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СТАТИСТИКА ПАРАМЕТРА ПИКОВАТОСТИ В СПЕКТРЕ ВЕТРОВЫХ ВОЛН ПО ДАННЫМ ГИДРОДИНАМИЧЕСКОГО МОДЕЛИРОВАНИЯ

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Функция спектральной плотности является универсальной характеристикой, позволяющей описать разнообразие свойств волнения и решить многочисленные прикладные задачи. Параметр пиковатости γ – наиболее трудно определяемый и в то же время наиболее важный для практических расчетов, т.к. определяет величину экстремальной волновой нагрузки на сооружение в море, и увеличивает вероятность образования необычных волн (так называемых волн убийц). В тех случаях, когда об условиях волнообразования ничего неизвестно, рекомендуется принимать γ=3.3, что соответствует средним условиям в Мировом океане. Данные измерений показывают, что γ может изменяется от 1 до 20. Столь широкий разброс оценок параметра γ неприемлем при решении конкретных прикладных задач. Однако инструментальных (контактных) измерений спектров волнения и, соответственно, оценок параметра пиковатости крайне мало. Более того, на морях вокруг России регулярные измерения волнения не проводятся. Это означает что непрерывные (за период в несколько десятков лет) расчёты волнения по гидродинамическим моделям могут служить основой соответствующих статистических обобщений параметра γ. Выполнены соответствующие расчёты для некоторых морей вокруг России. В качестве примера приведены результаты для морей с различным режимом волнения − Берингова и Белого. Дана оценка двумерных и условных распределений (высот волн и параметра пиковатости), моментов распределений и других статистик.

Ключевые слова: ветровое волнение, зыбь, климатические спектры, параметр пиковатости и его статистика.

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WIND WAVE SPECTRA PEAKEDNESS AND ITS STATISTICS BASED ON HINDCASTING

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Wind wave spectrum peakedness γ is one of the most important parameters as it determines extreme wave loads and may be responsible for the freak wave generation. Usually, γ=3.3 is supposed as the mean value, but measurements show that peakedness is varying from 1 to 20. This means that real values of peakedness have to be specified and γ statistics is needed for applied research. The ideal decision of the specified problem is continuous wave measurements (for the period of several decades). Unfortunately, the information about peakedness for various regions is very scarce, mainly due to a restricted number of direct wave spectra measurements. Moreover, regions where regular wave measurements are not being performed still remain, including some seas around Russia. This means that continuous wave hindcasting by numerical models is the only database for statistical generalization. Statistics of wave peakedness based on hindcasting for some seas around Russia were calculated. Data for seas with quite different wave climates, namely for Bering and White sea, is presented. Statistics include two-dimensional distributions (wave heights-peakedness), regression (peakedness on wave height), conditional distributions, etc.

Key words: wind waves, swell, climatic spectra, peakedness and its statistics.

1. Introduction. The heights of sea waves depend on a set of external factors (wave formation conditions), particularly on wind velocity, its duration, fetch, etc. Under invariable conditions, wind waves are a quasistationary, quasihomogeneous process. Changes of conditions are related to a passage of cyclones (synoptic variability), annual rhythmic (seasonal variability) and long-term variations (year-to-year variability). This,

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in turn, allows defining a wave climate (or regime) as an ensemble of conditions of a wave surface taking into account the specified variability and hence to describe it in terms of long-term (regime) statistical characteristics.

Calculation of a wave climate is based on the results of hindcasting by numerical hydrodynamic models. Such approach has gained the greatest spread and has been approved and exercised for numerous scientific and applied problems. The models are based on the equation of wave energy balance in spectral form, therefore they are called spectral, and a wave climate based on these results is «a spectral wave climate» [1]. Wave Watch for the Bering Sea and SWAN model for the White Sea were used for wave climate hindcasting. A two-level nested domain was employed and a 40-year period was simulated at 3-hour intervals. A coarse grid (2°×2°) for N. Pacific and nested high resolution grid (93×66 = 6138 cells) were used for the Bering Sea. A coarse grid (2°×1.5°) for the Barents Sea and N. Atlantic and nested grid (145×150 = 21750 cells) were used for the White Sea.

The input for calculation is improved NCEP/NCAR reanalysis wind fields [2]. Time series of directional spectra $S(f, \theta)$ in each grid point (x, y) is a result of a numerical simulation. Total number of spectra in each grid is 116880. They are the base for the further statistical generalizations, including climatic wave spectra and their parameters.

2. Climatic wave spectra. Climatic wave spectra are statistics needed for seaworthy qualities of ships. The climatic spectrum is the spectrum with certain probability, depending on conditions of wave formation. In the first publications climatic wave spectra were calculated for the sequence of not overlapped intervals of wave heights and periods, i.e. for each cell values of corresponding spectra were averaged [3]. As a result for the same combination of heights and periods, spectra of the various physical natures was averaged (e.g. wind waves, swell and their combination). This approach is acceptable for some applied problems (for example, the estimation of fatigue parameters of the object which is operating in the fixed point of the sea during many years, or an estimation of wave energy resources). For a lot of applied problems, it is necessary to take into consideration not only the probability of spectra for some combinations of wave heights and periods but the specificity of spectral structure, i.e. it is necessary to consider wave formation conditions. This means, that selection of functionally similar classes is important, i.e. genetic classification of spectra is needed: wind waves, swell, mixed situations etc. Automatic classification of directional spectra $S(\omega, \Theta)$ or $S(f, \Theta)$ demands elaboration of special procedures. The task in view decision includes two basic stages:

- Classifications of the directional spectra taking into account wave formation conditions;
- Approximation of spectra of each class by a set of parameters.

The approach to classification and some results are published in the papers [4, 5] and here only the information needed for peakedness estimation is presented. The important thing is that it is possible to select some basic classes of spectra almost for any sea. Total number of classes may vary, but in the open areas at least 5 classes exist. Below is short characteristic of each class.

One-peaked spectra (classes I, II). One wave system prevails — either the wind waves (class I) or the swell (class II). The separation between wind waves and swell is based on the non-dimensional steepness defined as

$$\Delta = \frac{h}{\lambda_p} = \frac{2\pi h}{g\tau_p^2} = \frac{8\pi}{g}\sqrt{m_0} f_p^2, \tag{1}$$

here λ_p, τ_p, f_p are the wave length, period and frequency associated with the spectral peak. The rule $\Delta > 0.011$ in the relation (1) is derived for selection of a wind wave system and otherwise, a swell is expected.

Two-peaked spectra (classes III, IV). Two wave systems exist simultaneously. For two-peaked spectra, two sub-classes are separated with respect to the swell age.

For the wind waves and “fresh” swell (class III) with close frequencies and different directions the wind waves and the “matured” swell (class IV) include all other two-peaked spectra with arbitrary relation between the frequencies f_0 and f_1. There are two pronounced maxima (f_0, θ_0) and (f_1, θ_1) separated both by frequency and direction.

Multipeaked spectra (class V). Complicated wave fields with two or more swells (class V). In this case, the spectrum has more than two pronounced peaks.

The classes of spectra considered above are valid for any area of the World Ocean. The probability of classes depends only on the regional conditions. In the frame of declared task it is needed to consider the ensemble of one peaked spectra and estimate their probability. The results are presented in table 1.

Table 1

Probabilities (%) of one-peaked spectra

Вероятность (%) однопиковых спектров

Sea	Bering	White
Probability (total, wind waves, swell)	35, 20, 15	48, 45, 3

It is seen from the table, that input of one-peaked spectra to spectral wave climate is essential. In the closed White Sea the total probability is more (48 %), than in the open Bering Sea. But input of swell is only 3%, in contrast to Bering Sea where probability of swell is 15 %.

3. Peakedness of one-peaked spectra. Function of spectral density is a universal characteristic, allowing to describe a variety of properties of wind waves and to solve numerous applied problems. Reviews of available approximations of a frequency spectrum can be found in many publications. Basically, all variety of available approximations of a frequency one-peaked spectrum of wind waves or swell can be reduced to expression of type

$$S(\omega) = A\omega^{-k} \exp[-B\omega^{-n}]. \tag{2}$$

In the case of fully developed wind waves spectrum (2) is known as Pierson-Moskowitz with parameters k = 5, n = 4. Fully developed wave conditions are realized quite rarely. As a rule for the limited fetch JONSWAP approximation is used, but it was also measured in variety wave making conditions at seas and oceans, and included in the majority of the standard documents necessary for calculations of wave loadings on vessels and constructions. Classical expression of JONSWAP spectrum looks like:

$$S(\omega) = \alpha\omega^{-5} g^2 \exp\left[-B\omega^{-4}\right]\gamma^{\beta(\omega)},$$

Where $\beta(\omega) = \exp\left[-\frac{\left(\omega-\omega_p\right)^2}{2\sigma^2\omega_p^2}\right]$ (3)

$$\sigma = \begin{cases} 0.07, \ \omega \le \omega_p \\ 0.09, \ \omega > \omega_p \end{cases},$$

the JONSWAP spectrum (3) is based on the PM spectrum with an peakedness (enhancement) factor γ added to control the sharpness of the spectral peak. This enhancement is only significant in the region near the spectral peak. Width of the peak region is represented by σ. Frequency of the peak of spectrum $\omega_p = 2\pi / \tau_p$, accordingly τ_p — the period of the peak of a spectrum, *g* — peakedness of a spectrum. The main difference between JONSWAP and Pierson—Moskowitz spectra is in energy magnitude on frequency of a maximum of a spectrum (fig. 1).

There are different recommendations for the estimation of *g*. If nothing is known about wave conditions then adopt *g* = 3.3. This value may be regarded as some generalization of wind wave spectra obtained during JONSWAP experiment [6]. Below are some approximations for estimation of peakedness mean value. Particularly, in the book [7]

$\gamma = 4.42\left(\tilde{f}_p^{0.429}\right)$ where

$$\tilde{f}_p = f_p V/g \tag{4}$$

V — wind velocity.

Classification Society Det Norske Veritas [8] suggested the relation:

$$\gamma = \exp\left(5.75 - \frac{1.15T_p}{\sqrt{H_s}}\right) \qquad \text{for } 3.6 \le \frac{T_p}{\sqrt{H_s}} \le 5. \tag{5}$$

T_p — period associated with spectral peak, H_s — significant wave height. Out of pointed limit, it is recommended to adopt *g* = 5 for $\frac{T_p}{\sqrt{H_s}} \le 3.6$ and *g* = 1 for $\frac{T_p}{\sqrt{H_s}} \ge 5$.

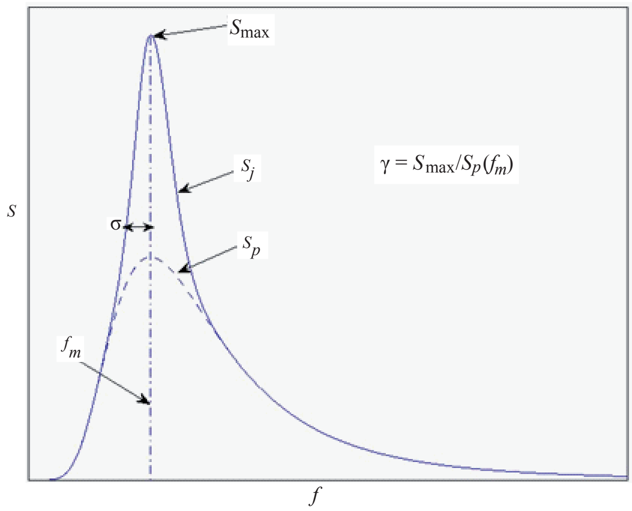

Fig. 1. The principal difference between Pierson—Moscowitz (*Sp*) and JONSWAP (*Sj*) spectra.

Рис. 1. Принципиальная разница между спектрами Пирсона—Московица (*Sp*) и JONSWAP (*Sj*).

Wave measurements near the coast of France resulted in the following approximation for high wave periods T_p (this means also for high waves) [9]:

$$\gamma = 0.219H_s + 0.43 \tag{6}$$

Our results for Baltic Sea showed that

$$\gamma = \begin{cases} 1.024\sqrt{H_s}, & h_s \geq 1.75 \text{ м} \\ 1.28 + 2.0\exp(-2H_s), & H_s \langle 1.75 \text{ м} \end{cases}. \tag{7}$$

Approximations (4)—(7) estimate mean values of peakedness, i.e. only one moment of distribution. Simulated data allow estimating the probability of peakedness for wind waves and swell spectra.
The main statistic is two-dimensional distributions of wave height and peakedness $f(h, g)$. From these statistics one-dimensional distributions of both wave heights and peakedness, as well as the moments of distributions, are estimated. In tables, 2—5 statistical estimations of peakedness of wind waves and a swell for two seas (Bering and White Sea) with a quite different wave climate are presented.

The main statistic is two-dimensional distributions of wave height and peakedness $f(h, g)$. From these statistics one-dimensional distributions of both wave heights and peakedness, as well as the moments of distributions, are estimated. In tables, 2—5 statistical estimations of peakedness of wind waves and a swell for two seas (Bering and White Sea) with a quite different wave climate are presented.

A lot of conclusions follow from the tables. One of the main is that in spite of different climate, mean statistics are close. The median value of g for wind waves is about 1.5 for both seas. But for swell g is about 2.5 in the Bering Sea and 1.5 in the White Sea. This means, that low-frequency swell from the Pacific freely penetrates the Bering Sea. In the White Sea, there are no sources of low-frequency swell. Max value of g is about 6 for wind wave and 10 for swell in the Bering Sea. In the White Sea, these values are 8 and 10. It is also important, that for the wind waves peakedness increases with wave height, while for swell it decreases. Presented data show that the value 3.3 adopted in many publications is a very rough generalization.

Parameters of three-dimensional Weibull distributions of peakedness (8) and lognormal distribution of wave heights (9) are shown beneath the tables.

$$F(\gamma) = 1 - \exp(-\left|\frac{\gamma - \gamma_0}{a}\right|^k), \tag{8}$$

where a – scale parameter; k – shape parameter; g_0 – bias.

$$F(x) = \frac{s}{\sqrt{2\pi}} \int_x^{\infty} \frac{1}{x} \exp\left[-\frac{1}{2}\ln^2 \left(x / x_{0,5}\right)^s\right] dx. \tag{9}$$

where $x_{0.5}$ — median of wave height, and 1/s – standard deviation (or rms) of wave heights logarithms.

Table 2

Joint probability (%) of wave height (3 % probability, in m) and peakedness *g*. Marginal probability *f*(*h*), *f*(*g*) cumulative probability *F*(*h*), *F*(*g*). Conditional mean values (regressions) and rms of wave height $m_h(g)$—$\sigma_h(\gamma)$ for prescribed *h*, and peakedness $m_\gamma(h)$—$\sigma_\gamma(h)$ for prescribed *g*. Parameters of conditional Weibull distribution (8) of wave heights and peakedness. WIND WAVES

Совместная повторяемость (%) высот волн 3%-ной обеспеченности *h* (м) и параметров пиковатости *g*. Безусловные (маргинальные) повторяемости *f*(*h*), *f*(*g*) и обеспеченности *F*(*h*), *F*(*g*) высот волн 3%-ной обеспеченности и параметров пиковатости. Условные средние (регрессии) $m_h(g)$ и среднеквадратические отклонения $\sigma_h(\gamma)$ параметра пиковатости для фиксированного значения высот волн *h*; и условные средние $m_\gamma(h)$ и среднеквадратические отклонения $\sigma_h(\gamma)$ высот волн для фиксированного значения параметра пиковатости *g*. Расчётные параметры аппроксимации распределений высот волн и параметра пиковатости трехпараметрическим законом Вейбулла

Берингово море. Ветровые волны

h	Peakedness γ Wind waves				$f(h)$	$F(h)$	$m_\gamma(h)$	$\sigma_\gamma(h)$	$a_\gamma(h)$	$k_\gamma(h)$	$\gamma_0(h)$
	1-2	2-4	4-6	≥6							
0-1	2.0	0.7	0.04	—	2.7	100.0	1.7	0.7	0.7	0.8	1.0
1-2	11.1	6.3	0.2	—	17.6	97.3	1.9	0.6	0.9	0.9	1.0
2-3	10.9	8.5	0.2	—	19.7	79.7	2.0	0.7	1.0	0.9	1.0
3-4	10.7	6.5	0.08	0.04	17.3	60.0	1.9	0.6	0.9	1.1	1.0
4-5	7.3	4.9	0.04	—	12.2	42.7	1.9	0.6	0.9	1.2	1.0
5-6	5.9	6.1	0.08	—	12.1	30.5	2.0	0.6	1.1	1.1	1.0
6-7	3.0	6.4	—	—	9.4	18.5	2.2	0.5	1.2	1.5	1.0
7-8	1.0	2.9	0.04	—	4.0	9.1	2.4	0.6	1.7	3.2	0.7
8-9	0.7	1.2	0.04	—	1.9	5.1	2.3	0.6	1.0	1.0	1.3
9-10	0.3	0.9	—	—	1.2	3.3	2.3	0.6	1.1	1.6	1.2
10-11	0.2	0.4	—	—	0.6	2.1	2.2	0.4	0.8	1.8	1.5
11-12	0.04	0.4	—	—	0.4	1.5	2.7	0.5	1.8	3.6	0.9
12-13	—	0.2	—	—	0.2	1.1	2.9	0.4	0.6	1.6	2.3
13-14	—	0.5	—	—	0.5	0.8	2.9	0.3	0.7	1.4	2.2
14-15	—	0.2	—	—	0.2	0.3	—	—	—	—	—
15-16	—	0.08	—	—	0.08	0.12	—	—	—	—	—
≥16	—	0.04	—	—	0.04	0.04	—	—	—	—	—
$f(\gamma)$	53.2	46.0	0.8	0.04							
$F(\gamma)$	100.0	46.8	0.8	0.04							
$m_h(g)$	3.5	4.6	3.2	—							
$\sigma_h(\gamma)$	1.8	2.7	2.1	—							
$\alpha_\tau(\gamma)$	3.3	4.6	2.5	—							
$k_h(\gamma)$	2.1	2.1	1.2	—							
$h_0(\gamma)$	0.2	0.0	0.8	—							

Wave heights log-normal distributions:
$h_{0.5}$ = 3.5 (m); s = 1.2
3-parameters Weibull distribution of peakedness:
a_γ = 1.0; k_γ = 1.0; γ = 1.0.
Regression between wave heights and peakedness:
$\overline{\gamma}(h) = 1.72h^{0.15}$,
($h_{3\,\%} = 1.33h_{sign}$).

Below are some shortly outlined applied problems, where the information about peakedness is useful:

- Estimation of energy and, as a result, the extreme loads (in peak of wave spectrum) on ships and constructions operated in the sea;
- Development of freak waves is connected with the kurtosis of wave height distribution, this, in turn, is associated with the peakedness of spectrum [10];
- Influence of the wave spectrum form on the bottom sediment dynamics [11].

Table 3

Joint probability (%) of wave height (3 % probability, in m) and peakedness *g*. Marginal probability *f*(*h*), *f*(*g*) cumulative probability *F*(*h*), *F*(*g*). Conditional mean values (regressions) and rms of wave height $m_h(g)$—$\sigma_h(\gamma)$ for prescribed *h*, and peakedness $m_\gamma(h)$—$\sigma_\gamma(h)$ for prescribed *g*. Below are also calculated parameters of conditional Weibull and log-normal distributions of wave heights and peakedness. The Bering Sea. SWELL

Совместная повторяемость (%) высот волн 3%-ной обеспеченности *h* (м) и параметров пиковатости *g*. Безусловные (маргинальные) повторяемости *f*(*h*), *f*(*g*) и обеспеченности *F*(*h*), *F*(*g*) высот волн 3%-ной обеспеченности и параметров пиковатости. Условные средние (регрессии) $m_h(g)$ и среднеквадратические отклонения $\sigma_h(\gamma)$ параметра пиковатости для фиксированного значения высот волн *h*; и условные средние $m_\gamma(h)$ и среднеквадратические отклонения $\sigma_\gamma(h)$ высот волн для фиксированного значения параметра пиковатости *g*. Расчётные параметры аппроксимации распределений высот волн и параметра пиковатости трехпараметрическим законом Вейбулла

Берингово море. Волны зыби

h	Peakedness γ swell					$f(h)$	$F(h)$	$m_\gamma(h)$	$\sigma_\gamma(h)$	$a_\gamma(h)$	$k_\gamma(h)$	$\gamma_0(h)$
	1-2	2-4	4-6	6-8	≥8							
0-1	5.0	8.6	2.8	1.0	0.3	17.7	100.0	3.1	1.7	2.1	0.8	1.0
1-2	11.5	19.2	3.9	0.9	0.3	35.8	82.3	2.7	1.4	1.8	0.9	1.0
2-3	10.4	11.4	1.6	0.07	0.05	23.5	46.5	2.3	1.0	1.4	0.9	1.0
3-4	6.4	5.9	0.6	0.05	—	12.8	23.0	2.2	0.9	1.2	0.9	1.0
4-5	3.1	3.3	0.09	0.02	—	6.5	10.1	2.2	0.7	1.2	1.3	1.0
5-6	1.3	1.1	0.02	—	—	2.4	3.6	2.0	0.7	1.0	1.0	1.0
6-7	0.5	0.3	—	—	—	0.9	1.3	1.9	0.6	0.9	1.0	1.0
7-8	0.09	0.14	—	—	—	0.2	0.4	2.1	0.6	0.9	1.2	1.2
≥8	0.05	0.11	—	—	—	0.2	0.2	2.3	0.4	1.3	2.5	1.1
$f(\gamma)$	38.4	49.9	9.0	2.0	0.7							
$F(\gamma)$	100.0	61.6	11.7	2.7	0.7							
$m_h(\tau)$	2.4	2.2	1.5	1.2	1.1							
$\sigma_h(\gamma)$	1.4	1.3	0.9	0.7	0.6							
$a_\tau(\gamma)$	2.4	1.8	1.3	0.9	0.9							
$.k_h(\gamma)$	2.0	1.6	1.6	1.5	1.2							
$h_0(\gamma)$	0.0	0.3	0.3	0.2	0.2							

Wave heights log-normal distribution:
$h_{0.5}$ = 1.9 (м); s = 1.2
3-parameters Weibull distribution of peakedness:
a_γ = 1.6; k_γ = 0.9; γ_0 = 1.0.
Regression between wave heights and peakedness:
$\overline{\gamma}(h) = 2.77h^{-0.15}$.
($h_{3\%} = 1.33h_{sign}$).

4. Conclusions. Automatic classification of wave spectra is presented. Probability of one, two and multi-peaked spectra is estimated. For the one-peaked class of spectra a set of statistics are calculated and presented in exclusive tables:

- Joint Probability and statistics of the distribution of wind wave heights and peakdness γ of the spectrum.
- Joint Probability and statistics of the distribution of swell heights and peakdness γ of the spectrum.
- Marginal distributions of wave heights and their approximations (Weibull and lognormal probability laws are used).
- Regression: peakedness on wave height, and wave heights on peakedness.
- Similarity and difference of peakedness statistics for Bering and White seas are outlined.

Table 4

Joint probability (%) of wave height (3% probability, in m) and peakedness *g*. Marginal probability *f*(*h*), *f*(*g*) cumulative probability *F*(*h*), *F*(*g*). Conditional mean values (regressions) and rms of wave height $m_h(g)$— $\sigma_h(\gamma)$ for prescribed *h*, and peakedness — for prescribed *g*. Below are also calculated parameters of conditional Weibull and log-normal distributions of wave heights and peakedness. The White Sea. WIND WAVES

Совместная повторяемость (%) высот волн 3%-ной обеспеченности *h* (м) и параметров пиковатости *g*. Безусловные (маргинальные) повторяемости *f*(*h*), *f*(*g*) и обеспеченности *F*(*h*), *F*(*g*) высот волн 3%-ной обеспеченности и параметров пиковатости. Условные средние (регрессии) $m_h(g)$ и среднеквадратические отклонения $\sigma_h(\gamma)$ параметра пиковатости для фиксированного значения высот волн *h*; и условные средние $m_\gamma(h)$ и среднеквадратические отклонения $\sigma_h(\gamma)$ высот волн для фиксированного значения параметра пиковатости *g*. Расчётные параметры аппроксимации распределений высот волн и параметра пиковатости трехпараметрическим законом Вейбулла

Белое море. Ветровые волны

h	Peakedness γ Wind waves					$f(h)$	$F(h)$	$m_\gamma(h)$	$\sigma_\gamma(h)$	$a_\gamma(h)$	$k_\gamma(h)$	$\gamma_0(h)$
	1-2	2-4	4-6	6-8	≥8							
0-1	38.3	8.2	0.2	0.08	0.07	46.9	100.0	1.5	0.6	0.5	0.7	1.0
1-2	20.4	12.7	0.2	0.04	+	33.2	53.1	1.9	0.6	0.9	1.0	1.0
2-3	5.9	7.8	0.06	0.01	—	13.7	19.9	2.1	0.5	1.1	1.9	1.0
3-4	1.8	2.9	—	—	—	4.6	6.2	2.2	0.5	1.3	2.7	0.9
4-5	0.5	0.7	+	—	—	1.2	1.5	2.3	0.6	1.2	2.4	1.1
5-6	0.09	0.2	+	0.01	—	0.3	0.3	2.7	1.3	1.6	1.9	1.1
≥6	—	0.02	0.02	—	—	0.04	0.04	3.7	0.8	1.5	1.6	2.2
$f(\gamma)$	66.9	32.4	0.4	0.14	0.08							
$F(\gamma)$	100.0	33.1	0.6	0.2	0.08							
$m_h(\tau)$	1.1	1.8	1.7	1.5	0.6							
$\sigma_h(\gamma)$	0.8	1.0	1.5	1.6	0.3							
$a_\tau(\gamma)$	1.0	1.7	1.6	1.3	0.3							
$.k_h(\gamma)$	1.6	1.9	1.5	0.8	1.0							
$h_0(\gamma)$	0.0	0.0	0.1	0.2	0.3							

Wave heights log-normal distribution:
$h_{0.5}$ = 1.1 (м); s = 1.5
3-parameters Weibull distribution of peakedness:
a_γ = 0.8; k_γ = 0.8; γ_0 = 1.0.
Regression between wave heights and peakedness:

$$\overline{\gamma}(h) = 1.72h^{0.28}$$

($h_{3\%} = 1.33h_{sign}$).

Table 5

Joint probability (%) of wave height (3% probability, in m) and peakedness *g*. Marginal probability *f*(*h*), *f*(*g*) cumulative probability *F*(*h*), *F*(*g*). Conditional mean values (regressions) and rms of wave height $m_h(g)$—$\sigma_h(\gamma)$ for prescribed *h*, and peakedness $m_\gamma(h)$ — for prescribed *g*. Below are also calculated parameters of conditional Weibull and log-normal distributions of wave heights and peakedness. The White Sea. SWELL

Совместная повторяемость (%) высот волн 3%-ной обеспеченности *h* (м) и параметров пиковатости *g*. Безусловные (маргинальные) повторяемости *f*(*h*), *f*(*g*) и обеспеченности *F*(*h*), *F*(*g*) высот волн 3%-ной обеспеченности и параметров пиковатости. Условные средние (регрессии) $m_h(g)$ и среднеквадратические отклонения $\sigma_h(\gamma)$ параметра пиковатости для фиксированного значения высот волн *h*; и условные средние $m_\gamma(h)$ и среднеквадратические отклонения $\sigma_h(\gamma)$ высот волн для фиксированного значения параметра пиковатости *g*. Расчётные параметры аппроксимации распределений высот волн и параметра пиковатости законом Вейбулла (8)

Белое море. Волны зыби

h	Peakedness γ swell					$f(h)$	$F(h)$	$m_\gamma(h)$	$\sigma_\gamma(h)$	$a_\gamma(h)$	$k_\gamma(h)$	$\gamma_0(h)$
	1-2	2-4	4-6	6-8	≥8							
0-1	64.5	17.1	3.9	3.0	2.3	90.8	100.0	2.1	1.7	1.1	0.6	1.0
1-2	4.2	2.9	1.1	0.3	0.14	8.6	9.2	2.6	1.7	1.6	0.7	1.0
≥2	0.4	0.14	0.03	—	—	0.6	0.6	1.9	1.0	0.9	0.9	1.0
$f(\gamma)$	69.0	20.2	5.0	3.3	2.4							
$F(\gamma)$	100.0	31.0	10.7	5.7	2.4							
$m_h(\tau)$	0.4	0.6	0.7	0.5	0.5							
$\sigma_h(\gamma)$	0.3	0.4	0.5	0.4	0.4							
$a_\tau(\gamma)$	0.4	0.6	0.7	0.5	0.4							
$k_h(\gamma)$	1.4	1.4	1.2	1.0	0.8							
$h_0(\gamma)$	0.0	0.0	0.0	0.0	0.0							

Wave heights log-normal distribution:
$h_{0.5}$ = 0.4 (м); s = 1.4
3-parameters Weibull distribution of peakedness:
a_γ = 1.2; k_γ = 0.6; γ_0 = 1.0.
Regression between wave heights and peakedness:
$$\overline{\gamma}(h) = 2.20h^{-0.02}$$
($h_{3\%} = 1.33h_{sign}$).

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