

p -adic fractal dimension. The example of the Sierpinski gasket is considered. It is shown that a fractal object, which has the fixed usual fractal dimension ($d_f = \log 3 / \log 2$ in this case) can be split in a class of objects with different p -adic properties. From the p -adic point of view there exists a class of the Sierpinski gaskets. These Sierpinski gaskets are connected with different ways in which n approaches infinity in the iteration process of the constructing of the Sierpinski gasket. Further, we investigate chaotic regimes of dynamical systems using p -adic numbers. We will see that some values of parameters, which generate chaotic regimes (with respect to the ordinary real metric), correspond to regular regimes in one of p -adic metrics (the convergence to a fixed point for all initial conditions). Thus, a homogeneous real chaos is split in different types of chaos characterizing by the p -adic regularity.

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N. Krivulin: Estimation of the Lyapunov Exponent in (max, +)-Algebra Models of Stochastic Dynamical Systems

We consider dynamical systems governed by the stochastic difference equation $x(k) = A(k) \otimes x(k-1)$, state transition matrix, and the symbol \otimes denotes (max,+)-algebra matrix-vector multiplication. Such models frequently arise in the description of the dynamics of actual systems including queueing networks, production systems, and transportation networks.

In many applications, one is interested in investigating the limiting behaviour of the quantity $\|A_k\|_{\oplus}/k$, where $A_k = A(k) \otimes \dots \otimes A(1)$, and $\|\cdot\|_{\oplus}$ denotes the (max,+)-algebra matrix norm. The limit $\lim_{k \rightarrow \infty} \|A_k\|_{\oplus}/k = \gamma$, which determines the growth rate of $x(k)$, is normally called, in a similar way as in the conventional linear system theory, maximum Lyapunov characteristic exponent. In practical problems, it is often used in reference to the mean cycle time of a system.

Our main result is essentially based on Kingman's subadditive ergodic theorem, and exploits some elements of the probability theory associated with (max,+)-algebra. It can be summarized as follows. Let $A(1), A(2), \dots$, be i.i.d. random matrices. If $E\|A(1)\|_{\oplus} < \infty$, $\lambda_{\oplus}(E[A(1)]) > -\infty$, where $\lambda_{\oplus}(\cdot)$ represent the spectral radius (the greatest eigenvalue) of a matrix in the (max,+)-algebra sense, and E denotes the expectation operator, then the Lyapunov exponent γ exists w.p.1, and it holds that

$$\lambda_{\oplus}(E[A(1)]) \leq \gamma \leq E\|A(1)\|_{\oplus} \quad \text{w.p.1.}$$

We give examples which show that although the above bounds are normally rather simple to evaluate, they appear to be quite comparable to those involving more sophisticated techniques. As an application, the bounds on the mean cycle time for a queueing network model are calculated.