SERIAL AND PARALLEL ALGORITHMS FOR TANDEM QUEUEING SYSTEM SIMULATION

S.M. Ermakov, N.K. Krivulin
Faculty of Mathematics and Mechanics,
St.Petersburg State University

The simulation of a queueing system is normally an iterative process which involves generation of random variables associated with current events in the system, and evaluation of the system state variables when new events occur [1, 3]. In a system being simulated the random variables may represent the interarrival and service time of customers, and determine a random routing procedure for customers within the system with non-deterministic routing. As state variables, the arrival and departure time of customers, and the service initiation time can be considered.

The methods of generating random variables present one of the main issues in computer simulation, which has been studied intensively in the literature (see, e.g. [1]). We however assume that for the random variables involved in simulating a queueing system, appropriate realizations are available when required, and we therefore concentrate only on algorithms of evaluating the system state variables from these realizations as well as performance measures commonly used in the analysis of queues.

The usual way to represent dynamics of queueing systems is based on recursive equations describing evolution of system state variables. Furthermore, these equations, which actually determine a global structure of changes in the state variables consecutively, have proved to be useful in designing efficient simulation algorithms [2] – [4].

Efficient algorithms based on recursive representations for simulation of tandem queueing systems with infinite buffers are presented and their performance is analyzed. Specifically, parallel algorithms designed for implementation on single instruction, multiple data parallel processors are considered. These algorithms are based on a simple computational procedure which exploits a particular order of evaluating the system state variables from the recursive equations.

It is shown that the basic simulation procedure which underlies the algorithms allows of simulating the first N customers in a tandem queue with M servers in the time

$$T_P = O\left(2M + 2N + 2\left\lfloor \frac{L_1 - 1}{P} \right\rfloor (L_2 - P)\right),$$

where P is the number of processors, $L_1 = \min(M+1, N)$, and $L_2 = \max(M+1, N)$. In this case it requires only $O(\min(M+1, N))$ memory locations. Finally, for tandem queueing systems the procedure using P processors achieves the speedup

$$S_P = O\left(\frac{M+1}{1 + \lfloor M/P \rfloor}\right) \quad \text{as } N \to \infty.$$

Specifically, if the number of processors $P = M + 1$, the speedup becomes linear.

The procedure is applied to the development of algorithms of evaluating tandem system performance measures including the average total time of customers, the average waiting time of customers, the average utilization of servers, the average number of customers, and the average queue length at each server. Time and memory requirements of the algorithms are also examined.

References

[1] S. M. Ermakov, *Die Monte-Carlo-Methode und verwandte Fragen*. VEB Deutscher Verlag der Wissesc hhaften, Berlin, 1975

[2] S. M. Ermakov and N. K. Krivulin, *Efficient Algorithms for Tandem Queueing Systems Simulation*, Appl. Math. Lett., (1994) (submitted for publication)

[3] N. K. Krivulin, *Optimization of Discrete Event Dynamic Systems by Using Simulation*, Ph.D. Dissertation, St.Petersburg State University, St.Petersburg, 1990. (in Russian)

[4] N. K. Krivulin, *Unbiased Estimates for Gradients of Stochastic Network Performance Measures*, Acta Applic. Math., 33 (1993), pp. 21–43.

APPLICATION OF SYMBOLIC DYNAMICS FOR SIMULATION OF GLOBAL STRUCTURE OF DYNAMICAL SYSTEM

I.V.Ilyin
Department of Higher Mathematics,
St.-Petersburg State Technical University

Let us consider a smooth dynamical system

$$\dot{x} = f(x) \tag{1}$$

on compact smooth manifold M. We shall assume that the number of components of chain recurrent set is finite. Our aim is to construct an algorithm for global structure analysis of the system (1).

Our considerations will be based on the construction of symbolic image [1] of the system (1), which is a finite approximation of (1) in form of oriented graph. It can be constfuçted in the following way. We consider the solution $X(t, x)$ of the system (1), $X(0, x) = x$. Let us fix non-zero t. $X(x) = X(t, x)$ is a diffeomorfism of M. Let $M(1), ..., M(n)$ be a covering of M by the closed cells. Symbolic image G consists of n vertexes $1, ..., n$ and a finite set of oriented edges. We shall say that the oriented edge (i, j) exists if intersection of $X(M(i))$ and $M(j)$ is non-empty. A vertex i of G is called recurrent if a closed finite path through i in G exists. Vertexes i and j of G are called equivalent if a closed finite path in G passing through i and j exists. We shall decompose the set of the recurrent vertexes of G into classes $G(1), ..., G(q)$ of equivalent vertexes. If $G(j) = \{i(1), ..., i(k)\}$ then we denote the union of $M(i(1)), ..., M(i(k))$ by $R(j)$. Let us fix $E > 0$. Let $S(j)$ be the union of those cells $M(i)$, that have non-empty intersection with E-neighborhood $V(E, R(j))$ of $R(j)$. We say that the classes $G(j(1)), ..., G(j(l))$ form one class $H(i)$ of the vertexes of G if the unit of $S(j(1)), ..., S(j(l))$ is a connected set and the set $\{j(1), ..., j(l)\}$ of vertexes of G is maximal with such condition. Let $H(1), ..., H(p)$ be all these classes. For every $H(j)$ we denote the union of the cells $M(i)$, i lies in $H(j)$, by $T(j)$.