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Weighted average price management of manufacturer sales on commodity exchanges

Sergey A. Vavilov*

Department of Economic Cybernetics, Faculty of Economics
St. Petersburg State University, 62 Chaikovskogo St.
St. Petersburg, 191123, Russia

Konstantin S. Kuznetsov†

Department of Applied Cybernetics
Faculty of Mathematics and Mechanics
St. Petersburg State University, 28 Universitetsky Prospekt, Peterhof
St. Petersburg, 198504, Russia

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Abstract

This paper proposed a method to control the weighted average price of manufacturer sales. The study of the posed problem is necessary to hedge manufacturer profits from price slumps in commodity markets. The elaborated method is applied to managing the trading process on real world commodity exchanges to prove its effectiveness.

Keywords: Decision analysis; finance; risk management; stochastic processes; sales management.

1. Introduction

Instability and uncertainty in world price dynamics including the trading of gas, oil, timber and other widely demanded consumer goods makes it necessary to revise various traditional mutual relations in business between buyers and manufactures of the goods. In Sec. 1, the idea concerns the refusal of long-term goods

†Corresponding author.
Email addresses: *savavilov@inbox.ru, †kostas.92@mail.ru

delivery contracting in large volumes with fixed prices. A buyer conscious of the indeterminacy in market pricing prefers to acquire goods in relatively small volumes at specialized commodity exchanges with obligatory delivery within small periods of time. There is strong evidence of the mutual relations between gas corporations, which typically refer to manufacturers and suppliers with European consumers. Presently in Europe, there are 13 storage hubs for gas and 7 commodity exchanges associated with them. They provide trading by different volumes of gas with the delivery obligation within two days from the instant of striking a bargain. Moreover, high liquidity and the total volume covering a considerable part of the world trade by gas characterize trading activities. It is reasonable to suppose that a similar turn of events forces a seller–manufacturer to react to his own strategy of sales management. Such a strategy is typically used to protect manufacturer profit from market price decreases. The management increase of the weighted average price of goods sold by a manufacturer can be regarded as one such strategy. One can easily see that a similar strategy not only implies non-uniform sales volumes but also executes reverse operations, meaning the purchasing of the same goods in certain volumes and at certain instants of time. The goal of the present study is to describe one of the possible sales management algorithms for the opportunity to construct the abovementioned strategy in explicit form and to demonstrate its effectiveness using examples of gas trading on European specialized commodity exchanges. This paper is organized as follows. In Sec. 2, mathematical formalization of the posed problem is given. Section 3 is devoted to management construction of the proposed method with the stable increase of the weighted average price of the sale of goods, which can be achieved only with the positive performance of certain conditions. Finally, in Sec. 4, the application of the proposed management scheme is given for gas trading at the specialized European Energy Exchange.[1] In Sec. 5, we discuss the obtained results and the weaknesses of the proposed method, which are mostly connected with the necessity to follow the sales volume that is usually planned in advance within a given time interval considered as the trading horizon. The objective is to match the volume of goods sold with the volume of goods intended for sale. On the other hand, the amount of goods sold in the application process of the below elaborated management scheme is shown to be a random process. In response to this contradiction, we introduce the succession of trading portfolios that should be closed if the conditions specified below are fulfilled.

The additional materials are represented in Appendix B, Appendix C, which describe results of trading simulator for the proposed control system. By making use of this simulator and the corresponding input data containing the prices of

[1]European Energy Exchange AG, 2017, https://www.eex.com/(accessed 23 September 2017).

struck bargains, we can model trading at the European Energy Exchange within the proposed strategy for the period of time from 01-05-2010 until 06-06-2016.

In this study, we follow continuous financial modeling based on geometrical Brownian motion (Oksendal, 1998). Moreover, we use the earlier elaborated management method as an alternative to the self-financing strategy. Originally this approach was proposed in (Vavilov, 2001; Vavilov and Ermolenko, 2007), and its subsequent extended version was published in (Vavilov and Ermolenko, 2008). It was applied to the operations of financial markets when long positions occur. In contrast to the aforementioned publications, we use modification of the previous technique for the case of short positions and with application to various commodity markets, and the method of widening the price band in the trading process is described in this study.

2. Mathematical Formalization of the Posed Problem

This study is based on the fact that for the given time interval $[0, T]$, the bargain prices with respect to the unit of goods traded on the commodity exchange follow a stochastic differential equation describing the process of geometrical Brownian motion as follows:

$$dx_t = c_t x_t dt + \sigma_t x_t dW_t, \tag{1}$$

where c_t, σ_t are coefficients of drift and volatility, respectively, which are represented by some random functions, and W_t is a standard Wiener process. Further, the observable realization of each stochastic process in contrast to the process itself are denoted by the same symbol but only with the wave, for instance $\tilde{x}_t$ and x_t.

Definition. Under the goal of management, we understand the construction of such function $\tilde{a}_t(\tilde{a}_0 = 0)$, where $\tilde{a}_t$ is the volume of goods sold on the interval of time $[0, t], t \in [0, T]$, which would provide fulfilment of condition

$$\tilde{x}_T^{av} > \max_{t \in [0, T]} \tilde{x}_t, \tag{2}$$

where $\tilde{x}_t^{av}$ is the weighted average price of the sold goods on the same time interval $[0, t]$.

Moreover, the feedback usage in the function $\tilde{a}_t$ construction process implies that the only information that may be taken into account is the data about the market price of the bargains on the time interval $[0, t]$. In other words, we proceed from the fact that coefficients in Eq. (1) cannot be evaluated with *a priori* accuracy in real time. One can easily see that the fulfilment of condition (2) is only possible at the expense of increasing the speculative profit. The latter profit occurs as the result of the gradual reinvestment of the cash obtained in the processing of sales.

Reinvestment means the acquisition of the same goods with further resale based on the analysis of the market price dynamics from the very start of the management process. In the next section, we construct one possible management scheme, which provides the validity of condition (2) when certain conditions are fulfilled.

One should note that the fulfilment of condition (2) on most of interval $[0, T]$ still causes the chosen strategy to be the least admissible. The advanced forecast of substantial market price growth with respect to shorter time intervals in comparison with a given time horizon for trading seems improbable.

3. Description of the Desired Management Method

This study follows continuous stochastic modelling in finances (Oksendal, 1998) but within an approach that is an alternative to a self-financing strategy (Vavilov and Ermolenko 2008). We also expand the previously discussed technique for the case of short position with application to trading operations on commodity markets. Moreover, in contrast to previous works, the widening of the initial price band in this study is admissible in the trading process.

Consider function f_t, which is set by the equation

$$f_t = a_t x_t + m_t, \tag{3}$$

where $a_t = a(t, \omega)$ is a random function defining the unit quantity of goods sold for the time interval $[0, t]$, and function $m_t = m(t, \omega)$ will be described below.

We assume that at each instant of time t, function f_t is subject to some control that leads to equality fulfilment

$$df_t = a_t dx_t + l(t, x_t) dt, \tag{4}$$

where dx_t is defined by the right-hand side of Eq. (1), and the quantity $l(t, x_t)dt$ may be interpreted as the amount of cash having been obtained as a result of sales in the time interval $[t, t + dt]$ subsequently deduced from the management scheme. Simultaneously, the other part of the profits is reinvested for acquiring the same type of goods. Consequently, $l(t, x_t)$ acts as a control function.

Applying the procedure of calculating the stochastic differential to the left- and right-hand sides of relationship (3), which implies the existence of stochastic differentials da_t and dm_t, one arrives at the relationship

$$df_t = a_t dx_t + x_t da_t + dx_t da_t + dm_t.$$

This relationship makings use of Eq. (4) and may be rewritten as follows:

$$dm_t = -x_{t+dt} da_t + l(t, x_t) dt,$$

where x_{t+dt} is considered $x_{t+dt} := x_t + dx_t$ or in the integral form,

$$m_t = -\int_0^t x_{\tau+d\tau} da_\tau + \int_0^t l(\tau, x_\tau) d\tau. \tag{5}$$

We define the "speculative profit" as the result of the effected trading operations on the time interval $[0, t]$ as follows:

$$\tilde{p}_t = \tilde{a}_t \tilde{x}_t - \int_0^t \tilde{x}_{\tau+d\tau} d\tilde{a}_\tau. \tag{6}$$

Keeping in mind Eqs. (3), (4) and (5), the previous ratio is equivalent to the relationship

$$\tilde{p}_t = \tilde{f}_t - \int_0^t l(\tau, \tilde{x}_\tau) d\tau. \tag{7}$$

The concept of "speculative profit" needs a detailed explanation. Suppose that at the instants of time $1, 2, \ldots, n$, the quantity of commodity units $b_1, b_2, \ldots, b_n$ were sold or bought at prices $x_1, x_2, \ldots, x_n$ while trading operations occurred within the short position. Thus, for arbitrary j, the inequality $\sum_{i=1}^{j} b_i \leq 0$ is valid; however, b_i is taken as negative when the commodity is sold and as positive when the commodity is acquired. Thus, the amount of cash obtained as the result of these operations constitutes the sum equal to $[-\sum_{i=1}^{n} b_i x_i]$. In the continuous version, this quantity may be written as $[-\int_0^t \tilde{x}_{\tau+d\tau} d\tilde{a}_\tau]$. Here, $x_{\tau+d\tau}$ means that evaluation of the integral implies the integrand calculation on the right boundaries of the partition interval. Otherwise, when the integrand is calculated with respect to the left boundary, the trading operations are not executed according to the current prices but in accordance with the past values. On the other hand,

$$-\int_0^t \tilde{x}_{\tau+d\tau} d\tilde{a}_\tau = \tilde{a}_t \tilde{x}_t - \int_0^t \tilde{x}_{\tau+d\tau} d\tilde{a}_\tau + |\tilde{a}_t| \tilde{x}_t.$$

Thus, the obtained profit may be represented as the sum of the speculative profit and the real capital corresponding to the quantity $|\tilde{a}_t|$ of commodity units sold over time interval $[0, t]$.

Further, we introduce the notion of the price band $[a, b]$. Originally, it is supposed that the observable price of the commodity unit $\tilde{x}_t$ belongs to the mentioned band on the entire time horizon $[0, T]$ of control; consequently, $\tilde{x}_t \in [a, b]$. Below, we specify the expansion method of a price band in the trading operations execution process in the case when it is necessary. Such a situation occurs when the unit price of a commodity $\tilde{x}_t$ crosses one of the band borders. For convenience and brevity, the calculations of the unit price of a commodity will be made dimensionless by normalization of the price with respect to the upper boundary of the band and defining the band as an interval $(\beta, 1)$, where $\beta = a/b < 1$, as shown in Fig. 1.

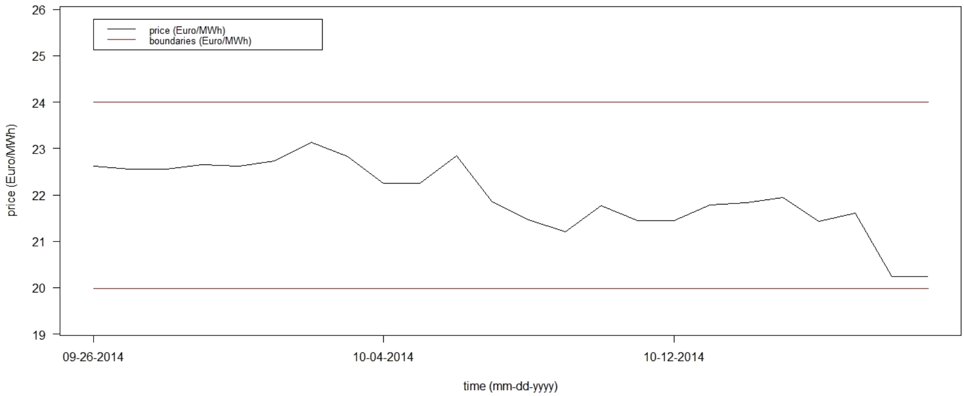

Fig. 1. The choice of price band.

Further, we consider the function $\varphi(x)$ that is the first Eigen function corresponding to the first Eigenvalue λ_1 of the following Sturm–Liouville problem:

$$\frac{d^2\varphi}{dx^2} + \frac{\lambda_1^2}{x^2}\varphi = 0, \tag{8}$$

$$\varphi(1) = \varphi'(\beta) = 0. \tag{9}$$

This function can be taken as follows: $\varphi(x) = \sqrt{x}\sin(b\ln x)$, while $\lambda_1^2 = b^2 + \frac{1}{4}$, $b = \frac{z^*}{\ln\beta} < 0$, where z^* is the minimal strictly positive root of the transcendental equation

$$tgz = -\frac{2z}{\ln\beta}, \tag{10}$$

where $-\frac{2}{\ln\beta} > 1$. It means that the band width is considered sufficiently small, namely, $\frac{1}{e^2} < \beta < 1$.

The graphical solution to Eq. (10) is shown in Fig. 2. Moreover, $\varphi'(1) = b < 0$.

One can easily verify the given solution to problems (8) and (9) by considering that function $\varphi(x) = c_1\sqrt{x}\sin(b\ln x) + c_2\sqrt{x}\cos(b\ln x)$, where c_1 and c_2 are arbitrary constants, is the general solution to Eq. (8) when ratio $\lambda_1^2 = b^2 + \frac{1}{4}$ is valid (Kamke, 1977). The chart of function $\varphi(x)$ as the solution to problems (8) and (9) is presented in Fig. 3.

The quantity dynamics of the commodity units sold on the time interval $[0, t]$ is set as follows:

$$\tilde{a}_t = \int_0^t \frac{u_0(\tau)}{\varphi(\tilde{x}_\tau)}d\tau \cdot \varphi'(x_t)\bigg|_{x_t=\tilde{x}_t}, \tag{11}$$

where $u_0(\tau) > 0$ is an arbitrary piecewise continuous function.

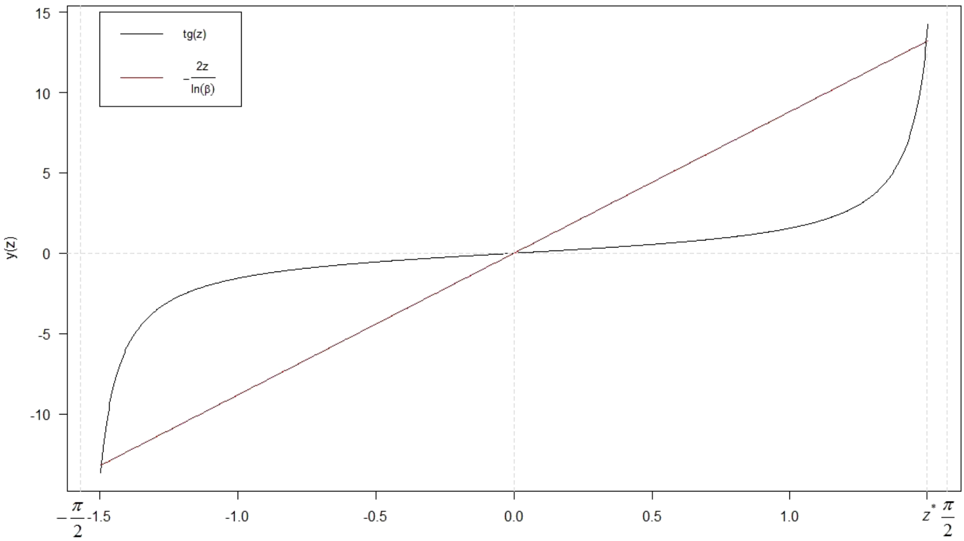

Fig. 2. Graphical solution to the transcendental equation.

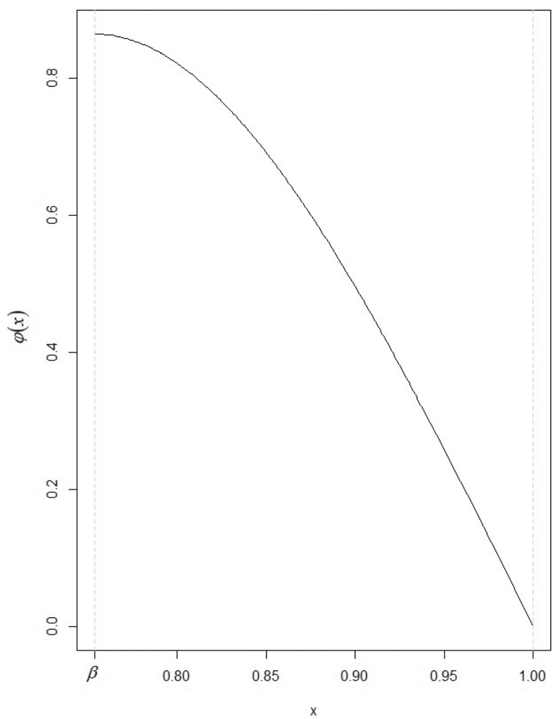

Fig. 3. $\varphi(x)$ solution to problem (8), (9).

Theorem 1. *Assume that price x_t follows stochastic differential equation* (1), *while the volatility factor $\sigma_t = \sigma(t, \omega)$ is a random function that does not depend on process x_t. Then, if the quantity of the commodity units $\tilde{a}_t$ is defined by formula* (11), *the speculative profit is determined by the equality*

$$\tilde{p}_t = \int_0^t \frac{u_0(\tau)}{\varphi(\tilde{x}_\tau)} d\tau \cdot \varphi(\tilde{x}_t) - \int_0^t u_0(\tau) e^{-\frac{1}{2}\lambda_1^2 \int_\tau^t \tilde{\sigma}_s^2 ds} d\tau, \tag{12}$$

while the relationship

$$V_T = \tilde{p}_T + |\tilde{a}_T| \cdot \tilde{x}_T, \tag{13}$$

corresponds to the overall amount of profit obtained on the time interval $[0, T]$.

The proof of Theorem 1 is given in Appendix A. Note that the weighted average price of goods sold in accordance with (13) may be calculated with ratio

$$x_T^{av} = \tilde{x}_T + \frac{\tilde{p}_T}{|\tilde{a}_T|}. \tag{14}$$

From equality (14), condition (2) is reduced to the validity of the following inequality:

$$\tilde{x}_T + \frac{\tilde{p}_T}{|\tilde{a}_T|} > \max_{t \in [0, T]} |\tilde{x}_t|. \tag{15}$$

The first Eigen function is not equal to zero inside the price band $[\beta, 1]$ and provides the absence of singularities in the denominators of formulas (11) and (12). Furthermore, the sales management defined by formula (11) does not explicitly contain the coefficients of differential Eq. (1) in accordance with previous findings.

Relationship (14) shows that for equal quantities of sold commodity units, the largest weighted average price corresponds to the most considerable "speculative profit". The analysis of formula (12) demonstrates that two factors promote increase of the weighted average price. The first factor addresses the weighted average price growth when the time interval $[0, t]$ increases under conditions that the observable market price $\tilde{x}_t$ stays inside the same price band. The second factor of increase is caused by the influence of the volatility coefficient when the second integral in formula (12) decreases exponentially in the growth process. Thus, these two factors sufficiently fulfil condition (15), as shown with the process of trading operations consideration on the world commodity exchanges.

By making use of formula (12) one can see that the slump of market prices leads to the sharp increase of the weighted average price. Thus, the proposed control system executes the role of the earlier declared hedging strategy.

The expansion of a price band is now necessary. Suppose that at instant t^* the observable value of price $\tilde{x}_t$ crosses the boundary of the band as shown in Fig. 4.

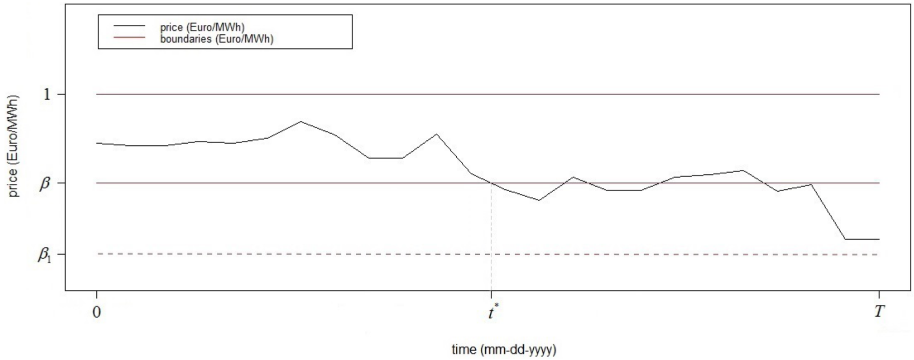

Fig. 4. The expansion of price band.

In this case, the new price band corresponding to the interval $(\beta_1, 1)$ is considered. The first Eigen function of the new Sturm–Liouville problem (8), (9) is denoted as $\varphi_1(x)$. On the time interval $[0, t^*]$, instead of $u_0(\tau)$, we choose the new control function $u_1(\tau)$ such one that the equality,

$$\int_0^t \frac{u_1(\tau)}{\varphi_1(\tilde{x}_\tau)} d\tau \cdot \varphi_1'(x_t) \Big|_{x_t=\tilde{x}_t} = \tilde{a}_t^*,$$

occurs for each $t \in [0, t^*]$, where the time dependence $\tilde{a}_t^*$ corresponds to the trading bargains already executed on the time interval $[0, t^*]$. The problem posed in such a way may be reduced to the Volterra integral equation of the first kind

$$\int_o^t K(t, \tau)x(\tau)d\tau = f(t) \tag{16}$$

with respect to the unknown function $x(\tau) = u_1(\tau)$, kernel $K(t, \tau) = \frac{\varphi_1'(x_t)|_{x_t=\tilde{x}_t}}{\varphi_1(\tilde{x}_\tau)}$ and the right-hand side $f(t) = \tilde{a}_t^*$.

The latter equation implies the introduction of the notion of a quasi-solution and may be solved on the basis of standard methods for the class of ill-posed problems in space L_2 on the compact set of functions with bounded variation (Tikhonov and Arsenin, 1977). Correspondingly, when $t > t^*$, the function of control $u_1(t) > 0$ may be chosen arbitrarily in the class of the piecewise continuous functions as previously discussed.

4. Realization of the Constructed Management Scheme on an Example of Real World Market Trading

Figure 5 shows the closing prices of daily sessions per gas unit in European currency at the European Energy Exchange for the time period from 10-01-2014 until 09-30-2015.

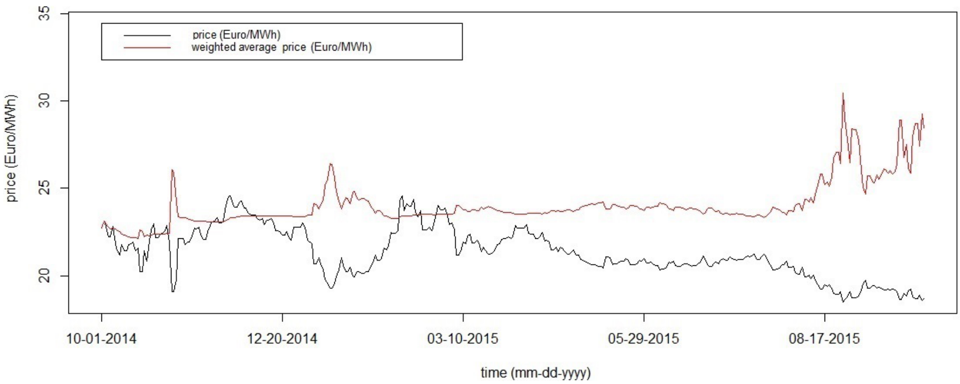

Fig. 5. Charts of prices.

Here, the gas unit corresponds to one *megawatt* · *hour* of energy. In the same chart, the dynamics of the weighted average price per the unit of gas are constructed within the proposed management scheme (11). The original function $u_0(t)$ is taken as a constant while the price band is chosen symmetrically with respect to the price of the first bargain executed by the control system and with the width of the price band equal to 2 euro. Further, both bounds of the price band are subject to widening of 1 euro if one of these bounds is crossed by the current market price. Moreover, the original function $u_0(t)$ is replaced by function $u_1(t)$ on the time interval $[0, t^*]$ as the result of the solution constructed for Eq. (16). Additionally, Fig. 6 illustrates the dynamics of gas volumes sold and bought. A volume of a bargain as negative implies a sale and as positive implies a purchase.

This chart clearly demonstrates the role of "reverse operations" in the trading process.

The next chart presents the dynamics of the total portfolio volume.

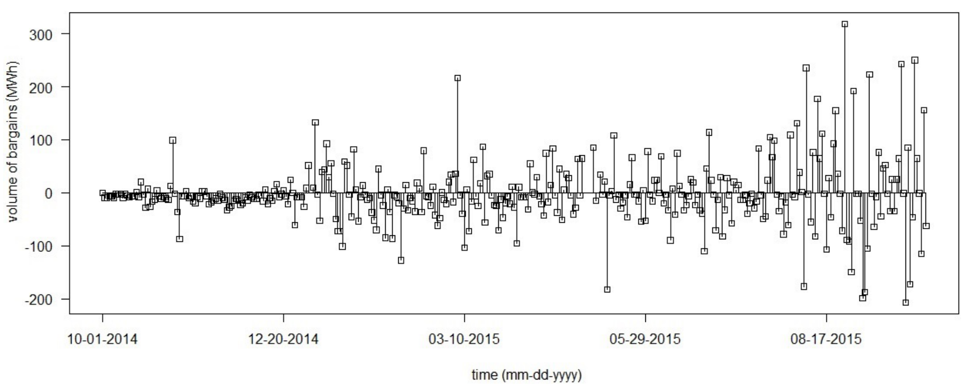

Fig. 6. Dynamics of volumes of gas having being bought and sold.

5. Analysis of the Weak Points and Reconciling of Mutually Contradictory Claims

The analysis of formulas (11) and (12) shows that increasing the weighted average price leads to diminishing the volume of sold goods and consequently to decreasing the overall income. On the other hand, the amount of goods sold usually matches the volume of goods intended for sale. Formula (11) demonstrates that the sharp increase of market prices leads to decrease of the weighted average prices in comparison. Thus, sometimes we reconcile two mutually contradictory goals. One goal is the increase of the weighted average price of the goods sold. The second goal is maximizing the income on the given time horizon with a priory known volume of goods intended for sale. On the other hand, under certain circumstances, increasing the weighted average price may be more important, for instance when the trading time horizon and the volume of goods intended for sale are generally indistinct in advance. At any rate, a reasonable trading strategy implies the presence of a sequence of portfolios close one after another if definite conditions are fulfilled.

By taking into consideration the abovementioned arguments, we impose conditions of closure of a current portfolio if at least one of the following inequalities:

$$\tilde{x}_t^{av} - \max_{\tau \in [0,t]} \tilde{x}_\tau > \varepsilon_1 \geq 0, \tag{17}$$

$$\tilde{x}_t^{av} - \max_{\tau \in [0,t]} \tilde{x}_\tau < \varepsilon_2 \leq 0 \tag{18}$$

is valid at some instant of time $\hat{t}$. Verification of conditions (17) and (18) occurs with respect to the discrete set of known prices.

Condition (17) prevents the excessive growth of the weighted average price that may lead to the relatively small volume of goods sold. Inequality (18) is to restrict the substantial growth of current market prices with respect to their weighted average magnitude. In the process of opening a new portfolio, we initially suppose that function $u_0(\tau)$ is a constant. Evaluation of this constant is defined by the volume of goods that should be sold at the end of the overall trading time horizon and in accordance with formula (11) under the condition that market price variations are not substantial for the corresponding time interval. The latter condition becomes reasonable if portfolio closure occurs relatively often.

Appendices B and C contain counting results of created software which allows performing virtual gas trading on the European Energy Exchange from 01-05-2010 until 06-06-2016, choosing parameters of the management scheme specified above.

Thus, in the trading process, we introduce two limit cases. The first case occurs when $\varepsilon_1 = +\infty$; $\varepsilon_2 = -\infty$. In this situation, the initial portfolio is never being

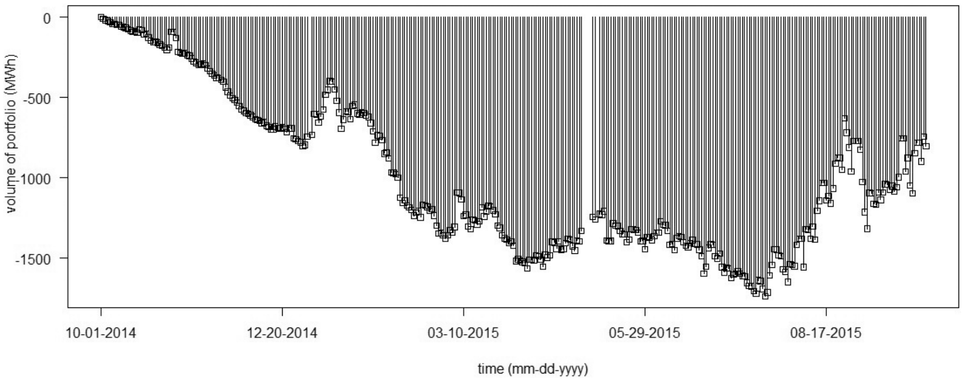

Fig. 7. Dynamics of portfolio volume.

close to the end of the predetermined trading time horizon. On the time horizon that constitutes several months, substantial growth of the weighted average price typically occurs; however, the growth is simultaneous to the substantial divergence of the volume of goods sold with respect to the planned volume. Nevertheless, a long growth trend of market prices can lead to the beginning of strong, exceeding weighted average price growth that prompts a trader to close an intermediate portfolio. The second limit case corresponds to the parameter values $\varepsilon_1 = \varepsilon_2 = 0$. Under these conditions, we address the great quantity of portfolios that are closed one after another. For the chosen parameters, the volume of goods sold with high precision typically coincides with the volume of goods intended for trading. On the other hand, under these circumstances, the weighted average price of sales for the entire time interval $[0, T]$ is much less in comparison with the first limit case. Under this choice of parameters $\varepsilon_1 = \varepsilon_2 = 0$ the "weighted average strategy" of the sale of goods does not imply the execution of "reverse operations". Scrutinizing the trading reports clearly demonstrates that the abovementioned strategies may be taken as decision-making platforms and consequently as two basic strategies for sales management. Moreover, in contrast to the "weighted average strategy", the previous approach $(\varepsilon_1 = +\infty; \varepsilon_2 = -\infty)$ cannot be completely automated in the situation when the planned volume of goods for sale exists. The point is that in this case, the decision of closing the intermediate portfolios is completely the prerogative of trader activity. On the other hand, it is impossible to predict in advance situations connected the global changes in market behaviour with the own possibilities of the trading company. Gas levels in tanks, the gas delivery schedule, availability of corresponding quotas, etc. to define the possibilities. Traders may also decide to close an intermediate portfolio because of its relatively high weighted average price simultaneously with the substantial volume of sold goods while keeping in mind the time interval that continued until the finish of the total trading time horizon. Therefore, similar software may serve only as a tool for supporting decisions being accepted by a trader depending on changing market behaviour, the momentary possibilities of the seller and the parameters given in advance of trading.

6. Conclusions

The analysis of the reports indicates the presence of anomalous prices in the process of the control system functioning. Prices are discarded in the management process of a given intermediate portfolio despite their negligible quantity. Such prices can be easily detected because of the tremendous volumes of gas units that are unexpectedly proposed for buying or selling in comparison with the current trading volumes being executed by the control system. This effect rarely occurs, and it can be explained by different reasons beginning from the enormous jumps in the market price up to the resonance phenomenon occurring in the control system. The point is that the automatic widening of the price band may lead to the actual coincidence of one of its border prices with the current market price. Nevertheless, from the practical point of view, predicaments do not present any fatal hazard in the case of the adequate interference of traders in the trading process. Adequate interferences imply either ignoring the corresponding anomalous price by excluding it from the register or compulsory closing of the intermediate portfolio.

Appendix A. Proof of the Theorem 1

Seek unknown f_t as the function of two variables $f_t = f(t, x_t)$, where x_t follows Eq. (1). Applying to $f(t,x)$ Ito's formula and comparing it with ratio (4), one arrives at the equations

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma_t^2 x_t^2 \frac{\partial^2 f}{\partial x_t^2} = l(t, x_t), \tag{A.1}$$

$$a_t = \frac{\partial f}{\partial x_t}. \tag{A.2}$$

The control function $l(t, x_t)$ is set according to the relationship

$$l(t, x_t) = r(t)\varphi(x_t), \tag{A.3}$$

where $\varphi(x)$ in the first Eigen function corresponding to the first eigenvalue λ_1 of the Sturm–Liouville problem (8), (9). The structure of the function $r(t)$ will be clarified below. Introduce the following initial and boundary conditions:

$$f(0, x_t) = 0, \tag{A.4}$$

$$\frac{\partial f}{\partial x_t} \to 0 \quad \text{as } x_t \to \beta, \tag{A.5}$$

$$f(t, x_t) \to 0 \quad \text{as } x_t \to 1. \tag{A.6}$$

The relationship (A.3) is used to find the solution to the initial boundary value problems (A.1), (A.4), (A.5) and (A.6) in the form

$$f(t, x_t) = k(t)\varphi(x_t),$$

where $k(t)$ is the unknown function.

As the result of trivial transformations, one arrives at the relationship

$$f(t, x_t) = \int_0^t e^{\frac{1}{2}\lambda_1^2 \int_\tau^t \sigma_s^2 ds} r(\tau) d\tau \cdot \varphi(x_t). \tag{A.7}$$

Relationships (A.2) and (A.7) define the amount of goods sold according to the formula

$$\tilde{a}_t = \left(\frac{\partial f}{\partial x_t}\right)\bigg|_{x_t = \tilde{x}_t} = \int_0^t e^{\frac{1}{2}\lambda_1^2 \int_\tau^t \tilde{\sigma}_s^2 ds} r(\tau) d\tau \cdot \varphi'(x_t)\bigg|_{x_t = \tilde{x}_t}. \tag{A.8}$$

The drawback of formula (A.8) is that it contains the volatility factor σ_t. The following procedure is used with a partition of the time interval $[0, t]$ in n as follows: $0 = t_0 < t_1 < \cdots < t_n = t$. Function $r(\tau)$ is defined as the limit of point wisely converging sequence of functions determined by the relationship

$$r_n(\tau) = \frac{u_n(\tau)}{\varphi(\tilde{x}_\tau)} e^{-\frac{1}{2}\lambda_1^2 \int_\tau^{t_i} \tilde{\sigma}_s^2 ds}, \tag{A.9}$$

where $\tau \in (t_{i-1}, t_i]$, $u_n(\tau)$ are given functions, while the sequence $u_n(\tau)$ as $n \to +\infty$ is supposed to converge point wisely to the function $u(\tau)$ when a uniform partition takes place.

Substituting in (A.8) instead of $r(\tau)$ sequence (A.9), one arrives to the relationship

$$\tilde{a}_{t_j}^n = \sum_{i=1}^{j} \int_{t_{i-1}}^{t_i} \frac{u_n(\tau)}{\varphi(\tilde{x}_\tau)} d\tau \cdot \varphi'(x_t)\bigg|_{x_t = \tilde{x}_{t_j}},$$

or

$$\tilde{a}_{t_j} = \int_0^{t_j} \frac{u_n(\tau)}{\varphi(\tilde{x}_\tau)} d\tau \cdot \varphi'(x_t)\Big|_{x_t=\tilde{x}_{t_j}}.$$

Ultimately, realizing the limit transition as $n \to \infty$ and within the framework of the uniform partition, one arrives at the formula describing the continuous distribution of the amount of goods sold in time under the observable realization of the price $\tilde{x}_t$ as

$$\tilde{a}_t = \int_0^{t} \frac{u(\tau)}{\varphi(\tilde{x}_\tau)} d\tau \cdot \varphi'(x_t)\Big|_{x_t=\tilde{x}_t}. \qquad (A.10)$$

By making use of the same arguments and taking into account relationships (7), (A.3), (A.7) and (A.9), we arrive to the formula for the amount of "speculative profit" corresponding to the observable relationship of price $\tilde{x}_t$ at instant t:

$$\tilde{p}_t = \int_0^{t} \frac{u(\tau)}{\varphi(\tilde{x}_\tau)} d\tau \cdot \varphi(\tilde{x}_t) - \int_0^t e^{-\frac{1}{2}\lambda_1^2 \int_\tau^t \tilde{\sigma}_s^2 ds} u(\tau) d\tau. \qquad (A.11)$$

Thus, formulas (A.10) and (A.11) correspond to the desired relationships (11) and (12).

Appendix B. Results of the Created Software in Graphical Form

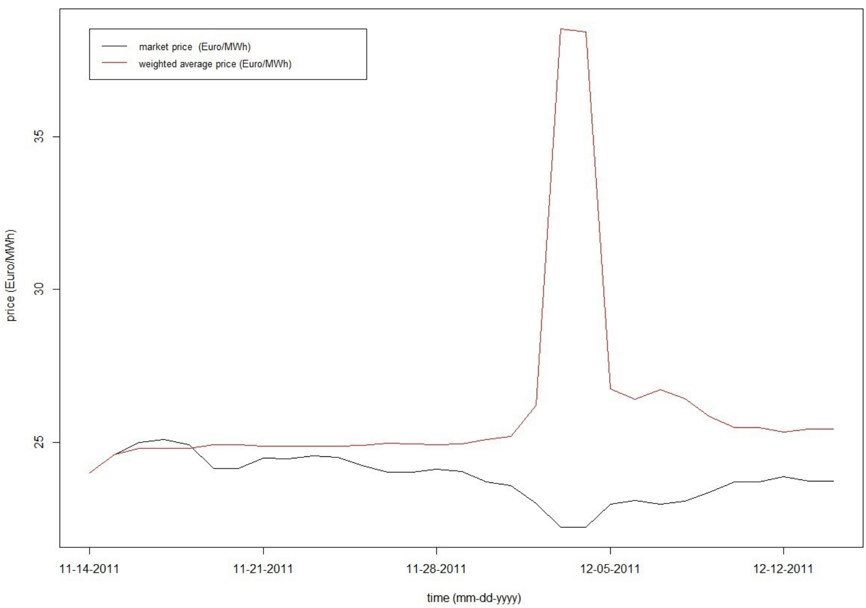

Fig. B.1. Results of created software in graphical form.

Appendix C. Counting Results of Created Software in Table Form

Table C.1. Counting results of created software in table form.

Date (dd.mm.yyyy)	Price band (upper boundary; bottom boundary) (euro)	Market price (euro)	Weighted average price (euro)	Volume of bargain (MWh)	Volume of portfolio (MWh)	Dynamics of profit (euro)
14.11.2011 (new portfolio)	(24.99;22.99)	23.99	23.99	0	0	0
15.11.2011	(24.99;22.99)	24.6	24.6	−717	−717	17638.2
16.11.2011	(25.99;21.99)*	25	24.8	−684	−1401	34738.2
17.11.2011	(25.99;21.99)	25.09	24.8	−43	−1444	35817.07
18.11.2011	(25.99;21.99)	24.92	24.8	8	−1436	35617.71
19.11.2011	(25.99;21.99)	24.13	24.92	219	−1217	30333.24
20.11.2011	(25.99;21.99)	24.13	24.92	−12	−1229	30622.8
21.11.2011	(25.99;21.99)	24.49	24.87	−140	−1369	34051.4
22.11.2011	(25.99;21.99)	24.45	24.87	−2	−1371	34100.3
23.11.2011	(25.99;21.99)	24.56	24.86	−51	−1422	35352.86
24.11.2011	(25.99;21.99)	24.51	24.86	0	−1422	35352.86
25.11.2011	(25.99;21.99)	24.23	24.9	85	−1337	33293.31
26.11.2011	(25.99;21.99)	24.01	24.96	79	−1258	31396.52
27.11.2011	(25.99;21.99)	24.01	24.95	−11	−1269	31660.63
28.11.2011	(25.99;21.99)	24.11	24.92	−54	−1323	32962.57
29.11.2011	(25.99;21.99)	24.04	24.93	19	−1304	32505.81
30.11.2011	(25.99;21.99)	23.7	25.09	154	−1150	28856.01
01.12.2011	(25.99;21.99)	23.57	25.18	62	−1088	27394.67
02.12.2011	(25.99;21.99)	22.99	26.21	349	−739	19371.16
03.12.2011	(25.99;21.99)*	22.22	38.52	558	−181	6972.4
04.12.2011	(25.99;21.99)	22.22	38.43	−1	−182	6994.62
05.12.2011	(25.99;21.99)	22.98	26.75	−563	−745	19932.36
06.12.2011	(25.99;21.99)	23.09	26.4	−79	−824	21756.47
07.12.2011	(25.99;21.99)	22.98	26.72	70	−754	20147.87
08.12.2011	(25.99;21.99)	23.07	26.43	−66	−820	21670.49
09.12.2011	(25.99;21.99)	23.37	25.83	−201	−1021	26367.86
10.12.2011	(25.99;21.99)	23.69	25.48	−196	−1217	31011.1
11.12.2011	(25.99;21.99)	23.69	25.47	−9	−1226	31224.31
12.12.2011	(25.99;21.99)	23.88	25.34	−111	−1337	33874.99
13.12.2011	(25.99;21.99)	23.73	25.43	72	−1265	32166.43
14.12.2011	(25.99;21.99)	23.72	25.42	−3	−1268	32237.59

Note: Price band widening is indicated with symbol "*". Bargain volume with a minus sign implies sales and a plus sign implies purchases.

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