# 4.CONTRIBUTED TALKS(CT)

GEOMETRY OF PER-ALTERNATE TRIANGULAR MATRICES

**Kiam Heong Kwa**
**University of Malaya, MALAYSIA**
Aug 6 (Wed), 10:30–10:55, (2B, 9B208)

In this talk, we study bijective adjacency invariant maps on per-alternate upper triangular matrices over an arbitrary field. Contrary to those on full matrices, it is found that such maps not only carry rank-2 matrices to rank-2 matrices, but may also fix all rank-2 matrices.

(This is a joint work with Wai Leong Chooi and Ming Huat Lim from University of Malaya.)
Keywords : Per-alternate triangular matrices, bijective adjacency invariant maps, rank-2 preservers
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CONVEXITY OF LINEAR IMAGES OF REAL MATRICES WITH PRESCRIBED SINGULAR VALUES AND SIGN OF DETERMINANT

**Pan-Shun Lau**
**The University of Hong Kong, HONG KONG**
Aug 6 (Wed), 10:55–11:20, (2B, 9B208)

For any $s = (s_1, ..., s_n) \in \mathbb{R}^n$, let $O(s)$ denote the set

$$\{U\mathrm{diag}(s_1, ..., s_n)V : U, V \in \mathrm{SO}(n)\},$$

where $\mathrm{diag}(s_1, ..., s_n)$ is the diagonal matrix with $s_1, ..., s_n$ as diagonal entries, and $\mathrm{SO}(n)$ the set of all real orthogonal matrices of order $n$ with positive determinant. It is clear that $O(s)$ is the set of all real $n \times n$ matrices with singular values $|s_1|, ..., |s_n|$ and their sign of determinant equal to the sign of $\prod_{i=1}^{n} s_i$. In this paper we consider linear maps $L$ from $\mathbb{R}^{n\times n}$ to $\mathbb{R}^2$, and prove that for any $s \in \mathbb{R}^n$ with $n \geq 3$, the linear image $L(O(s))$ is always convex. We also give an example to show that $L(O(s))$ may fail to be convex if $L$ is a linear map to $\mathbb{R}^3$. Our study is motivated by a result of RC Thompson which gave some necessary and sufficient conditions on the existence of a real square matrix with prescribed sign of determinant, prescribed diagonal elements and prescribed singular values. To prove our convexity result, we first consider two types of semi-group actions on $\mathbb{R}^n$ to obtain a new necessary and sufficient condition on Thompson's result. Then for $s, s' \in \mathbb{R}^n$, we apply this new condition to study inclusion relations of the form $L(O(s)) \subset L(O(s'))$ which hold for all linear maps $L$ under consideration. Such inclusion relations are then applied to give our convexity result on $L(O(s))$. The techniques we used are motivated by a result of YT Poon which gave an elegant proof on the convexity of the $c$-numerical range. We also extend the results to real non-square matrices. This is a joint work with NK Tsing.
Keywords : Singular values, linear images.
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APPROXIMATION PROBLEMS IN THE RIEMANNIAN METRIC ON POSITIVE DEFINITE MATRICES

**Rajendra Bhatia, Tanvi Jain***
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Aug 6 (Wed), 11:20–11:45, (2B, 9B208)

There has been considerable work on matrix approximation problems in the space of matrices with Euclidean and unitarily invariant norms. The purpose of this talk is to initiate the study of approximation problems in the space of positive definite matrices with the Riemannian metric. In particular, we focus on the reduction of these problems to approximation problems in the space of Hermitian matrices and in Euclidean spaces.
Keywords : Matrix approximation problem, positive definite matrix, Riemannian metric, convex set, Finsler metric
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DIRECT ALGEBRAIC SOLUTIONS TO TROPICAL OPTIMIZATION PROBLEMS

**Nikolai Krivulin**
**Saint Petersburg State University, RUSSIA**
Aug 6 (Wed), 10:30–10:55, (2B, 9B215)

Multidimensional optimization problems are considered within the framework of tropical (idempotent) algebra. The problems consist of minimizing or maximizing functions defined on vectors of a finite-dimensional semimodule over an idempotent semifield, and may have constraints in the form of linear equations and inequalities. The objective function can be either a linear function or a nonlinear function that is given by the vector operator of multiplicative conjugate transposition.
We start with an overview of known optimization problems and related solution methods. Certain problems that were originally stated in different terms, but can readily be reformulated in the tropical algebra setting, are also included.
First, we present problems that have linear objective functions and thus are idempotent analogues of those in conventional linear programming. Then, problems with nonlinear objective functions are examined, including Chebyshev and Chebyshev-like approximation problems, problems with minimization and maximization of span seminorm, and problems that involve the evaluation of the spectral radius of a matrix. Some of these problems admit complete direct solutions given in an explicit vector form. The known solutions to other problems are obtained in an indirect form of iterative algorithms that produce a particular solution if any or show that there is no solution.

Furthermore, we consider new optimization problems that extend certain known problems to take into account more general objective functions and more complicated systems of constraints. To obtain direct, explicit solutions to the new problems, we propose algebraic approaches, which offer the results in a compact vector form that is suitable for both further analysis and practical implementation, and guarantees low computational complexity of the solutions. For many problems, the approaches yield complete solutions to the problems of interest.
The solution of certain problems without constraints involves obtaining sharp lower or upper bounds for the objective function. We use the bound in an equation for the function, which is solved to find all vectors that yield the bound. To extend the solution to problems with linear equation and inequality constraints, we first obtain a general solution to the equation or inequality, and then substitute it into the function to arrive at one of the unconstrained problems with known solution.
To solve other problems, with and without constraints, we introduce an auxiliary variable to indicate the minimum value of the objective function. The problem is reduced to the solving of a linear inequality with a parameterized matrix, where the auxiliary variable plays the role of a parameter. We exploit the existence condition for solutions of the inequality to evaluate the parameter, and then take the general solution of the inequality as a complete solution to the initial optimization problem. We conclude with a brief discussion of application of the result to solve real-world problems in job scheduling, location analysis, decision making and other fields.
Keywords : Tropical Algebra, Idempotent Semifield, Optimization Problem, Nonlinear Objective Function, Linear Constraint
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## SOLVING LARGE-SCALE NONSYMMETRIC ALGEBRAIC RICCATI EQUATIONS BY DOUBLING

**Peter Chang-Yi Weng**
**Academia Sinica, TAIWAN**
Aug 6 (Wed), 10:55–11:20, (2B, 9B215)

We consider the solution of the large-scale nonsymmetric algebraic Riccati equation $XCX-XD-AX+B=0$, with $M \equiv [D,-C;-B,A] \in \mathbb{R}^{(n_1+n_2)\times(n_1+n_2)}$ being a nonsingular M-matrix. In addition, $A, D$ are sparse-like, with the products $A^{-1}u$, $A^{-\top}u$, $D^{-1}v$ and $D^{-\top}v$ computable in $O(n)$ complexity (with $n = \max\{n_1, n_2\}$), for some vectors $u$ and $v$, with three cases (1) $B = ee^\top$ and $C = qq^\top$ (2) $B$ and $C$ are of low-ranked and (3) $B$ and $C$ are low-ranked corrections of nonsingular diagonal matrices. We adapt an efficient method called structure-preserving doubling algorithm associated with the appropriate applications of the Sherman-Morrison-Woodbury formula and the sparse-plus-low-rank representations of various iterates. The resulting large-scale doubling algorithm has an $O(n)$ computational complexity and memory requirement per iteration and converges essentially quadratically, as illustrated by the numerical examples. As two examples in cases (1) and (2) respectively, one of NARE of dimension n=100,000, with 10 billion variables in the solution $X$, was solved using MATLAB on the Dell PowerEdge R910 computer to the accuracy of $O(10^{-14})$; the other of NARE of dimension n=1,000,000, with 1 trillion variables in the solution $X$, was computed using MATLAB on the HP SL390s computer to the accuracy of $O(10^{-13})$ within 1910 seconds.
Keywords : doubling algorithm, Krylov subspace, M-matrix, nonsymmetric algebraic Riccati equation, transport theory
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## A STRUCTURE-PRESERVING FLOW FOR SYMPLECTIC MATRIX PAIRS AND ITS DYNAMICS

**Shih-Feng Shieh**
**National Taiwan Normal University, TAIWAN**
Aug 6 (Wed), 11:20–11:45, (2B, 9B215)

The main purpose of this talk is the study of numerical methods for the maximal solution of the matrix equation $X + A^*X^{-1}A = Q$, where $Q$ is Hermitian positive definite. We construct an ODE of symplectic matrix pairs with a special structure, whose solution flow passes through all iteration points generated by the known numerical methods, including the fixed-point iteration, the structure-preserving doubling algorithm (SDA), and Newton's method provided that $A^*Q^{-1}A = AQ^{-1}A^*$. It turns out that this flow forms a natural measurement for the given numerical methods. On the other hand, the curve defined by the flow forms an invariant manifold for the SDA. We also show that the SDA has chaotic behavior whenever the matrix pair $\left(\begin{bmatrix} A & 0 \\ Q & -I \end{bmatrix}, \begin{bmatrix} 0 & I \\ A^* & 0 \end{bmatrix}\right)$ has eigenvalues of module 1.
Keywords : nonlinear matrix equation, structure-preserving flow, symplectic matrix pair, fixed-point iteration, structure-preserving doubling algorithm, Newton's method
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## ON BLOCK TRIANGULAR MATRICES WITH SIGNED DRAZIN INVERSE

**Wenzhe Wang**
**Harbin Engineering University, CHINA**
Aug 6 (Wed), 11:45–12:10, (2B, 9B215)

The sign pattern of a real matrix $A$, denoted by sgn $A$, is the $(+,-,0)$-matrix obtained from $A$ by replacing each entry by its sign. Let $\mathcal{Q}(A)$ denote the set of all real matrices $B$ such that sgn $B$ = sgn $A$. We say that a square matrix $A$ has signed Drazin inverse if sgn $\widetilde{A}^d$ = sgn $A^d$