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ESTIMATION OF WATER DISCHARGE DURING DEVELOPMENT OF GLACIAL AND SUBGLACIAL OUTBURST FLOODS

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Discussed is the mathematical model of the formation of hydrograph of the catastrophic flood of lakes overlapped in general case by a glacier of arbitrary thickness. This model is a further development of the Yu.B. Vinogradov's mathematical model created in 1976, which still has not lost its topicality. Some improvements to this model have resulted in descriptions of the evolution of outburst flooding of subglacial and intraglacial water bodies. The use of the bathymetric data *eo ipso* increasing the modelling accuracy is the main advantage of the model. Evaluation of the correctness of the modeling has been carried out for the process of intraglacial lake outburst in Dålk Glacier, East Antarctica, in 2017.

Mathematical modeling, outburst floods, subglacial reservoirs, hazardous hydrological phenomena

INTRODUCTION

As it was likewise defined by W.M. Davis [*1962*] in his concept of landscape-development stages, the life cycle of a body of water developing within the cryosphere begins from its inception, which is followed first by the steady-state, and then by its degradation, until the water body completely ceases to exist. The final phase is often accompanied by the onset of period of outburst floods generally termed glacier lake outburst flood (GLOF). This phenomenon reflects the natural glacial and subglacial processes dynamics. Predicting glacial outburst floods in itself is a challenge, and is almost impossible with respect to intraglacial or subglacial reservoirs. This is associated primarily with the fact that water bodies characterized by rapid discharges are usually located in hard-to-reach areas where systematic observations are unavailable. The characteristics of glacial outburst floods are therefore commonly derived from the aposteriori information.

Given that most implications of outbursts of glacial and subglacial lakes may entail essential damage to the surrounding area and even can potentially cause human casualties [*Vinogradov, 1977; Fowler, 1999; Richardson and Reynolds, 2000; Björnsson, 2002; Chernomorets et al., 2003, 2007; Popovnin et al., 2003; Popov et al., 2017*], the question as to whether there are alternative ways of obtaining relevant information is critical. Physical modeling, which is certainly capable of providing most accurate representations, can be one of the options, however in practice this technique is often seen to be too costly or sophisticated. This can be exemplified by the experiments on artificial mudflow dammed natural stream channel. To the best of the authors' knowledge, the pioneering attempts were guided by S.P. Kavetsky [*1957*]. The large-scale works conducted on August 27, 1972 and August 19, 1975 by the KazNIGMI staff and led by Yu.B. Vinogradov at the unique site for studing the mudflow phenomena on the Chemolgan river (Karasay, Almaty region, USSR) yielded most important results and findings [*Vinogradov, 1976*]. These were presented in a 19- min scientific research-based movie titled "A Word about Mudflow" filmed by the Kazakh Research Institute of Hydrometeorology (KazNIGMI). Some slides from the movie are shown in Fig. 1.

Advances in computing technologies during the past decade have enabled mathematical modeling to become another alternative way to obtain information about the dynamics of certain natural phenomena and processes. The foundations of contemporary modeling of catastrophic lake water discharge were laid in the 70s of the last century. J. Nye's pioneering fundamental research into this problem is presented in [*Nye, 1976*], whose model is underpinned by the glacier hydrology (theory of channelized water flow through glaciers) discussed in detail in [*Röthlisberger, 1972*]. Numerous scientific works have been devoted to the study of rapid water discharges from lakes in mountainous areas. In respect to polar regions, this subject matter has gained the greatest traction after the discovery of subglacial lake Vostok [*Ridley et al., 1993; Kapitsa et al., 1996*] and subglacial floods (jökulhlaups) [*Wingham et al., 2006*]. Numerous overviews and detailed descriptions of contemporary models which focus on various aspects of this process, have been amply discussed in the scientific publications [*Björnsson, 1992, 2002; Clarke, 2003; Evatt et al., 2006; Fowler, 2009; Pattyn, 2013*], as well as in the dedicated monograph [*Glazovsky and Macheret, 2014*].

The mathematical model proposed in this paper is an extension of the earlier version developed by Yu.B. Vinogradov [*1976*]. Its main advantage consists in the comprehensive approach, since, on the one hand, it adheres to the strictness of physical laws,

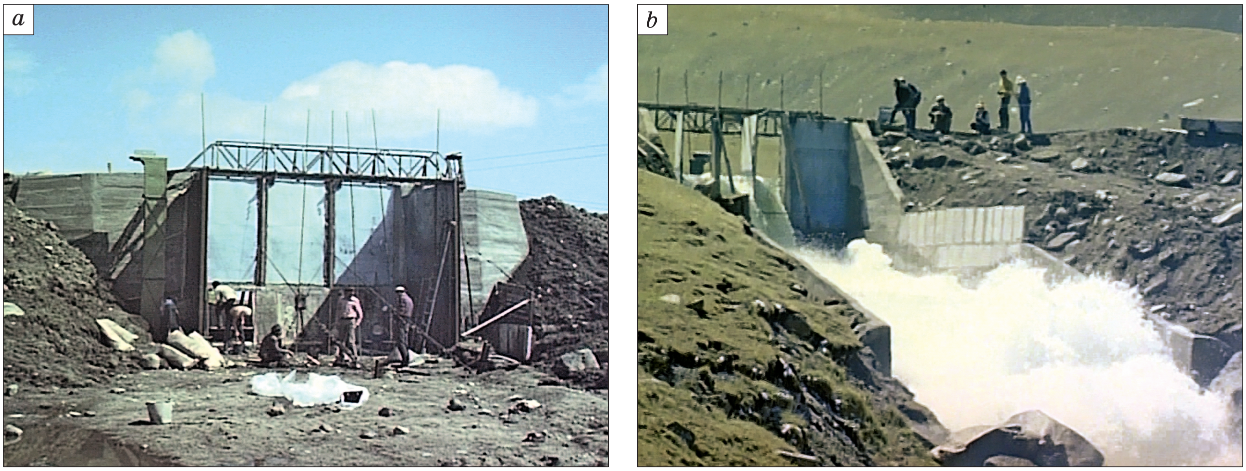

Fig. 1. Modeling of artificial mudflow in the Chemolgan river basin.

a – a dam used for constructing a water reservoir; *b* – water discharge from a reservoir; *c* – mudflow set in motion.

and, on the other hand, includes initial and boundary conditions that can be obtained from field measurements. From among the existing calculus methods, this model uses data and parameters that can be derived from empirical relations while performing field works without involving calculations. Basically, this model was used to estimate the volume of outburst floods in mountainous areas [*Gnezdilov et al., 2007; Kidyaeva et al., 2018*], however, our research has shown its appropriateness for studying this phenomenon in polar regions.

DESCRIPTION OF THE MATHEMATICAL MODEL

We use the Yu.B. Vinogradov's mathematical model [*1976*] for evaluation of the processes occurring during devastating GLOFs. Let us consider a water body with depth H_w which is overlapped by the glacier with ice thickness H_r and assume that a sloping channel with sectional area ω generally develops in its bottom part during the outburst flood. Consider the movement of an infinitely small volume of water along the tunnel (flow path) at a distance l, corresponding to the elevation difference Δh (Fig. 2).

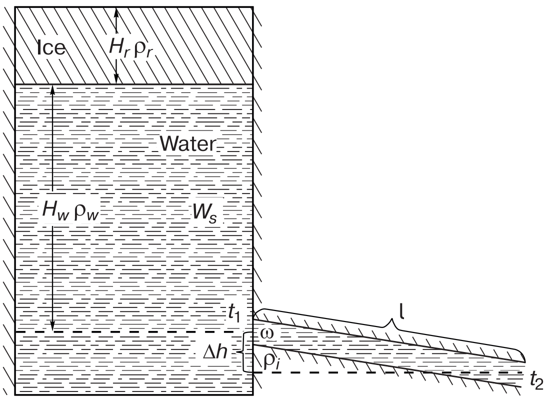

Fig. 2. Schematic representation of a water body and discharge water channel in the discussed models.

ρ_w – water density in lake; H_w – lake depth (to the mid point of the channel slope); l – channel slope depth; Δh – difference between heights of the channel slope; ω – cross-sectional area of the channel slope; t_1, t_2 – inflowing/outflowing water temperature in the sloping channel; ρ_i – density of the tunnel substratum; W_s – volume of lake water between the overburden and mid point of the channel slope; H_r – overburden thickness; ρ_r – overlying glacier density generally inconsistent with ρ_i.

At the initial moment of time, infinitely small volume of water has some potential energy E_p, which, upon reaching the terminal point, goes into the kinetic energy of water movement, as well as the energy consumed on heating the flow and on melting of the ice walls of the channel:

$$E_p = m_w \frac{v^2}{2} + c_w m_w \Delta t + \lambda V_i \rho_i, \qquad (1)$$

where m_w is the weight of infinitely small volume of water; v is its flow velocity; c_w is specific heat capacity of water (4190 J/(kg·°C)); λ is the specific heat of fusion of ice ($3.34 \cdot 10^5$ J/kg); V_i is the volume of melted ice; ρ_i is its density (917 kg/m^3) [*Paterson, 1981*]; Δt is the temperature difference at the tunnel closure t_2 and in the lake t_1 ($\Delta t = t_2 - t_1$). Here and below the dimensions are given in the SI units.

The potential energy E_p of the elementary volume of water consists of the potential energies of the position and pressure. The first value is related to the vertical movement of the elementary volume of water at Δh, and the second value – with the pressure in excess due to the overburden (the water thickness above and the overlapping glacier). Given that, purely hypothetically, the density of the overlying glacier and the channel walls may differ, we will designate the potential energy of pressure as ρ_r, and most likely, $\rho_r = \rho_i$. Thus,

$$E_p = m_w g \Delta h + m_w g H_w + m_w g H_r \frac{\rho_r}{\rho_w}, \qquad (2)$$

where ρ_w is density of water (1000 kg/m^3); g is free fall acceleration (gravity factor) (9.8 m/s^2).

By equating (1) and (2) and considering that $m_w = Q\rho_w \Delta T$, we express V_i as

$$V_i = \frac{Q\rho_w \Delta T}{\lambda \rho_i}\left[g\left(\Delta h + H_w + H_r \frac{\rho_r}{\rho_w}\right) - \frac{v^2}{2} - c_w \Delta t \right], \qquad (3)$$

where Q is water discharge per time interval ΔT.

We obtain the ratio in order to change the channel cross-sectional area $d\omega$, $d\omega = V_i/l$. Following Yu.B. Vinogradov [*Vinogradov, 1976, 1977; Vinogradov and Vinogradova, 2010*], we will assume that $v \to 0$. Then

$$d\omega = \frac{\rho_w}{l\lambda\rho_i}\left[g\left(\xi + H_w\right) - c_w \Delta t \right] dW,$$
$$\xi \equiv \Delta h + H_r \frac{\rho_r}{\rho_w}, \qquad (4)$$

where $dW = Q\Delta T$ in itself, represents the considered elementary volume of water.

In his model, Yu.B. Vinogradov proposed to express the lake depth H_w through the volume of water W as a power function written as $H_w = aW^m$ [*Vinogradov, 1976, 1977; Vinogradov and Vinogradova, 2010*]. Figure 3 shows real dependences $H_w = \mathbf{F}(W)$ built for three Antarctic subglacial water bodies and underpinned by well studied bathymetries. These are subglacial lakes (Vostok [*Popov and Chernoglazov, 2011*], Concordia [*Thoma et al., 2009*] and Ellsworth [*Ross et al., 2011*]), as well as a series of water bodies located in the area of Progress station (Larsemann Hills, East Antarctica).

As it follows from Fig. 3, these absolutely different water bodies have generally identical distribution patterns, and therefore the presence of functional dependence $H_w = \mathbf{F}(W)$ appears perfectly reasonable. However, the proposed approximation by power function (given for each lake) does not always reflect it realistically (Fig. 3).

To obtain more accurate ratios between ω and W we integrate (4) allowing for the remarks stated, given the conditions corresponding to the GLOF's starting point and to a certain time T. Since the channel has not formed at the initial moment of time, the lower limit of integration over its cross-sectional area $d\omega$ is equal to zero. At this time point, the volume of water in the lake to the channel's midpoint depth is known and equal to W_s. This is the upper limit of integration with respect to dW. We therefore have

$$\omega = \frac{\rho_w}{l\lambda\rho_i}\left[\left(g\xi - c_w \Delta t\right)\left(W_s - W\right) + g\int_W^{W_s} \mathbf{F}(W)\, dW \right]. \qquad (5)$$

Let us turn now to the integral remaining in the right part of the expression (5) which corresponds to region under the curve of function defined by tabular data $H_w = \mathbf{F}(W)$ in the interval between W and W_s. It would be logical to assume that the best result within the frames of the considered model can be derived from calculating specifically on real curves shown in Fig. 3 as an example. This can best be achieved by numerical integration using any of the known methods. In the calculations, the authors used the trapezoid method [*Mochalov and Tsukerman, 1982; Volkov, 1987*]. We use $\mathbf{S}(W)$ to denote the result of numerical integration of arbitrary function $H_w = \mathbf{F}(W)$ at certain W. Then, taking into account its calculus method, we will write the ratio (5) as

$$\omega = \frac{\rho_w}{l\lambda\rho_i}\left[\left(g\xi - c_w \Delta t\right)\left(W_s - W\right) + \mathbf{S}(W) g \right]. \qquad (6)$$

In [*Vinogradov, 1976, 1977*], the author provided a substantiation of the use the model of a "short tube" which allowed to estimate the discharge

$$Q = \alpha \omega^{1.25} \sqrt{\mathbf{F}(W)},$$

where α is some dimensionless empirical coefficient.

Then the ratio (6) takes the form

$$Q = \alpha \left\{ \frac{\rho_w}{l\lambda\rho_i}\left[\left(g\xi - c_w \Delta t\right)\left(W_s - W\right) + \mathbf{S}(W) g \right] \right\}^{1.25} \times$$
$$\times \sqrt{\mathbf{F}(W)}. \qquad (7)$$

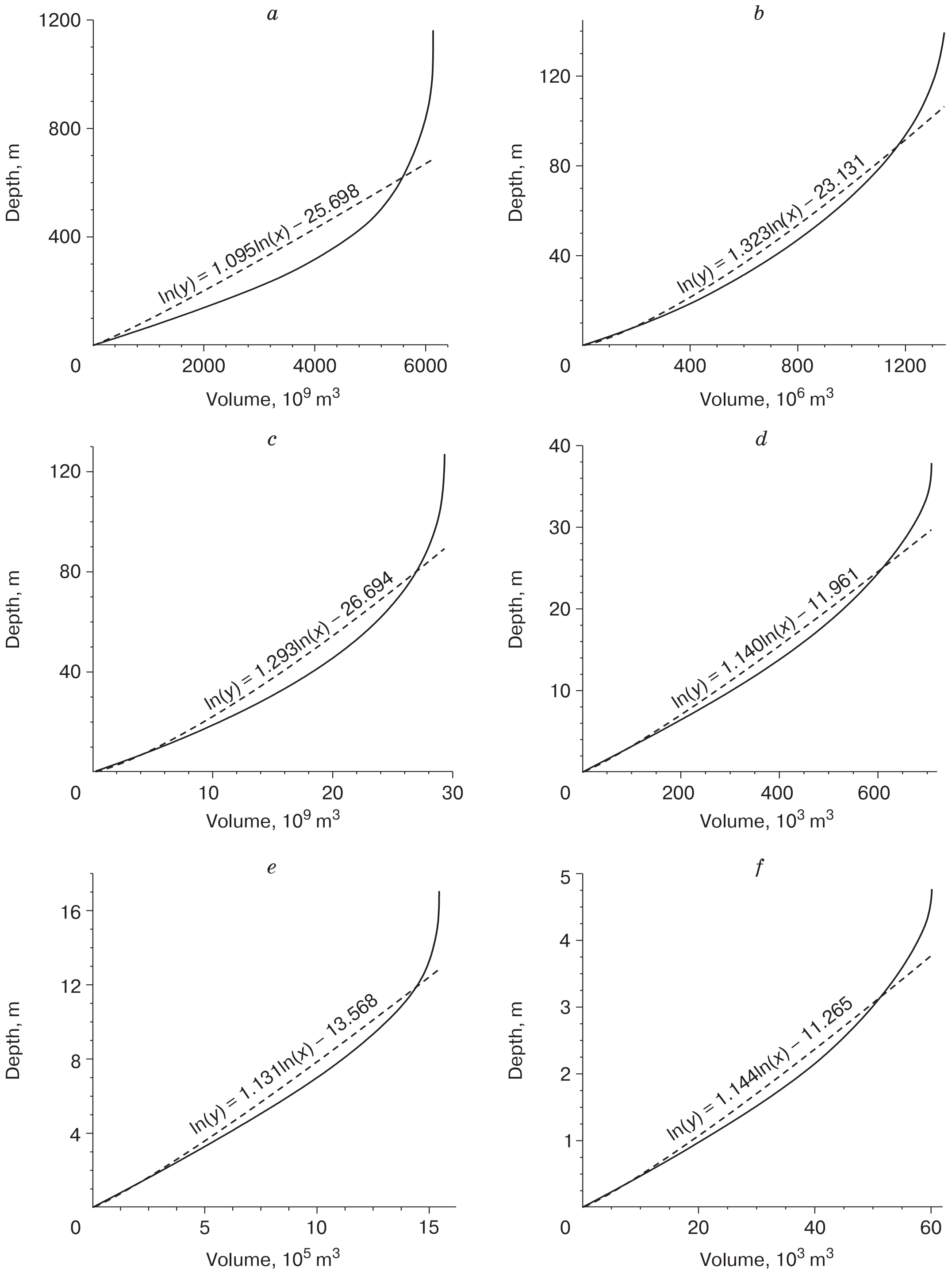

Fig. 3. A relationship between the volume of water masses and depth for lakes: Vostok (*a*), Ellsworth (*b*), Concordia (*c*), lake within Dålk Glacier (*d*), Scandrett (*e*) and Discussion (*f*) lakes.

Dashed line shows the power law fitting of real data.

The approximate dependence α on the tunnel with length l, derived from the field measurements, is given in [*Vinogradov and Vinogradova, 2010*]. In the interval between 1.9 and 50 km, it can be approximated by the relation

$$\lg \alpha = -1.124 \lg l + 0.7289 \quad (8)$$

within the accuracy of 2 %, with l expressed in kilometers. For limit values (see figure in [*Vinogradov and Vinogradova, 2010*]) we have $\lim_{l\to 0}\alpha(l) = 2.7$, $\lim_{l\to\infty}\alpha(l) = 0.07$. By omitting further calculations, the ratio for the difference in water temperatures in the lake and the tunnel, taking into account the new expression for H_w:

$$t_1 - t_2 = -\Delta t = t_1\left\{1 - \exp\left[-\frac{4000\alpha^{0.30} l}{Q^{0.55}\rho_w c_w}\mathbf{F}(W)^{0.15}\right]\right\}. \quad (9)$$

Upon substituting (9) into (8), our model reaches its final form

$$Q = \alpha \mathbf{F}(W)^{0.5}\left\{\frac{\rho_w g}{l\lambda\rho_i}\left[(\xi + \varsigma)(W_s - W) + \mathbf{S}(W)\right]\right\}^{1.25},$$

$$\varsigma = t_1\frac{c_w}{g}\left\{1 - \exp\left[-\frac{4000\alpha^{0.30} l}{Q^{0.55}\rho_w c_w}\mathbf{F}(W)^{0.15}\right]\right\}. \quad (10)$$

The solution of this equation cannot be obtained analytically, however given that (10) might be represented by $\mathbf{F}(Q, W) = 0$, it is solved numerically, in particular, by the Newton's method [*Volkov, 1987*]. We thus obtain a hydrograph, i.e. the rate of flow (discharge) Q over time T. This problem is solved numerically as well. To do this, we divide the total amount of water W_s flowing from the lake into J parts ($J \to \infty$), with each having volume $\delta W = W_s/J$. Then, according to (10), the discharge Q_j corresponds to the j-th volume $W_j = j\delta W$. Besides, the volume of water δW corresponding to this discharge transferred through the cross section of a tunnel over the time is $\delta T_j = \delta W/Q_j$. The total time T_j (since the water of lake broke the ice blockage) has amounted to

$$T_j = \sum_{i=0}^{j}\frac{\delta W}{Q_i} \quad (0 \leqslant j \leqslant J), \quad W_s = \sum_{i=0}^{J}\delta W. \quad (11)$$

Expressions (10) and (11) are found appropriate for compiling a computer program (which actually is a product of this study) applicable to calculating the desired hydrograph [*Popov et al., 2018b*].

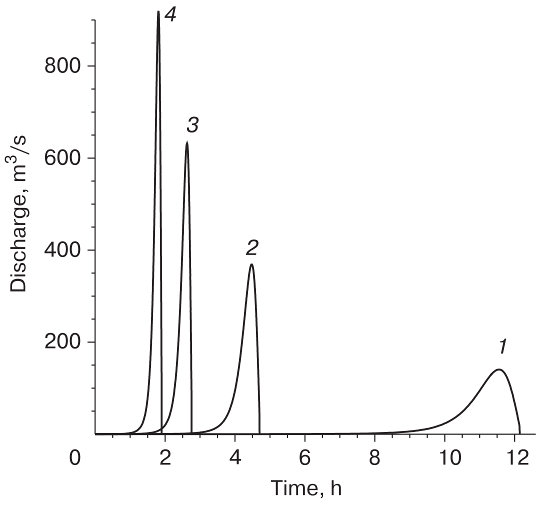

Fig. 4. Modeled glacier lake outburst discharge hydrographs for varying thicknesses of the overlapping glacier.

1 – 0 km; *2* – 1 km; *3* – 2 km; *4* – 3 km.

MODELING OF GLACIAL AND SUBGLACIAL LAKE OUTBURSTS

In order to evaluate correctness of the above discussed model, we consider in detail the outburst flow of englacial reservoir formed in the area of Dålk glacier, East Antarctica [*Popov et al., 2017*]. According to the tachymetry and GPR surveys, the lake volume is 708 690 m^3 [*Boronina et al., 2018*]. This water body is physically described as: rounded in shape; the coastline is slightly indented; the slopes are mostly steep, sometimes vertical; the linear dimensions are 188 × 250 m; the average depth is about 27 m [*Boronina et al., 2018; Popov et al., 2018a*]. Proceeding from the preliminary assumptions, the water body is located within the glacier (i.e. intraglacial), and during the initiation of outburst flooding, an inclined (sloping) channel forms in its bottom portion with a length of 1134 m, which corresponds to the distance from the lake border to the glacier front. According to the airborne geophysical data, the elevation difference in the tunnel between the inflow and outflow is 764 m [*Popov and Pozdeev, 2002; Popov and Kiselev, 2018*]. In a number of cases, the outburst flood discharge hydrograph was estimated from a series of calculations. The first hydrograph was calculated for the real situation when the ice thickness on the lake was 0 m, i.e. the amount of ice varies from insignificant to nonexisting (Fig. 4, curve *1*). Other synthetic hydrographs were calculated for a hypothetical situation when the thickness of the overlying ice is 1, 2 and 3 km, respectively (Fig. 4, curves *2–4*).

The temperature in the water body for all the simulation cases is assumed to be 0 °C, based on the assumption that it is overlapped by either a thick glacier or a small layer of seasonal ice, i.e. lake water is at the phase transition temperature (its dependence on pressure was ignored). The density of the dam (ice blockage) broken through by the sufficiently accumulated water is 910 kg/m^3 [*Paterson, 1981*]. The empirical parameter α was calculated based on the tunnel length l, using the ratio (8).

The model calculations have shown, that the time from the initiation of water outflow to its peak discharge tends to increase, as the overlying ice thickness decreases. The largest flow rate (among all the considered scenarios of outburst floods) is observed in the fourth case with the overlying glacier 3 km in thickness. The flow reached maximum after 1 hour 48 minutes from the start of discharge and its drainage capacity was about 920 m^3/s. When the peak is passed, the outflow rate first sharply falls, then completely ceases after 1 h 54 min.

A similar situation is observed with intermediate glacier thicknesses over the lake. The hydrograph shape remains to be asymmetric showing a gentle rise combined with sharp decline. However, the maximum discharge rates are tending to be lower: about 630 and 370 m^3/s at glacier thickness of 2 and 1 km, respectively. The time of their inception also increases as the glacier decreases in thickness. Thus, the glacier lake discharge was peaking after 2 h 38 min (the overlying ice thickness: 2 km), and after 4 h 28 min (ice thickness: 1 km). A similar pattern is characteristic of the period of the discharge decline which started after reaching its peak value: 8 min after the maximum value in the second calculation case (glacier thickness: 2 km); and about 15 min in the third case (glacier thickness: 1 km).

The situation somehow differs in the absence of ice cover on the lake. The lake outflow hydrograph is also characterized by negative asymmetry with a gentle rise, whereas the discharge declines less sharply. The glacial lake discharge reaches its maximum (141 m^3/s) after 11 h 27 min from the GLOF onset, and complete termination of the drainage occurred after 12 h 9 min from the release of outflow. The difference between the discharge volumes is quite understandable if we keep it in mind that in the first case we have water pressure in excess of about 270 atm of the glacier overburden pressure. The considered model can also be applied to glacial lake break through the ice-snow and snow dam, by changing the λ and ρ_i values for the relation (10), accordingly.

CONCLUSION

The results presented have allowed an inference that the application of the adapted Yu.B. Vinogradov's model is appropriate for calculating the outflow hydrographs formed during the glacial lake outburst floods (both intraglacial and subglacial lakes). This also enables evaluation of the outburst flood discharge variation over time, its maximum volume and the total time of hydraulic transmissivity, i.e. to quantify this process. Besides, in the case when the flood path is formed not at the lake bottom, it is possible to estimate the volume of flood discharge. The model also allows to calculate the hydrograph at any distance from the outburst flood center. Importantly, the calculated hydrograph is not only of great scientific and applied interest, but it equally allows to use the data obtained as correct initial conditions for the models of extensive flood-affected areas and predicting the propagation of surges of devastating floods.

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