrak{g}$. The asymptotics of the multiplicity $m_\lambda(\{V_i\}, \{N_i\})$ of irreducible representations in the tensor product $\prod_{i=1}^m V_i^{\otimes N_i}$ is derived in the limit $N_i \to \infty$, when $N_1 : \cdots : N_m$ is finite. Let $\chi_\lambda(e^t)$ be the character of the representation with highest weight λ. The character measure $p_\lambda^{(N)}(t) = \frac{m_\lambda(\{V_i\},\{N_i\})\chi_\lambda(e^t)}{\prod_i \chi_{\nu_i}(e^t)^{N_i}}$ is introduced as a natural probability measure on irreducible components of tensor product. The asymptotics of multiplicity is used to compute the asymptotics of the character and Plancherel measures in this limit. The type of the limiting distribution changes from the Gaussian to the radial part of the Gaussian depending on whether t is strictly inside the Weyl chamber or at the origin. If t is on a wall of the Weyl chamber we will have an intermediate distribution. For the representations with highest weights in the vicinity of $\sum_i N_i\nu_i$ the character distribution converges to Poisson type distribution. Based on joint works with N. Reshetikhin and V. Serganova.

Andrei Pronko (PDMI RAS)

Thermodynamics of the five-vertex model with 'scalar product' boundary conditions

We derive the phase diagram and the free energy of the five-vertex model with the 'scalar product' boundary conditions. We use some Hankel determinant formula for the partition function. The key ingradient in our calculation is the connection of this formula with the tau-function of the sixth Painleve equation. We find that not only the free energy but also the phase diagram is essentially different to the case of the periodic boundary conditions.

Nikolai Reshetikhin (UC Berkeley)

Limit shapes integrable statistical mechanics : Hamiltonian integrable systems ?

We will argue that limit shape for integrable models in statistical mechanics on cylinders are infinite dimensional Hamiltonian integrable systems. As a specific example we will take the 6-vertex model and will show that differential equations for limit shapes possess infinitely many conserved quantities which Poisson commute with respect to natural Poisson brackets. This property also holds for an inhomogeneous case.

Senya Shlosman (IITP RAS, SkolTech and CNRS)

Glassy states of the Ising model on trees and Lobachevsky plane

The ferromagnetic Ising model on trees and on Lobachevsky plane has many ground states, as is the case for the Ising model with random interactions on the standard lattices. As a result, the free state of the Ising model below the spin-glass temperature has a structure of a spin-glass state : it is a mixture of a continuum (!) many extremal states. I will describe precisely the structure of this mixture. Joint work with D. Gandolfo, Ch. Maes, and J. Ruiz.

Christian Webb (Aalto University)

How much can the eigenvalues of a random Hermitian matrix fluctuate ?

One of the most classical results of random matrix theory is Wigner's theorem, which says that the eigenvalues of a large random Hermitian matrix, drawn from the Gaussian Unitary Ensemble, are approximately distributed according to the semicircle distribution. I will discuss some recent joint work with T. Claeys (UCLouvain), B. Fahs (Imperial College London), and G. Lambert (University of Zürich) describing how much the eigenvalues can fluctuate around the semicircle distribution.