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ELECTROMAGNETIC WAVES SCATTERING ON AN ARRAY COMPOSED OF THIN DIELECTRIC OBJECTS

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This research concerns the scattering of electromagnetic waves by thin dielectric impediments in 2D geometry. Dielectric and geometric properties of the impediments are modeled by varying the inhomogeneous component of the refractive index. It is assumed that the impediments have arbitrary finite lengths, and their widths are much lesser in comparison with the incident wavelength. In contrast to the previous approaches the proposed one enables us to solve the scattering problem simultaneously on several objects in the case where the impediments structure is not a regular one. A system of integral equations to provide a solution to the problem is derived. A unique solvability of the obtained system is discussed. Bibliography: 9 titles.

1. Statement of the problem

This study suggests a generalization of the previously proposed [9] modeling equation to the case of electromagnetic waves scattering simultaneously on several dielectric obstacles. Consider the case of horizontal polarization where the electromagnetic field is generated by an electric current of frequency ω flowing along the y axis. It is assumed that the geometry of the problem does not depend on the Cartesian coordinate y. Dropping the time multiplier $\exp(-i\omega t)$, one can reduce the original Maxwell equations to a scalar Helmholtz equation with respect to the electric field component $V = E_y(x, z)$:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} + k^2(1 + i\alpha + n(x, z))V = Q_0(x)\delta(z), \tag{1}$$

where $k = \omega/c$ is the wave number, $\alpha > 0$ is a small dissipation parameter. The introduction of this parameter ensures the uniqueness of the solution owing to the limiting absorption principle. The electromagnetic field is generated by the volume source $Q_0(x)\delta(z)$ on the right-hand side of Eq. (1).

The dielectric properties of the scattering bodies are modeled by variation of the refractive index $n(x, z)$. It is assumed that the function $n(x, z)$ is compactly supported and vanishes outside the bodies of impediments. The discontinuity jumps of the refractive index inside the impediments, as well as on their boundaries, are admissible.

One of the distinguishing features concerning the proposed statement of the problem lays in the fact that the electromagnetic field is generated by a volume source on the right-hand side of the Helmholtz equation rather than by a flat incident wave. Thus, varying function $Q_0(x)$ one can model a point source or a directional one. Moreover, the present study enables us to consider a finite number of obstacles arranged parallel to the axis x. Restrictions on the mutual arrangement of the impediments are not imposed. The schematic description of the considered problem is depicted in Fig. 1.

The paper is structured as follows. In the next section, we derive a system of integral equations that model the scattering on the array of thin dielectric bodies. The properties of the derived system are examined in Sec. 3. The existence and uniqueness of a solution to

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the obtained system are studied in Sec. 4. In Sec. 5 we discuss possible applications of the proposed model to solve some unstudied classical and modern diffraction problems.

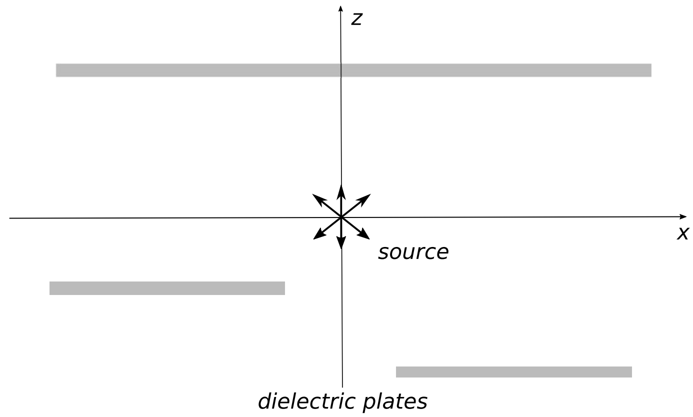

Fig. 1. Schematic description of the considered problem.

2. Derivation of the system of integral equations

Following [9], we use dimensionless variables and represent the problem under consideration in the form of an integral equation:

$$\widehat{V}(\lambda, z) + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \widehat{G}(z, z', \lambda) R(z', \lambda - \lambda') \widehat{V}(\lambda', z') d\lambda' dz' = \widehat{Q}_0(\lambda) \widehat{G}(z, 0, \lambda), \tag{2}$$

where

$$\widehat{V}(\lambda, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} V(x, z) e^{-i\lambda x} dx,$$

$$\widehat{Q}_0(\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} Q_0(x) e^{-i\lambda x} dx,$$

$$R(z, \lambda - \lambda') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} n(x, z) e^{-ix(\lambda - \lambda')} dx.$$

Owing to the principle of limiting absorption, the Green function $\widehat{G}(z, z', \lambda)$ can be written down as

$$\widehat{G}(z, z', \lambda) = -\frac{a - id}{2(a^2 + d^2)} e^{-(a+id)|z - z'|},$$

where $a = \sqrt{\frac{\sqrt{(\lambda^2-1)^2+\alpha^2}-(1-\lambda^2)}{2}}$, $d = -\sqrt{\frac{\sqrt{(\lambda^2-1)^2+\alpha^2}+(1-\lambda^2)}{2}}$.

We model the refractive index $n(x, z)$ in accordance with the representation

$$n(x, z) = \sum_{j=1}^{m} b_j(x) \eta_j(z), \tag{3}$$