ferred to as an almost invariant aggregate if transitions from x_t to x_{t+1} where $x_t \in \mathcal{A}$ and $x_{t+1} \notin \mathcal{A}$ are exceedingly rare. A Markov chain is referred to as nearly uncoupled if there are two or more disjoint almost invariant aggregates contained in its state space. Nearly uncoupled Markov chains are characterised by long periods of relatively constant behaviour, punctuated by sudden, extreme changes. We present algorithms for producing almost invariant aggregates of a nearly uncoupled Markov chain, given in the form of a stochastic matrix. These algorithms combine a combinatorial approach with the utilisation of the stochastic complement – a tool from the theory of stochastic matrices.

Companion Matrices and Spectrally Arbitrary Patterns

Kevin Vander Meulen, Brydon Eastman, In-Jae Kim, Bryan Shader

We explore some sparse spectrally arbitrary matrix patterns. We focus on patterns with entries in $\{*, \#, 0\}$, with $*$ indicating that the entry is a nonzero real number and $\#$ representing an arbitrary real number. As such, the standard companion matrix of order n can be considered to be a spectrally arbitrary $\{*, \#, 0\}$-pattern with $(n-1)$ $*$ entries and n $\#$ entries. We characterize all patterns which, like the standard companion matrices, uniquely realize every possible spectrum for a real matrix. We compare some of our work to recent results on companion matrices (including Fiedler companion matrices), as well as existing research on spectrally arbitrary zero-nonzero patterns.

Applications of Tropical Mathematics

Tropicalizing the simplex algorithm

Xavier Allamigeon, Pascal Benchimol, Stephane Gaubert, Michael Joswig

We present an analogue of the simplex algorithm, allowing one to solve tropical (max-plus) linear programming problems. This algorithm computes, using signed tropical Cramer determinants, a sequence of feasible solutions which coincides with the image by the valuation of the sequence produced by the classical simplex algorithm over the ordered field of Puiseux series. This is motivated in particular by (deterministic) mean payoff games (which are equivalent to tropical linear programming).

Tropical eigenvalues

James Hook, Francoise Tisseur

Tropical eigenvalues can be used to approximate the order of magnitude of the eigenvalues of a classical matrix or matrix polynomial. This approximation is useful when the classical eigenproblem is badly scaled. For example

$$A = \begin{bmatrix} 10^5 & -10^4 \\ i & 10^{-2} \end{bmatrix}, \quad \mathrm{Val}(A) = \begin{bmatrix} 5 & 4 \\ 0 & -2 \end{bmatrix}.$$

The classical matrix A has eigenvalues $\lambda_1 = (-1 + 0.000001i) \times 10^5$ and $\lambda_2 = (-0.01 - i) \times 10^{-1}$. By taking the componentwise-log-of-absolute-value A corresponds to the max-plus matrix $\mathrm{Val}(A)$, which has tropical eigenvalues $\mathrm{t}\lambda_1 = 5$ and $\mathrm{t}\lambda_2 = -1$.

In this talk I will give an overview of the theory of tropical eigenvalues for matrices and matrix polynomials emphasising the relationship between tropical and classical spectra.

Tropical matrix semigroups

Marianne Johnson

In this talk I will discuss some of my recent research with Mark Kambites and Zur Izhakian. In particular, I will give some structural results about the semigroup of 'tropical matrices', that is, the set of all matrices with real entries, with respect to a 'tropical' multiplication defined by replacing the usual addition of numbers by maximisation, and replacing the usual multiplication of numbers by addition. I will give some information about the *idempotent* tropical matrices (that is, the matrices which square (tropically) to themselves) and their corresponding groups.

Extremal properties of tropical eigenvalues and solutions of tropical optimization problems

Nikolai Krivulin

We consider linear operators on finite-dimensional semimodules over idempotent semifields (tropical linear operators) and examine extremal properties of their spectrum. It turns out that the minimum value for some nonlinear functionals that involve tropical linear operators and use multiplicative conjugate transposition is determined by their maximum eigenvalue, where the minimum and maximum are thought in the sense of the order induced by the idempotent addition in the carrier semifield. We exploit the extremal properties to solve multidimensional optimization problems formulated in the tropical mathematics setting as to minimize the functionals under constraints in the form of linear equations and inequalities. Complete closed-form solutions are given by using a compact vector representation. Applications of the results to real-world problems are presented, including solutions to both unconstrained and constrained multidimensional minimax single facility location problems with rectilinear and Chebyshev distances.

Integrality in max-algebra

Peter Butkovic, Marie MacCaig

The equations $Ax \leqslant b$, $Ax = b$, $Ax \leqslant \lambda x$, $Ax = \lambda x$,