Mean-field style results for real-time databases with optimistic concurrency control

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We analyze the closed queueing systems modelling large databases run with Optimistic Concurrency Control. We establish the validity of the standard *Poisson departure flow* approximation for large such systems using the *mean-field style* approach. Further, we present a novel way to optimize the throughput for multi-class systems.

An analysis of a (max,+)-algebra product of random matrices

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We examine a (max,+)-algebra product of random matrices, which arises from stability analysis and performance evaluation of acyclic fork-join queueing networks. Consider a network with n nodes. The network dynamics can be described by the stochastic vector difference (max,+)-algebra equation

$$\boldsymbol{x}(k) = A(k) \otimes \boldsymbol{x}(k-1) \qquad \text{with} \qquad A(k) = (I \oplus \mathcal{T}_k \otimes G)^p \otimes \mathcal{T}_k,$$

where $\boldsymbol{x}(k)$ is an n-vector of the kth service completion times in the nodes, $\mathcal{T}_k = \mathrm{diag}(\tau_{1k}, \ldots, \tau_{nk})$ is a diagonal random matrix of the kth service times, I is the identity matrix, G is the adjacency $(\varepsilon - 0)$-matrix of the network graph, and p is the length of the longest path in the graph. It is assumed that for each i, $i = 1, \ldots, n$, the service times τ_{ik}, $k = 1, 2, \ldots$, form a stationary sequence of nonnegative random variables (r.v.'s) with finite mean $E\tau_{ik} < \infty$. One of the problems of interest is to evaluate the growth rate of the vector $\boldsymbol{x}(k)$ as $k \to \infty$, which can be considered as service cycle time of the network. To get information about the growth rate, we investigate the limiting behaviour of the product $A_k = A(k) \otimes \cdots \otimes A(1)$. First we prove that there exists $\lim_{k\to\infty} A_k/k = A$ w.p.1. Furthermore, we show that if for each i, the times τ_{ik}, $k = 1, 2, \ldots$, are i.i.d. r.v's with finite variance $D\tau_{ik} < \infty$, then it holds

$$E[\mathcal{T}_1] \leq A \leq E\left[\bigoplus_{0 \leq r+s \leq p} G^r \otimes \mathcal{T}_1 \otimes G^s\right],$$

where $E[\cdot]$ denotes the component-wise matrix expectation operator with $E\varepsilon = \varepsilon$.