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## RECURSIVE EQUATIONS BASED MODELS OF QUEUEING SYSTEMS


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As a representation of dynamics of queueing systems, recursive equations have been introduced by Lindley in his classical investigation of the $G/G/1$ queue. In the last few years the representations based on recursive equations have been extended to a variety of systems which consist of single server queues and operate according the first come, first served discipline. Specifically, there are the equations designed for open and close tandem queues with both infinite and finite buffers. Recursive equations have been also derived to represent more complicated systems of $G/G/1$ queues, including acyclic fork-join networks and closed networks with a general deterministic routing mechanism.

Recursive equations find a wide application in recent works on both analytical study and simulation of queueing systems. As an analytical tool, they were exploited to investigate system performance measures and estimates of their gradients. Recursive equations based representations have provided the means for establishing the stability conditions and deriving bounds on system performance in a class of queueing systems. Finally, these representations made it possible to develop efficient algorithms of queueing systems simulation as well as powerful methods of estimating gradients of system performance measures.

A brief overview of advances in the area is given. Specifically, included are new results concerning recursive representations for the $G/G/m$ queue [5], simulation algorithms [2], and evaluation of sensitivity of network performance measures [2] - [4]. Finally, limitations on the use of the queueing system models based on the recursive equations are discussed.


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## SYSTEM MODELING: PRINCIPLES AND TRAINING

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It known that system modeling is the synthesis of science and art [3]. The process of model's construction can't be completely formalized and the transition from the verbal decription of the system to its mathematical model requires investigatoris experience and intuition. This aspect makes students training in construction of mathematical models much more difficult to have in some sense the uniform not formal description of a sufficiently wide range of systems. One of the possible [1] approaches to the solution of this problem is represented in the present work [2].

We assume that the investigating system can be described by the account set $Z$ of element units (or simply units) interacting with each other in some way. Then each unit $i \in Z$ in the process of functioning is supposed to be in the finite number of states $| S_i |$ where $S_i$ is the set of the states of the $i$-th unit. The units themselves and their states are chosen in such a way that the state of the system as a whole first would be completely defined by the state of the units and, secondly, it would substantially determine the efficiency of the system on the whole. Thus, the state of the entre system on the whole is given by the set of the states of the units $\{S_i \mid i \in Z.\}$

It is supposed that the whole set of units $Z$ (in the general case infinite) can be divided into the finite number $n$ of groups so that all the units of one group have one and the same set of the states. If $Z_i$ is the set of units of $i$th-group then $\{Z = Z_1 + ... + Z_n, Z_i \cap Z_j \neq \emptyset, i \neq j\}$. Two units from $Z$ are attributed to different groups if the statement of the problem urges you to consider that the sets of their states are substantialy different. If $S_{z_i}$ is the set of the states of the units belonging to group $Z_i$ and $n_{ij}$ is the member of units in the state $j$ then the array of numbers $\{n_{ij} \mid i \in 1 : n, j \in S_{z_i}\}$ completely describes the state of the system on the whole. As regards real systems, the dimension of the states vector $S_j$ considerably exceeds the dimension of the vector $n_{ij}$.

The choice of the units depends on the task which the investigator brings his attention to. You can choose as units both independent phisical objects of the real system (for example, demands, channels, etc) and separate quantitave variables with their possible values as states. A change of the state of the units brings to a change of the state of