an advanced and more abstract computational machinery often depends on a long chain of more specialized algorithms and efficient data structures at various levels. On the software development side, for cross-border approaches to solving mathematical problems, the efficient interaction of systems specializing in different areas is indispensable; handling complex examples or large classes of examples often requires a considerably enhanced performance. Whereas the interaction of systems is based on a systematic software modularization and the design of mutual interfaces, a new level of computational performance is reached via parallelization, which opens up the full power of multi-core computers, or clusters of computers. In my talk, I will report on the ongoing collaboration of groups of developers of several well-known open source computer algebra systems, including GAP, which pays particular emphasis to group theory, Singular, a system for applications in algebraic geometry and singularity theory, and Polymake, a software for polyhedral geometry. In presenting computational tools relying on this collaboration, and some of the mathematical challenges which lead us to develop such tools, I will in particular highlight the Homalg project which provides an abstract structure and algorithms for abelian categories, aiming at concrete applications ranging from linear control theory to commutative algebra and algebraic geometry. I will also comment on progress in the design of parallel algorithms for basic tasks in commutative algebra and algebraic geometry such as primary decomposition, normalization, finding adjoint curves, or parametrizing rational curves.

Wolfram Decker
Universität Kaiserslautern
decker@mathematik.uni-kl.de

IP8

Progress Report on Geometric Complexity Theory

Geometric complexity theory is an approach to the fundamental lower bound problems of complexity theory via algebraic geometry and representation theory. This talk will give an overview of some progress in this field.

Ketan Mulmuley
University of Chicago
mulmuley@cs.uchicago.edu

IP9

Hodge Theory and Combinatorics

A conjecture of Read predicts that the coefficients of the chromatic polynomial of any graph form a log-concave sequence. A related conjecture of Welsh predicts that the number of linearly independent subsets of varying sizes form a log-concave sequence for any configuration of vectors in a vector space. All known proofs use Hodge theory for projective varieties, and the more general conjecture of Rota for possibly 'non-realizable' configurations is still open. In this talk, I will argue that two main results of Hodge theory, the Hard Lefschetz theorem and the Hodge-Riemann relations, continue to hold in a realm that goes far beyond that of Kahler geometry. This cohomology theory gives strong restrictions on numerical invariants of tropical varieties, and in particular those of graphs and matroids. Joint work with Karim Adiprasito and Eric Katz.

June Huh
Princeton University
huh@princeton.edu

CP1

Solving Multidimensional Optimization Problems Over Tropical Semifields

We consider multidimensional problems that are formulated in the framework of tropical mathematics to minimize or maximize functions defined on vectors of a finite-dimensional semimodule over an idempotent semifield. The objective functions can be linear or nonlinear; in the latter case they are defined using multiplicative conjugate transposition of vectors. Both unconstrained problems and problems with vector equality and inequality constraints are under consideration. We start with a brief overview of known problems and existing solution methods. Some of these problems can be solved directly in an explicit form under fairly general assumptions about the underlying semifield. For other problems, algorithmic solutions are known only in terms of particular semifields to have the form of iterative computational procedures, which produces a particular solution, or indicates that no solution exist. Furthermore, we examine new problems with nonlinear objective functions, including problems of Chebyshev approximation, problems of minimizing the span seminorm, and problems with evaluating the spectral radius of a matrix. To solve the problems, several techniques are proposed based on the reduction of the problem to a parameterized system of inequalities, the derivation sharp bounds for the objective function, and the application of extremal properties of the spectral radius. We use these technique to obtain direct exact solutions of the problems in a compact vector form, which is ready for further analysis and practical implementation. The solutions obtained are applied to solve optimization problems in Chebyshev approximation, project scheduling, location analysis and decision making.

Nikolai Krivulin
Saint Petersburg State University
Nikolai_Krivulin@hotmail.com

CP1

Real Algebraic Geometry of Root Invariance: Spaces of Robust Stability Problems

Control theory and analysis meets us with different stability definitions - Schur, Hurwitz, hyperbolicity, D-stabilities etc. But even for such basic classes of problems as PI and PID-controller synthesis geometry of parametric stability problems is not well understood. Here provided a unified algebraic-geometric framework for polynomial and matrix stability problems. Author builds stratified filtered (infinite-dimensional) real closed space of stability problems for each stability definition and problem class. That construction provides natural proofs of various old and