that each of these entries is now closed by proving non-existence. We will include a discussion of some characteristics which the last entries share, and some consequences which have arisen from this investigation.

Sequences involving *free* variables

H. Kharaghani (with W. H. Holzmann)

The existence of an Hadamard matrix of order n is equivalent to the existence of n-complementary sequences of length n involving n *free* variables. These sequences are used to generate a class of *almost extremal* orthogonal designs.

A Branch and Probability Bound Random Search Technique for Solving Construction Problems in Experimental Design

Nikolai K. Krivulin

We consider the problem which is to find the incidence matrix A to satisfy the equation

$$AA^T = M, \tag{1}$$

where matrix M is given. Clearly, it can be reformulated as a problem of minimizing the residual functional $f(A) = \|AA^T - M\|$ over a set of $(0-1)$-matrices, and then attacked with a combinatorial optimization method. Even though the solution of equation (1) does not actually exist, solving the optimization problem can lead to a matrix which may be of interest for practitioners, in particular, in the area of experimental design.

In order to solve the optimization problem, we propose a technique based on the Branch and Probability Bound (BPB) method [A.A. Zhigljavsky, *Theory of Global Random Search*, vol. 65 of *Mathematics and Its Applications Soviet Series*, Kluwer Academic Publishers, Dordrecht, 1991.] which actually combines the usual branch and bound scheme with random search and statistical procedures of parameter estimating and hypothesis testing. As in the branch and bound methods, each step of BPB algorithms consists in partitioning the feasible set into subsets followed by choosing the subsets most promising for the solution. However, the BPB approach assumes both partitioning into subsets and determining the subsets for further search to be performed as statistical

procedures based on the information obtained in the course of the previous search steps.
We present a BPB algorithm designed to find the exact solution of equation (1) provided that it exists, or an approximate solution (in the sense of minimum of the residual functional f), and give related numerical examples. Potentialities of the BPB technique to solve other construction problems in algebraic combinatorics are also discussed.

Galois Rings, Witt Vectors and Coordinate Sequences

Jyrki Lahtonen

I. Rings of Witt vectors

Let $\mathbf{F}$ be a finite field of characteristic p and $\mathbf{F}^k$ its cartesian power. Witt was the first to observe that $\mathbf{F}^k$ can be made into a commutative ring $W_k(\mathbf{F})$ of characteristic p^k by defining addition and multiplication of vectors as follows:

$$\mathbf{x}+\mathbf{y} = (x_0, x_1, \ldots, x_{k-1})+(y_0, y_1, \ldots, y_{k-1}) = (s_0(\mathbf{x},\mathbf{y}), s_1(\mathbf{x},\mathbf{y}), \ldots, s_{k-1}(\mathbf{x},\mathbf{y}))$$

and

$$\mathbf{x}\cdot\mathbf{y} = (x_0, x_1, \ldots, x_{k-1})\cdot(y_0, y_1, \ldots, y_{k-1}) = (p_0(\mathbf{x},\mathbf{y}), p_1(\mathbf{x},\mathbf{y}), \ldots, p_{k-1}(\mathbf{x},\mathbf{y})).$$

Here $s_t(\mathbf{x},\mathbf{y})$ and $p_t(\mathbf{x},\mathbf{y}), t = 0, 1, \ldots, k-1$ are certain recursively defined polynomials (with integer coefficients) of the variables $x_i, y_j \in \mathbf{F}, i, j \leq t$. In the case $p = k = 2$, i.e. when $W_2(\mathbf{F})$ has characteristic 4, these rules take the following very simple form:

$$\begin{aligned} (x_0, x_1) + (y_0, y_1) &= (x_0 + y_0, x_1 + y_1 + x_0 y_0) \\ (x_0, x_1) \cdot (y_0, y_1) &= (x_0 y_0, x_0^2 y_1 + y_0^2 x_1). \end{aligned}$$

It is known that the ring $W_k(\mathbf{F})$ is isomorphic to the Galois ring of the same size and characteristic. The isomorphism can be best expressed by identifying the Teichmuller set $\mathcal{T}$ inside a Galois ring R with the set of Witt vectors of the form $\mathbf{t}(x) = (x, 0, 0, \ldots, 0)$. The p-adic expansion of a Witt vector is then

$$\mathbf{t}(x_0) + p\mathbf{t}(x_1) + p^2\mathbf{t}(x_2) + \cdots + p^{k-1}\mathbf{t}(x_{k-1}) = (x_0, x_1^p, x_2^{p^2}, \ldots, x_{k-1}^{p^{k-1}}).$$

The importance of the Witt vector presentation is (at the moment) mostly pedagogical: it is remarkably easy to rebuild and learn the stucture theory of Galois rings using the Witt vector formalism, especially in the characteristic 4 case. In this talk I shall give several examples of this. Another advantage of