Petersburg Motives

Vladimir Chernousov: *On the norm principle for algebraic groups*

In the talk we present some new cases of the norm principle.

Jean-Louis Colliot-Thélène: *On the local-global principle for tori over arithmetic curves*

This is a report on joint work with D. Harbater, J. Hartmann, D. Krashen, R. Parimala and V. Suresh. We study local-global principles for tori over semi-global fields, which are one variable function fields over complete discretely valued fields. In particular, we show that for principal homogeneous spaces of tori defined over the underlying discrete valuation ring, the obstruction group to a local-global principle with respect to discrete valuations can be computed using methods coming from patching. We give a sufficient condition for the vanishing of the obstruction, as well as examples were the obstruction group is nontrivial or even infinite. A major tool is the notion of a flasque resolution of a torus.

An analogous analysis gives a counterexample to the local-global principle for a principal homogeneous space under a suitable semisimple simply connected group G. This is in sharp contrast with the classical number theory situation.

Jean Fasel: *Hermitian K-theory, algebraic cycles and characters*

The purpose of this talk is to explain the construction of an explicit isomorphism between (rational) Hermitian K-theory and rational MW-motivic cohomology, paralelling the Chern character linking K-theory and rational motivic cohomology. We'll briefly survey the construction of the Borel classes and explain the basic principles behind the construction of the character. This is a joint work with F. Déglise, A. Khan and F. Jin.

Grigory Garkusha: *Stable motivic homotopy theory via framed bispectra and framed spectral functors*

This is a joint work with Ivan Panin. In the early 2000-s Voevodsky introduced framed correspondences, the main purpose of which was to suggest an alternative approach to the classical Morel-Voevodsky stable motivic homotopy theory $SH(k)$. Using the theory of (big) framed motives introduced and developed by the speaker and Panin (basing on Voevodskys notes on framed correspondences), we suggest two models recovering $SH(k)$. The first model for $SH(k)$ is given by the triangulated category of framed bispectra, the second one uses the theory of framed spectral functors. Both models are genuinely local in the sense that they do not use any kind of motivic equivalences. We also introduce the triangulated category of framed motives and show that it recovers the category of effective motivic bispectra.

Nikita Karpenko: *A counter-example by Yagita*

According to a 2018 preprint by Nobuaki Yagita, the conjecture on a relationship between K- and Chow theories for a generically twisted flag variety of a split semisimple algebraic group G, due to the speaker, fails for G the spinor group $Spin(17)$. Yagita's tools include a Brown-Peterson version of algebraic cobordism, ordinary and connective Morava K-theories, as well as Grothendieck motives related to various cohomology theories over fields of characteristic 0. The talk presents a simpler proof using only the K- and Chow theories themselves and, in particular, extending the (slightly modified) example to arbitrary characteristic.

Alexander Kuznetsov: *Rationality of Fano 3-folds over non-closed fields*

In the talk I will discuss rationality criteria for Fano 3-folds of geometric Picard number 1 over a non-closed field k of characteristic 0. Among these there are 8 types of geometrically rational varieties. We prove that in one of these cases any variety of this type is k-rational, in four cases the criterion of rationality is the existence of a k-rational point, and in the last three cases the criterion is the existence of a k-rational point and curve of genus 0 and degree 1, 2, and 3 defined over k, respectively. The last result is based on recent results of Benoist and Wittenberg.

This is a joint work with Yuri Prokhorov.

Marc Levine: *Quadratic degrees and Gromov-Witten invariants of genus zero curves*

This is a joint work with Jesse Kass, Jake Solomon and Kirsten Wickelgren. Relying on the geometry of singular points, we define a relative orientation for the evaluation map from the modulo space of maps of genus zero curves with marked points to a del Pezzo surface S. The resulting push-forward of the fundamental class gives a section of the sheaf of Grothendieck-Witt rings on the unordered configuration space of S, whose rank recovers the classical count of rational curves in a linear system $|D|$ passing through $-D.K_S - 1$ points, and whose signature gives the Welschinger invariant counting the real such curves, counted with their Welschinger "mass". Our invariant is defined over an arbitrary perfect base-field of characteristic different from 2, 3.

Fabien Morel: *Playtime in geometric $\mathbb{A}^1$-topology*

In this talk I will present the fundamental ideas and problems behind what I call geometric $\mathbb{A}^1$-topology, that is to say how can we use $\mathbb{A}^1$-algebraic topology to "classify" smooth projective schemes over a field k, in parallel to classical geometric topology. After quickly recalling the classical approach, and considering $\mathbb{A}^1$-connected smooth projective k-schemes, I will address the basic facts and conjectures that one encounters on the way to geometric $\mathbb{A}^1$-topology, among them, for instance the notion of Poincare complex, the signature formula, and I will give concrete examples in low dimensions.

Dmitri Orlov: *Finite-dimensional DG algebras, derived noncommutative schemes, and geometric realizations*

The main aim of this talk is to describe derived noncommutative schemes that are related to finite-dimensional DG algebras. It will be explained that a famous Auslander construction for finite dimensional algebras can be extended to the case of DG algebras. We also show that all such derived noncommutative schemes have geometric realizations and can be embedded to categories of perfect complexes on smooth projective varieties with full semi-exceptional collections.

Raman Parimala: *A local-global principle for simply connected groups over arithmetic surfaces*

Classical Hasse principle theorem for number fields states that a principal homogeneous space under a semisimple simply connected linear algebraic group over a number field admits a rational point if it does over all its completions. Similar Hasse principle for function fields of curves over p-adic fields has been posed as a conjecture by Collliot-Thélène–Parimala–Suresh. We shall discuss the case when the p-adic field is replaced by a general complete discrete valued field. We explain some positive results on Hasse principle when the field is the function field of a smooth curve over a complete discrete valuation ring and the group is defined over the discrete valuation ring. There are examples of failure of Hasse principle when the curve is not smooth.

(Report on the work with J.-L. Colliot-Thélène, D. Harbater, J. Hartmann, D. Krashen and V. Suresh and on the work with P. Gille and V. Suresh)

Victor Petrov: *Hopf-theoretic approach to motives of twisted flag varieties*

Let G be a split semisimple algebraic group over a field and let A^* be an oriented cohomology theory in the sense of Levine–Morel. We provide a uniform approach to the A^*-motives of geometrically cellular smooth projective G-varieties based on the Hopf algebra structure of $A^*(G)$. Using this approach we provide various applications to the structure of motives of twisted flag varieties.

Oliver Röndigs: *On motivic Moore spectra*

The term "motivic Moore spectrum" refers to a cone of an element in the motivic stable homotopy groups of spheres. The talk discusses some properties of motivic Moore spectra, among them the question whether the ring structure on the motivic sphere spectrum descends to a ring structure on a motivic Moore spectrum. This discussion requires an understanding of some Toda brackets in the motivic stable homotopy groups of spheres.

Anastasia Stavrova: *A few corollaries of the Serre-Grothendieck conjecture*

We will discuss a few corollaries of the work of Ivan Panin on the Serre-Grothendieck conjecture for isotropic reductive groups.

Nikolai Vavilov: *Ramblings in the K-theory of algebraic groups*

Although absolute K_1 and K_2 modeled on Chevalley groups were introduced some 50 years ago, even in this simplest case many of the central problems remained open until very recently. For non-split reductive groups even definitions are non-trivial. After giving an overview of the state of art and some recent results of the St. Petersburg school in this direction, I concentrate on two unexpected recent advances: 1) the relative case (unrelativisation, etc.), 2) efficient versions of the structure theorems at the level of K_1 (reverse decomposition of unipotents, etc.), and results which are presently under way 3) combination of the above, effective versions of relative results.

Alexander Vishik: *Isotropic world*

The isotropic localisations of the algebro-geometric motivic category produce categories whose complexity is similar to a classical topological one. Such isotropic realisations parametrized by finitely generated extensions of the ground field permit to present global motivic information in a simple local form. I will discuss the general picture, possible applications and open problems here.

Kirill Zainoulline: *Localized cohomological operations on oriented cohomology*

Cohomological operations on oriented cohomology theories (e.g. Steenrod, Adams, Landweber-Novikov) have been extensively studied during the past decades (Brosnan, Merkurjev, Vishik) in the algebraic context. They turned out to be extremely useful in computing various geometric invariants of flag varieties (incompressibility, canonical dimension, torsion, motivic decomposition type, etc.). They also play an important role in the Riemann-Roch formalism (Panin-Smirnov, Levine-Morel).

In the present talk, we explain how to extend the cohomological operations on oriented cohomology to the setup of T-equivariant oriented cohomology theories using the localization techniques of Goresky-Kottwitz-MacPherson and Kostant-Kumar. These operations can be viewed as operations on global sections of the so called structure sheaves on moment graphs (corresponding to arbitrary Coxeter groups). They satisfy several natural properties, e.g. they commute with characteristic map and restrict to usual operations.

As an application, we extend the algorithm by Garibaldi-Petrov-Semenov for computing the Steenrod operations on Chow groups to the context of arbitrary multiplicative operations and oriented theories.

Paul Arne Østvær: *Framed motivic Gamma-spaces*

In joint work with Grigory Garkusha and Ivan Panin we combine framed correspondences and Gamma-spaces to give an elementary description of connective motivic spectra, very effective motivic spectra, and infinite motivic loop spaces.