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MATHEMATICS

Two-Dimensional Homogeneous Cubic Systems: Classification and Normal Forms: IV

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Abstract—This article is the fourth in a series of works devoted to two-dimensional cubic homogeneous systems. It considers a case when a homogeneous polynomial vector in the right-hand part of the system has a quadratic common factor with complex zeros. A set of such systems is divided into classes of linear equivalence, wherein the simplest system is distinguished on the basis of properly introduced structural and normalization principles, being, thus, the third-order normal form. In fact, such a form is defined by a matrix of its right-hand part coefficients, which is called the canonical form (CF). Each CF has its own arrangement of nonzero elements, their specific normalization and canonical set of permissible values for the nonnormalized elements, which relates CF to a selected class of equivalence. In addition, each CF is characterized by: (1) conditions imposed on the coefficients of the initial system, (2) nonsingular linear substitutions that transform the right-hand part of the system under these conditions into a selected CF, and (3) obtained values of CF's nonnormalized elements. Refs 9.

Keywords: homogeneous cubic system, normal form, canonical form.

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INTRODUCTION

This work is aimed at establishing the canonical forms of real homogeneous cubic systems possessing a common second-degree factor with complex zeros and consisting of five sections.

In the first section, the right-hand part of the initial system, defined by eight coefficients, uniquely decomposes in the product of the common factor $P_0^2(x)$ with a negative discriminant D_0 and a vector Hx, where H is some nonsingular matrix for which the discriminant of the characteristic polynomial is denoted as D.

Herewith, the invariance of signs for D_0 and D was found in [1]. There is a list of normalized structural forms and canonical forms with their sets of permissible parameter values , corresponding to the case of $D_0 < 0$.

The second and third sections are dedicated to the case of $D \geq 0$ and $D < 0$, respectively. For each of them, there are listed the canonical forms with the proper canonical sets of the allowable parameter values introduced in [2].

The theorems confirming linear nonequality of the reduced CFs and demonstrating explicitly for each CF were proven: (1) all systems belonging to the linear equivalence class generated by this CF, (2) a linear replacement reducing any such system to the chosen CF, and (3) the CF parameter values obtained by replacement from its canonical set.

The fourth section is dedicated to the minimal canonical sets introduced in [2], and the fifth one is devoted to the unique list of canonical forms and canonical sets for systems with the common second-degree factor that is the total results of [3] and of the present paper.

The appendix represents the classifications of systems with other unperturbed parts.

Since this work is the direct continuation of papers [1–3], all the above notations are thus retained therein. In connection with a large amount of references to the formulae obtained in [1–3], they will be marked with a superscript for brevity. For example, system (2.1) from [1] will be designated as $(2.1)^1$.

1. Extraction of $CF^{m,2}$ at a negative P_0^2 discriminant. System (1.1)[3], obtained after taking the common factor $P_0^2(x)$ from the right-hand part of the system (2.1)[1] $\dot{x} = Aq^{[3]}(x)$, at the condition $D_0 = \beta^2 - \alpha\gamma < 0$, can be written as:

$$\dot{x} = P_0^2(x)Hx, \qquad \begin{matrix} P_0^2 = x_1^2 + 2\beta x_1 x_2 + \gamma x_2^2, \\ D_0 = \beta^2 - \gamma < 0 \quad (\alpha = 1), \end{matrix} \qquad H = \begin{pmatrix} p_1 & q_1 \\ p_2 & q_2 \end{pmatrix}, \quad \delta_{pq} = \det H \neq 0. \qquad (1.1^<)$$

Let us extract the structural forms up to $SF_2^{7,2}$ inclusively from the list 1.1 from [2], related to the case of $l = 2$, $D_0 < 0$ (see [2, definition 1.2]), and normalize them according to the NPs in [2, section 1.2], simultaneously extracting the common factor P_0^2 with $\alpha = 1$, having a discriminant $D_0 < 0$ and a matrix H with the discriminant D of its characteristic polynomial and the permissible sets (see [2, definition 1.8]) on account of (2.19)[1].

Besides this fact, we are going to establish which of $NSF^{m,2,<}$ are the canonical forms.

List 1.1. Seventeen $NSF^{m,2,<}$ and $CF^{m,2,<}$ to $CF_2^{7,2,<}$ inclusively, where each of them has $(1, 2\beta, \gamma)$, H, D_0, D and nontrivial $ps^{m,2,<}$ ($\sigma = \pm 1$, u, v, $w \neq 0$).

$$CF_{8,+1}^{4,2,<,\geq} = \sigma\begin{pmatrix} u & 0 & u & 0 \\ 0 & 1 & 0 & +1 \end{pmatrix}, \quad (1, 0, 1), \quad \sigma\begin{pmatrix} u & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{matrix} -1, \\ (u-1)^2; \end{matrix}$$

$$CF_{34,+1}^{4,2,<,\gneqq} = \sigma\begin{pmatrix} 0 & u & 0 & u \\ 1 & 0 & +1 & 0 \end{pmatrix}, \quad (1, 0, 1), \quad \sigma\begin{pmatrix} 0 & u \\ 1 & 0 \end{pmatrix}, \quad \begin{matrix} -1, \\ 4u; \end{matrix}$$

$$CF_7^{5,2,<,\geq} = \sigma\begin{pmatrix} u & -u & u & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad (1, -1, 1), \quad \sigma\begin{pmatrix} u & 0 \\ 1 & 1 \end{pmatrix}, \quad \begin{matrix} -3/4, \\ (u-1)^2; \end{matrix}$$

$$CF_{22}^{5,2,<,\gtreqqless} = \sigma\begin{pmatrix} 0 & u & -u & u \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad (1, -1, 1), \quad \sigma\begin{pmatrix} 0 & u \\ 1 & 1 \end{pmatrix}, \quad \begin{matrix} -3/4, \\ 4u+1; \end{matrix}$$

$$CF_1^{6,2,<,\geq} = \sigma\begin{pmatrix} u & u & uv & 0 \\ 0 & 1 & 1 & v \end{pmatrix}, \quad (1, 1, v), \quad \sigma\begin{pmatrix} u & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{matrix} 1/4 - v, \\ (u-1)^2; \end{matrix}$$

$$CF_3^{6,2,<,\geq} = \sigma\begin{pmatrix} u & u(1-v) & 0 & -uv^2 \\ 0 & 1 & 1 & v \end{pmatrix}, \quad (1, 1, v), \quad \sigma\begin{pmatrix} u & -uv \\ 0 & 1 \end{pmatrix}, \quad \begin{matrix} 1/4 - v, \\ (u-1)^2; \end{matrix}$$

$$CF_{4,+1}^{6,2,<,\geq} = \sigma\begin{pmatrix} u & v & u & v \\ 0 & 1 & 0 & +1 \end{pmatrix}, \quad (1, 0, 1), \quad \sigma\begin{pmatrix} u & v \\ 0 & 1 \end{pmatrix}, \quad \begin{matrix} -1, \\ (u-1)^2; \end{matrix}$$

$$CF_6^{6,2,<,\lesseqqgtr} = \sigma\begin{pmatrix} u & u(v^{-2} - v) & 0 & uv^{-3} \\ 1 & 0 & v^{-1} - v^2 & 1 \end{pmatrix}, \quad (1, -v, v^{-1}), \quad \sigma\begin{pmatrix} u & uv^{-2} \\ 1 & v \end{pmatrix}, \quad \begin{matrix} (v^3/4 - 1)v^{-1}, \\ (u-v)^2 + 4uv^{-2}; \end{matrix}$$

$$CF_7^{6,2,<,\gtreqqless} = \sigma\begin{pmatrix} u & v & -v & u+v \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad (1, -1, 1), \quad \sigma\begin{pmatrix} u & u+v \\ 1 & 1 \end{pmatrix}, \quad \begin{matrix} -3/4, \\ (u+1)^2 + 4v; \end{matrix}$$

$$CF_{11,+1}^{6,2,<,\gtreqqless} = \sigma\begin{pmatrix} u & v & u & v \\ 1 & 0 & +1 & 0 \end{pmatrix}, \quad (1, 0, 1), \quad \sigma\begin{pmatrix} u & v \\ 1 & 0 \end{pmatrix}, \quad \begin{matrix} -1, \\ u^2 + 4v; \end{matrix}$$

$$CF_2^{7,2,<,\gtreqqless} = \sigma\begin{pmatrix} u & w & uv^{-1} - v(uv+w) & u + wv^{-1} \\ 1 & 0 & v^{-1} - v^2 & 1 \end{pmatrix}, \quad (1, -v, v^{-1}), \quad \sigma\begin{pmatrix} u & uv+w \\ 1 & v \end{pmatrix}, \quad \begin{matrix} (v^3/4 - 1)v^{-1}, \\ (u+v)^2 + 4w; \end{matrix}$$

$$NSF_5^{6,2,<,\geq} = \sigma\begin{pmatrix} u & 0 & u(v-1) & -uv \\ 0 & 1 & 1 & v \end{pmatrix}, \quad (1, 1, v), \quad \sigma\begin{pmatrix} u & -u \\ 0 & 1 \end{pmatrix}, \quad \begin{matrix} 1/4 - v, \\ (u-1)^2; \end{matrix}$$

$$NSF_{12}^{6,2,<,\gtreqless} = \sigma\begin{pmatrix} 0 & -uv & u & -uv^2 \\ 1 & 0 & v - v^{-2} & 1 \end{pmatrix}, \quad (1, v^{-1}, v), \quad \sigma\begin{pmatrix} 0 & -uv \\ 1 & v^{-1} \end{pmatrix}, \quad \begin{matrix} (1/4 - v^3)v^{-2}, \\ v^{-2} - 4uv; \end{matrix}$$

$$NSF_{13}^{6,2,<,\gtreqless} = \sigma\begin{pmatrix} u & 0 & uv & uv^2(1+v) \\ 1 & 1 & 0 & v(1+v)^2 \end{pmatrix}, \quad (1, -v, v^2 + v), \quad \sigma\begin{pmatrix} u & uv \\ 1 & 1+v \end{pmatrix}, \quad \begin{matrix} -v(3v+4)/4, \\ (u+v+1)^2 - 4u; \end{matrix}$$

$$NSF_{15}^{6,2,<,\gtreqless} = \sigma\begin{pmatrix} 0 & u & uv & uv(v-1) \\ 1 & 1 & 0 & -v(v-1)^2 \end{pmatrix}, \quad (1, v, v^2 - v), \quad \sigma\begin{pmatrix} 0 & u \\ 1 & 1-v \end{pmatrix}, \quad \begin{matrix} -v(3v-4)/4, \\ (v-1)^2 + 4u; \end{matrix}$$

$$NSF_{16}^{6,2,<,\gtrless} = \sigma\begin{pmatrix} 0 & u & u & uv \\ 1 & 1 & v & 0 \end{pmatrix}, \quad (1, 1, v), \quad \sigma\begin{pmatrix} 0 & u \\ 1 & 0 \end{pmatrix}, \quad \begin{matrix} 1/4 - v, \\ 4u; \end{matrix}$$

$$NSF_{1}^{7,2,<,\geq} = \sigma\begin{pmatrix} u & w & uv - u + w & v(w-u) \\ 0 & 1 & 1 & v \end{pmatrix}, \quad (1, 1, v), \quad \sigma\begin{pmatrix} u & w-u \\ 0 & 1 \end{pmatrix}, \quad \begin{matrix} 1/4 - v, \\ (u-1)^2; \end{matrix}$$

$$ps_1^{6,2,<} = \{v > 1/4\}, \quad ps_3^{6,2,<} = \{v > 1/4, v \neq 1\}, \quad ps_5^{6,2,<} = \{v > 1/4, v \neq 1\},$$

$$ps_6^{6,2,<} = \{v \in (0, \sqrt[3]{4}), v \neq 1\}, \quad ps_7^{6,2,<} = \{v \neq -u\}, \quad ps_{12}^{6,2,<} = \{v > 4^{-1/3}, v \neq 1\},$$

$$ps_{13}^{6,2,<} = \{v \notin [-4/3, 0]\}, \quad ps_{15}^{6,2,<} = \{v \notin [0, 4/3]\}, \quad ps_{16}^{6,2,<} = \{v > 1/4\},$$

$$ps_1^{7,2,<} = \{v > 1/4, w \neq u, u(1-v)\},$$

$$ps_2^{7,2,<} = \{v \in (0, \sqrt[3]{4}), v \neq 1, w \neq -uv, -u(v - v^{-2})\}.$$

We mention that, in List 1.1, only $CF_{8,+1}^{4,2,<}$ and $CF_1^{6,2,<}$ possess the diagonal matrix H. Moreover, the canonical sets of $CF_{8,+1}^{4,2,<}$ have the form of $cs_{8,+1}^{4,2,<,>} = \{u \neq 1\}$ and $cs_{8,+1}^{4,2,<,=} = \{u = 1\}$,because they exhibit no previous forms with $D_0 < 0$.

2. Case $D \geq 0$. So, we will assume that the matrix H has real eigenvalues λ_1, λ_2 in system ($1.1^<$).

Collection 2.1. The constants and the replacements used hereinafter in Section 2:

$$\psi_1(u) = (u^2 - 3u + 3)^{1/2}(u-3)^{-1}, \quad \psi_2(u) = (3u^2 - 3u + 1)(3u-1)^{-2},$$

$$\psi_3(u) = (u^2 + 3u + 3)(3u^2 + 3u + 1)(3u^2 + 8u + 3)^{-2}; \quad \psi_4 = \tilde{v}(\tilde{v} - (\tilde{v}^2 - 1)^{1/2})/2;$$

$$L_{8,+1}^{4,2,<,>} = \{r_1 = (\tilde{\alpha}|\lambda_2|)^{-1/2}, s_1, r_2 = 0, s_2 = (\tilde{\gamma}|\lambda_2|)^{-1/2}\};$$

$$L_{34,+1}^{4,2,<,>} = \{r_1 = -v^{1/2}r_2, s_1 = (v(4v-1))^{1/4}(2v^{1/2}+1)^{-1}, r_2 = (v(4v-1))^{-1/4}, s_2 = v^{-1/2}s_1\};$$

$$L1_7^{5,2,<,>} = \{r_1 = (\tilde{u}^2 - 3\tilde{u} + 3)^{1/2}(1-\tilde{u})^{-1}, s_1 = 0, r_2 = (1-\tilde{u})^{-1}s_2, s_2 = \psi_1^{-1}(\tilde{u})\};$$

$$L2_7^{5,2,<,>} = \{r_1 = \tilde{u}^{1/3}|\tilde{u}|^{1/6}(\tilde{u}-1)^{-1}, s_1 = |\tilde{u}|^{-1/2}, r_2 = (3\tilde{u}-1)\tilde{u}^{-1}r_1, s_2 = 0\};$$

$$L_{22}^{5,2,<,>} = \{r_1 = -(\tilde{u}^2 + 3\tilde{u} + 3)^{1/2}(\tilde{u}-1)^{-1}|\tilde{u}+1|^{-1/2}, s_1 = \tilde{u}(\tilde{u}+1)^{-1}r_1,$$
$$r_2 = -(3\tilde{u}^2 + 8\tilde{u} + 3)(\tilde{u}^2 + 3\tilde{u} + 3)^{-1}r_1, s_2 = (\tilde{u}+1)^{-1}r_2\};$$

$$L_1^{6,2,<,>} = \{r_1 = (\tilde{\alpha}|\lambda_2|)^{-1/2}, s_1, r_2 = 0, s_2 = \tilde{\alpha}(2\tilde{\beta})^{-1}r_1\};$$

$$L_{8,+1}^{4,2,<,=} = \{r_1 = -\beta r_2, s_1 = |p_1|^{-1/2}, r_2 = |p_1|^{-1/2}(\gamma - \beta^2)^{-1/2}, s_2 = 0\};$$

$$L_7^{5,2,<,=} = \{r_1 = 1/2, s_1 = -1, r_2 = \mp\sqrt{3}/2, s_2 = 0\};$$

$$L_{22}^{5,2,<,=} = \{r_1 = \pm\sqrt{14}/7, s_1 = \mp 5\sqrt{14}/28, r_2 = \sqrt{42}/14, s_2 = \sqrt{42}/28\};$$

$$L_3^{6,2,<,=} = \{r_1 = 1, s_1 = 1/2, r_2 = 0, s_2 = -(\tilde{v} + (\tilde{v}^2 - 1)^{1/2})/2\};$$

$$L_{4,+1}^{6,2,<,=} = \{r_1 = 0, s_1 = \tilde{\gamma}^{1/2}|\nu|^{-1/2}\xi^{-1}, r_2 = |\tilde{\gamma}\nu|^{-1/2}, s_2 = -\beta\tilde{\gamma}^{-1}s_1\}.$$

In [3], system $(1.5)^3$ is obtained from system $(1.1^<)$ with $\gamma - \beta^2 > 0$ at $D > 0$ ($\lambda_1, \lambda_2 \neq 0, \lambda_1 - \lambda_2 = \sigma_0\sqrt{D} \neq 0$) with a substitution J_1^2 (see Collection $(1.1)^3$), wherein $\tilde{\alpha}, \tilde{\gamma} > 0$ and $\tilde{\xi} = (\tilde{\alpha}\tilde{\gamma} - \tilde{\beta}^2)^{1/2} > 0$ in accordance with $(2.18)^1$ and $(2.19)^1$, and system $(1.6)^3$ with $\tilde{\alpha}, \tilde{\gamma}, \tilde{\xi} > 0$ is found at $D = 0$ ($\lambda_1, \lambda_2 = \nu = (p_1 + q_2)/2 \neq 0$) and $[q_1 \neq 0 \vee q_1 = 0, p_2 \neq 0]$ with a substitution $[J_{2a}^2 \vee J_{2b}^2]$. Finally, system $(1.1^<)$ with $\gamma - \beta^2 > 0$ at $D = 0$ and $q_1, p_2 = 0$ immediately takes the form $(1.5)^3$, but with $\lambda_1, \lambda_2 = \nu$, because the H matrix is diagonal with $p_1, q_2 = \nu \neq 0$.

Lemma 2.1. (i) *System* $(1.5)^3$ *at* $\tilde{\beta} = 0$ *with a substitution* $L_{8,+1}^{4,2,<,>}$ *is reduced to* $CF_{8,+1}^{4,2,<,>}$ *with* $\sigma = \operatorname{sgn}\lambda_2$, $u = \lambda_1\lambda_2^{-1}$, *and at* $\tilde{\beta} \neq 0$ *with a substitution* $L_1^{6,2,<,>}$ *to* $NSF_1^{6,2,<,>}$ *with* $\sigma = \operatorname{sgn}\lambda_2$, $u = \lambda_1\lambda_2^{-1}$, $v = \tilde{\alpha}\tilde{\gamma}(2\tilde{\beta})^{-2}$.

(ii) *System* $(1.6)^3$ *with a substitution* $L_{4,+1}^{6,2,<,=}$ *is reduced to* $NSF_{4,+1}^{6,2,<,=}$ $(u = 1)$ *with* $\sigma = \operatorname{sgn}\nu$, $v = \tilde{\gamma}(\nu\tilde{\xi})^{-1}$.

(iii) *System* $(1.1^<)$ *at* $D, q_1, p_2 = 0$ *with a substitution* $L_{8,+1}^{4,2,<,=}$ *is reduced to* $CF_{8,+1}^{4,2,<,=}$ $(u = 1)$ *with* $\sigma = \operatorname{sgn} p_1$.

Corollary 2.1. All six $NSF^{m,2,<}$ forms from the list 1.1 at $D > 0$ and $D = 0$ are not canonical.

Statement 2.1. The following forms from list 1.1 with the specified values of the parameters are reduced to the previous ones in accordance with SPs (see in [2, Section 1.1]) $SF_i^{m,2}$:

(i) $NSF_{34,+1}^{4,2,<,>}$ with $ps_{34,+1}^{4,2,<,>} = \{u > 0\}$ at $u = 1$ with a substitution $(2.2)^1$ $L = \begin{pmatrix} r_1 & s_1 \\ r_2 & s_2 \end{pmatrix}$ $(\det L \neq 0)$ with $s_1 = s_2$, $r_2 = -r_1$ is reduced to $SF_8^{4,2}$;

(ii) $NSF_7^{5,2,<,>}$ with $ps_7^{5,2,<,>} = \{u \neq 1\}$:

(a) at $u = 3$ at a substitution with $s_1 = 2s_2$, $r_1 = 0$ is reduced to $SF_8^{4,2}$;

(b) at $u = -1$ at a substitution with $s_1 = s_2(1 + \sqrt{7})/3$, $r_2 = -r_1(1 + \sqrt{7})/2$ is reduced to $SF_{34}^{4,2}$;

(iii) $NSF_{22}^{5,2,<,>}$ with $ps_{22}^{5,2,<,>} = \{u > -1/4\}$:

(a) at $u = 3/2$ at a substitution with $s_1 = s_2(\sqrt{7} - 1)/2$, $r_1 = -r_2(\sqrt{7} + 1)/2$ is reduced to $SF_8^{4,2}$;

(b) at $u = [6 \vee 4 \mp \sqrt{13}]$ at a substitution with $s_2 = [-3s_1 \vee (-1 \mp \sqrt{13})s_1/6]$, $r_1 = [4r_2/3 \vee (-1 \mp \sqrt{13})r_2/6]$ is reduced to $SF_7^{5,2}$;

(iv) $NSF_1^{6,2,<,>}(\tilde{\sigma}, \tilde{u}, v)$ with $ps_1^{6,2,<,>} = \{\tilde{u} \neq 1, v > 1/4\}$:

(a) at $\tilde{u} = -1$ at a substitution $L_{34,+1}^{4,2,<,>}$ is reduced to $NSF_{34,+1}^{4,2,<,>}$ with $\sigma = \tilde{\sigma}$, $u = (2v^{1/2} - 1)(2v^{1/2} + 1)^{-1}$;

(b) at $\tilde{u} \neq -1$, $[3 \vee 1/3]$, $v = [\psi_1^2(\tilde{u}) \vee \psi_2(\tilde{u})]$ with a substitution $[L1_7^{5,2,<,>} \vee L2_7^{5,2,<,>}]$ is reduced to $NSF_7^{5,2,<,>}$ with $\sigma = [\tilde{\sigma} \vee \tilde{\sigma}\operatorname{sgn}\tilde{u}]$, $u = [\tilde{u} \vee \tilde{u}^{-1}]$;

(c) at $\tilde{u} \neq -1$, $v = \psi_3(\tilde{u})$ with a substitution $L_{22}^{5,2,<,>}$ is reduced to $NSF_{22}^{5,2,<,>}$ with $\sigma = \tilde{\sigma}\operatorname{sgn}(\tilde{u} + 1)$, $u = -\tilde{u}(\tilde{u} + 1)^{-2}$;

(v) $NSF_3^{6,2,<,=}$ with $ps_3^{6,2,<,=} = \{u = 1, v > 1/4, v \neq 1\}$:

(a) at $v = 1/3$ at a substitution with $r_2 = -3r_1$, $s_2 = 0$ is reduced to $NSF_7^{5,2,<,=}$;

(b) at $v = (49 \mp 7\sqrt{46})/6$ at a substitution with $r_1 = r_2(11 \mp 2\sqrt{46})/6$, $s_1 = s_2(-38 \pm 5\sqrt{46})/6$ is reduced to $NSF_{22}^{5,2,<,=}$;

(vi) $NSF_{4,+1}^{6,2,<,=}(\tilde{\sigma}, 1, \tilde{v})$ with $ps_{4,+1}^{6,2,<,=} = \{u = 1\}$:

(a) at $v = \pm 2/\sqrt{3}$ with a substitution $L_7^{5,2,<,=}$ is reduced to $NSF_7^{5,2,<,=}$ $(u = 1)$ with $\sigma = \tilde{\sigma}$;

(b) at $v = \mp 7/\sqrt{3}$ with a substitution $L_{22}^{5,2,<,=}$ is reduced to $NSF_{22}^{5,2,<,=}$ $(u = -1/4)$ with $\sigma = \tilde{\sigma}$;

(c) at $|\tilde{v}| \geq 1$ with a substitution $L_3^{6,2,<,=}$ is reduced to $NSF_3^{6,2,<,=}$ $(u = 1)$ with $\sigma = \tilde{\sigma}$, $v = \psi_4(\tilde{v})$.

(See [4, Appendix 3.6.1, p. 141] to points 1–3; [4, Appendix 3.6.2, p. 148] to point 4; [4, Appendix 3.6.3, p. 152] to points 5 and 6.)

Note 2.1. Here and hereinafter, according to agreement 1.3 from [2], the notation

"$\zeta = [\varsigma_1 \vee v_1] \ldots \eta = [\varsigma_2 \vee v_2] \ldots$" means that either $\zeta = \varsigma_1$, $\eta = \varsigma_2$, or $\zeta = v_1$, $\eta = v_2$, and the reference to the appendix to [4] distinguishes a program therein, which confirms the results with the character calculations in Maple.

The above statements allow for $l = 2$, $D_0 < 0$, $D \geq 0$ writing out all canonical forms with their canonical sets.

List 2.1. Five $CF_i^{m,2,<,>}$, five $CF_i^{m,2,<,=}$ and their $cs_i^{m,2,<,\geq}$ ($\sigma = \pm 1$, $u, v \neq 0$).

$$CF_{8,+1}^{4,2,<,\geq} = \sigma\begin{pmatrix} u & 0 & u & 0 \\ 0 & 1 & 0 & +1 \end{pmatrix}, \quad CF_{34,+1}^{4,2,<,>} = \sigma\begin{pmatrix} 0 & u & 0 & u \\ 1 & 0 & +1 & 0 \end{pmatrix};$$

$$CF_7^{5,2,<,\geq} = \sigma\begin{pmatrix} u & -u & u & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad CF_{22}^{5,2,<,\geq} = \sigma\begin{pmatrix} 0 & u & -u & u \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad CF_1^{6,2,<,>} = \sigma\begin{pmatrix} u & u & uv & 0 \\ 0 & 1 & 1 & v \end{pmatrix},$$

$$CF_3^{6,2,<,=} = \sigma\begin{pmatrix} u & u(1-v) & 0 & -uv^2 \\ 0 & 1 & 1 & v \end{pmatrix}, \quad CF_{4,+1}^{6,2,<,=} = \sigma\begin{pmatrix} u & v & u & v \\ 0 & 1 & 0 & +1 \end{pmatrix};$$

$$cs_{8,+1}^{4,2,<,>} = \{u \neq 1\}, \quad cs_{8,+1}^{4,2,<,=} = \{u = 1\}; \quad cs_{34,+1}^{4,2,<,>} = \{u > 0,\, u \neq 1\};$$

$$cs_7^{5,2,<,>} = \{u \neq \pm 1, 3\}, \quad cs_7^{5,2,<,=} = \{u = 1\};$$

$$cs_{22}^{5,2,<,>} = \{u > -1/4,\, u \neq 3/2,\, 6,\, 4 \pm \sqrt{13}\},$$

$$cs_{22}^{5,2,<,=} = \{u = -1/4\}; \quad cs_1^{6,2,<,>} = \{u \neq \pm 1,\, v > 1/4,\, v \neq \psi_1^2(u),\, \psi_2(u),\, \psi_3(u)\};$$

$$cs_3^{6,2,<,=} = \{u = 1,\, v > 1/4,\, v \neq 1/3,\, 1,\, (49 \mp 7\sqrt{46})/6\}; \quad cs_{4,+1}^{6,2,<,=} = \{u = 1,\, |v| < 1\}.$$

Theorem 2.1. *Any system* $(2.1)^1$ *with* $l = 2$, *written as* $(1.1^<)$ *in accordance with* $(2.15)^1$ *and having* $D \geq 0$, *is linearly equivalent to the system generated by some representative of the corresponding canonical form from list* 2.1. *Below, there are for each* $CF_i^{m,2,<,>}$ *and* $CF_i^{m,2,<,=}$: (i) *conditions to the coefficients of the system* $(1.1^<)$,

(ii) *substitutions* $(2.2)^1$, *transforming the right-hand part of the system* $(1.1^<)$ *into the chosen form at the above conditions,*

(iii) *the values of* σ *factor and parameters from* $cs_i^{m,2,<,>}$ *or* $cs_i^{m,2,<,=}$ *are, as follows:*

$CF_{8,+1}^{4,2,<,>}$: (a) $D > 0$, *in* $(1.5)^3$ $\tilde{\beta} = 0$; (b) J_1^2, $L_{8,+1}^{4,2,<,>}$; (c) $\sigma = \operatorname{sgn}\lambda_2$, $u = \lambda_1\lambda_2^{-1}$;

$CF_{34,+1}^{4,2,<,>}$: (a) $D > 0$, *in* $(1.5)^3$ $\tilde{\beta} \neq 0$, $\nu = 0$; (b) J_1^2, $L_1^{6,2,<,>}$, $L_{34,+1}^{4,2,<,>}$ *with* $v = \tilde{\alpha}\tilde{\gamma}(2\tilde{\beta})^{-2}$; (c) $\sigma = \operatorname{sgn}\lambda_2$, $u = (\tilde{\alpha}\tilde{\gamma} - |\tilde{\beta}|)(\tilde{\alpha}\tilde{\gamma} + |\tilde{\beta}|)^{-1}$;

$CF_7^{5,2,<,>}$: (a) $D > 0$, *in* $(1.5)^3$ $\tilde{\beta} \neq 0$, $\tilde{\alpha}\tilde{\gamma}(2\tilde{\beta})^{-2} = [\psi_1^2(\tilde{u}) \vee \psi_2(\tilde{u})]$, *where* $\tilde{u} = \lambda_1\lambda_2^{-1} \neq -1$, $[3 \vee 1/3]$; (b) J_1^2, $L_1^{6,2,<,>}$, $[L1_7^{5,2,<,>} \vee L2_7^{5,2,<,>}]$; (c) $\sigma = [\operatorname{sgn}\lambda_2 \vee \operatorname{sgn}\lambda_1]$, $u = [\tilde{u} \vee \tilde{u}^{-1}]$;

$CF_{22}^{5,2,<,>}$: (a) $D > 0$, *in* $(1.5)^3$ $\tilde{\beta} \neq 0$, $\tilde{u} = \lambda_1\lambda_2^{-1} \neq -1$, $(-5 \pm \sqrt{13})/6$, $(-5 \mp \sqrt{13})/2$, $(-4 \pm \sqrt{7})/3$, $-3/2$, $-2/3$, $\tilde{\alpha}\tilde{\gamma}(2\tilde{\beta})^{-2} = \psi_3(\tilde{u})$; (b) J_1^2, $L_1^{6,2,<,>}$, $L_{22}^{5,2,<,>}$; (c) $\sigma = \operatorname{sgn}\nu$, $u = -\delta_{pq}(2\nu)^{-2}$;

$CF_1^{6,2,<,>}$: (a) $D > 0$, *in* $(1.5)^3$ $\tilde{\beta} \neq 0$, $\tilde{u} = \lambda_1\lambda_2^{-1} \neq -1$, $\tilde{v} = \tilde{\alpha}\tilde{\gamma}(2\tilde{\beta})^{-2} \neq \psi_1^2(\tilde{u}), \psi_2(\tilde{u}), \psi_3(\tilde{u})$; (b) J_1^2, $L_1^{6,2,<,>}$; (c) $\sigma = \operatorname{sgn}\lambda_2$, $u = \tilde{u}$, $v = \tilde{v}$;

$CF_{8,+1}^{4,2,<,=}$: (a) $D = 0$, $q_1 = 0$, $p_2 = 0$; (b) $L_{8,+1}^{4,2,<,=}$; (c) $\sigma = \operatorname{sgn} p_1$;

$CF_7^{5,2,<,=}$: (a) $D = 0$, $[q_1 \neq 0 \vee q_1 = 0,\, p_2 \neq 0]$, $\tilde{\gamma}(\nu\tilde{\xi})^{-1} = \pm 2/\sqrt{3}$, *where* $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ *from* $[(1.6_a)^3 \vee (1.6_b)^3]$; (b) $[J_{2a}^2 \vee J_{2b}^2]$, $L_{4,+1}^{6,2,<,=}$, $L_7^{5,2,<,=}$; (c) $\sigma = \operatorname{sgn}\nu$;

$CF_{22}^{5,2,<,=}$: (a) $D = 0$, $[q_1 \neq 0 \vee q_1 = 0, p_2 \neq 0]$, $\tilde{\gamma}(\nu\xi)^{-1} = \pm 7/\sqrt{3}$, *where* $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ *from* $[(1.6_a)^3 \vee (1.6_b)^3]$; (b) $[J_{2a}^2 \vee J_{2b}^2]$, $L_{4,+1}^{6,2,<,=}$, $L_{22}^{5,2,<,=}$; (c) $\sigma = \operatorname{sgn}\nu$;

$CF_3^{6,2,<,=}$: (a) $D = 0$, $[q_1 \neq 0 \vee q_1 = 0, p_2 \neq 0]$, $|\tilde{v}| \geq 1$, $|\tilde{v}| = 2/\sqrt{3}, 7/\sqrt{3}$, *where* $\tilde{v} = \tilde{\gamma}(\nu\xi)^{-1}$, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ *from* $[(1.6_a)^3 \vee (1.6_b)^3]$; (b) $[J_{2a}^2 \vee J_{2b}^2]$, $L_{4,+1}^{6,2,<,=}$, $L_3^{6,2,<,=}$; (c) $\sigma = \operatorname{sgn}\nu$, $v = \psi_4$;

$CF_{4,+1}^{6,2,<,=}$: (a) $D = 0$, $[q_1 \neq 0 \vee q_1 = 0, p_2 \neq 0]$, $|\tilde{v}| < 1$, *where* $\tilde{v} = \tilde{\gamma}(\nu\xi)^{-1}$, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ *from* $[(1.6_a)^3 \vee (1.6_b)^3]$; (b) $[J_{2a}^2 \vee J_{2b}^2]$, $L_{4,+1}^{6,2,<,=}$; (c) $\sigma = \operatorname{sgn}\nu$, $v = \tilde{v}$.

Proof. I. Consider a case $D > 0$. According to Lemma 2.1, p. i at $\tilde{\beta} \neq 0$, system $(1.5)^3$, obtained from $(1.1^<)$ with substitution J_1^2, is always reduced to $NSF_1^{6,2,<,>}(\tilde{\sigma}, \tilde{u}, \tilde{v})$ with $\tilde{\sigma} = \operatorname{sgn}\lambda_2$, $\tilde{u} = \lambda_1\lambda_2^{-1}$, $v = \tilde{\alpha}\tilde{\gamma}(2\tilde{\beta})^{-2} > 1/4$ (see [4, App. 3.6.2, p. 148]). According to Statement 2.1, p. iv, this $NSF_1^{6,2,<,>}$ can be reduced to one of the preceding $NSF^{m,2,<,>}$ from List 2.1.

The limitations that guarantee the reduction to $CF^{m,2,<,>}$ remain to be clarified.

(iv$_a$) At $\tilde{u} = -1 \Leftrightarrow \lambda_2 = -\lambda_1 \Leftrightarrow \nu = 0$, $CF_{34,+1}^{4,2,<,>}$ is obtained with $\sigma = \tilde{\sigma}$, $u = (2v^{1/2} - 1)(2v^{1/2} + 1)^{-1} = (\tilde{\alpha}\tilde{\gamma} - |\tilde{\beta}|)(\tilde{\alpha}\tilde{\gamma} + |\tilde{\beta}|)^{-1}$. Herewith, $0 < u < 1$, there are no limitations, because $NSF_{34,+1}^{4,2,<,>}$ is reduced to $SF_8^{4,2}$ only at $u = 1$, as follows from Statement 2.1, p. i.

(iv$_b$) At $\tilde{u} \neq -1$, $[3 \vee 1/3]$, $v = [\psi_1^2(\tilde{u}) \vee \psi_2(\tilde{u})]$, $CF_7^{5,2,<,>}$ is obtained with $\sigma = [\tilde{\sigma} \vee \tilde{\sigma}\operatorname{sgn}\tilde{u}]$ ($\tilde{\sigma}\operatorname{sgn}\tilde{u} = \operatorname{sgn}\lambda_1$), $u = [\tilde{u} \vee \tilde{u}^{-1}]$. Herewith, $u \neq -1, 3$, and there are no restrictions, because $NSF_7^{5,2,<,>}$ according to Statement 2.1, p. ii, is reduced to the previous forms only at $u = -1, 3$.

(iv$_c$) At $\tilde{u} \neq -1 \Leftrightarrow \nu \neq 0$, $v = \psi_3(\tilde{u})$, one has $NSF_{22}^{5,2,<,>}$ with $\sigma = \tilde{\sigma}\operatorname{sgn}(\tilde{u} + 1) = \operatorname{sgn}\nu$, $u = -\tilde{u}(\tilde{u} + 1)^{-2} = -\delta_{pq}(2\nu)^{-2}$. Furthermore, $\tilde{u} \neq (-4 \pm \sqrt{7})/3$, otherwise $u = 3/2$, $\tilde{u} \neq -3/2, -2/3$, otherwise $u = 6$, $\tilde{u} \neq (-5 \pm \sqrt{13})/6, (-5 \mp \sqrt{13})/2$, otherwise $u = 4 \mp \sqrt{13}$, because $NSF_{22}^{5,2,<,>}$, according to Statement 2.1, p. iii, at these u is reduced to the previous forms.

II. Let us consider a case $D = 0$. According to Lemma 2.1, p. ii, at $[q_1 \neq 0 \vee q_1 = 0, p_2 \neq 0]$ a system $(1.6)^3$ obtained from $(1.1^<)$ with a substitution $[J_{2a}^2 \vee J_{2b}^2]$, is always reduced to $NSF_{4,+1}^{6,2,<,=}$ $(\tilde{\sigma}, 1, \tilde{v})$ with $\tilde{\sigma} = \operatorname{sgn}\nu$, $\tilde{v} = \tilde{\gamma}(\nu\xi)^{-1}$ (see [4, App. 3.6.3, p. 152]). And this $NSF_{4,+1}^{6,2,<,=}$ with respect to Statement 2.1, p. vi, can be reduced to one of three preceding $CF^{m,2,<,=}$ from List 2.1.

In particular, in p. vi$_c$, at $|\tilde{v}| \geq 1$ and, additionally, $|\tilde{v}| \neq 2/\sqrt{3}, 7/\sqrt{3}$ there was obtained $CF_3^{6,2,<,=}$ with $\sigma = \operatorname{sgn}\nu$, $v = \psi_4(\tilde{v})$, because $\tilde{v} = -2/\sqrt{3} \Leftrightarrow v = 1$ and $v = 1/3$ at $\tilde{v} = 2/\sqrt{3}$, $v = (49 \pm 7\sqrt{46})/6$ at $\tilde{v} = \mp 7/\sqrt{3}$, and as follows from Statement 2.1, p. v, $NSF_3^{6,2,<,=}$ is reduced to the previous forms at only these v values.

Other results of the theorem are evident enough. □

3. Case $D < 0$. Let us now assume that the matrix H in the system $(1.1^<)$ has only the complex conjugate eigenvalues λ_1 and λ_2.

Collection 3.1. Constants, ranges and substitutions used hereinafter in Section 3:

$$\psi_5 = 4\hat{v}^5(\hat{u} - 1)^2 + 4\hat{v}^3(2\hat{u} + 1) + 1, \quad \psi_6 = \hat{v}^3(1 - \hat{u}) - 1 + \psi_5^{1/2}; \quad \psi_7^{\pm} = (v^3 - 2 \pm 2(1 - v^3)^{1/2})v^{-2};$$

$$\psi_8 = \tilde{\gamma}\nu + \tilde{\beta}\mu, \quad \psi_9 = \tilde{\gamma}^2\nu^2 + 2\tilde{\beta}\tilde{\gamma}\nu\mu + (9\tilde{\alpha}\tilde{\gamma} - 8\tilde{\beta}^2)\mu^2, \quad \psi_{10} = 2\tilde{\gamma}\nu - \tilde{\beta}\mu,$$

$$\psi_{11} = \tilde{\gamma}^3\nu^3 + 3\tilde{\gamma}(\tilde{\gamma}^2 + 2\tilde{\alpha}\tilde{\gamma} - 2\tilde{\beta}^2)\nu\mu^2 + \tilde{\beta}(4\tilde{\beta}^2 - 3\tilde{\alpha}\tilde{\gamma} + 3\tilde{\gamma}^2)\mu^3, \quad \underline{\theta}_1 \in \mathbb{R}^1 \text{ is zero of } \psi_{11}(\nu, 1),$$

$$\psi_{12} = \tilde{\gamma}^2\nu^2 - 4\tilde{\beta}\tilde{\gamma}\nu\mu + (4\tilde{\beta}^2 + 9\tilde{\gamma}^2)\mu^2, \quad \psi_{13} = \tilde{\gamma}^2\nu^2 + 2\tilde{\beta}\tilde{\gamma}\nu\mu - (3\tilde{\alpha}\tilde{\gamma} - 4\tilde{\beta}^2)\mu^2, \quad \psi_{14}^{\pm} = -\tilde{\beta} \pm \sqrt{3}\xi,$$

$$\psi_{15}^{\pm} = 2\xi \pm \sqrt{3}\tilde{\beta}, \quad \psi_{16} = 4\tilde{\alpha}\tilde{\gamma} - 3\tilde{\beta}^2, \quad \psi_{17} = \tilde{v}^2 - 3\tilde{v} + 9, \quad \psi_{18} = 4\tilde{v}^2 - 3\tilde{v} + 9 + (4\tilde{v} + 3)\psi_{17}^{1/2},$$

$$\psi_{19} = -(\tilde{v}^6 + 6\tilde{v}^{9/2} + 13\tilde{v}^3 + 12\tilde{v}^{3/2} + 4)(\tilde{v}^6 - 5\tilde{v}^3 + 4)^{-1}, \quad \psi_{20}^{\mp} = \tilde{u}(3\tilde{u} \mp \sqrt{3})(2 \mp \sqrt{3}\tilde{u})^{-1},$$

$$\psi_{21} = (\tilde{v} - 3 + (\tilde{v}^2 - 3\tilde{v} + 9)^{1/2})^2 (3\tilde{v})^{-1}; \quad \psi_{22} = \tilde{v}^2\tilde{w}^2 - 5\tilde{v}^4\tilde{w} + 2\tilde{v}\tilde{w} + 4\tilde{v}^6 + 4\tilde{v}^3 + 1,$$

$$\psi_{23} = 2\tilde{v}\tilde{w} - 5\tilde{v}^3 + 2 + 2\psi_{22}^{1/2}, \quad \psi_{24} = \tilde{v}^3 - \tilde{u}\tilde{v}^2 + \tilde{u}^2\tilde{v} - 3, \quad \underline{\theta_2(u)} > 0 \text{ is zero of } \psi_{24}(u,\theta),$$

$$\psi_{25}^{\pm} = (\tilde{v}^2 \pm (12\tilde{v} - 3\tilde{v}^4)^{1/2})(2\tilde{v})^{-1}, \quad \psi_{26} = 2\tilde{v}^3 + \tilde{u}\tilde{v}^2 - \tilde{u}^2\tilde{v} - 9, \quad \underline{\theta_3(u)} \in \mathbb{R}^1 \text{ is } \forall \text{ zero of } \psi_{26}(u,\theta),$$

$$\psi_{27} = 4\tilde{v}^2\tilde{w}^2 - 4\tilde{v}\tilde{w}(2\tilde{v}^3 - 3\tilde{u}\tilde{v}^2 - 2) + (\tilde{v}^3 - 3\tilde{u}\tilde{v}^2 + 2)^2, \quad \psi_{28} = 2\tilde{v}\tilde{w} - 2\tilde{v}^3 + 3\tilde{u}\tilde{v}^2 + 2 - \psi_{27}^{1/2},$$

$$\psi_{29} = \tilde{u}\tilde{v}^2 - \tilde{u}^2\tilde{v} - 1, \quad \psi_{30} = \theta_3^2(\tilde{u})(2\tilde{u} - \theta_3(\tilde{u}))^2\psi_{29}^{-1}(\tilde{u},\theta_3(\tilde{u}))(8\theta_3^3(\tilde{u}) - 5\psi_{29}(\tilde{u},\theta_3(\tilde{u})) - 32)^{-1},$$

$$\psi_{31} = -\psi_{29}(\tilde{u},\theta_3(\tilde{u}))(3\tilde{w} + 4\tilde{u}^2 + 2\theta_3(\tilde{u})\tilde{u} - 2\theta_3^2(\tilde{u}))(\theta_3(\tilde{u})(2\tilde{u} - \theta_3(\tilde{u}))\tilde{w})^{-1},$$

$$\psi_{32} = 3(\tilde{u}^3 - 3\tilde{u}^2 + 6\tilde{u} + 1)(\tilde{u}^2 - \tilde{u} + 1)((\tilde{u} - 2)(2\tilde{u} - 1)(\tilde{u} + 1))^{-1}.$$

$$\psi_{33} = 3(\tilde{u}\tilde{v}^6 - 2\tilde{u}^2\tilde{v}^5 + (4\tilde{u}^3 - 1)\tilde{v}^4 - \tilde{u}(4\tilde{u}^3 + 1)\tilde{v}^3 + \tilde{u}^2(\tilde{u}^3 - 6)\tilde{v}^2 + (6\tilde{u}^3 + 2)\tilde{v} + 5\tilde{u})(\tilde{v}(2\tilde{u} - \tilde{v})\psi_{24})^{-1},$$

$$\psi_{34} = (\tilde{v}^4 - \tilde{u}\tilde{v}^3 + 3\tilde{u}^2\tilde{v}^2 - \tilde{u}^3\tilde{v} - 2\tilde{v} - 5\tilde{u})\psi_{29}^{-1}.$$

$$\psi_{35} = (2\tilde{u} - \theta_2(\tilde{u}))(3\theta_2(\tilde{u})\tilde{u}^3 - 3\theta_2^2(\tilde{u})\tilde{u}^2 + \theta_2^3(\tilde{u})\tilde{u} - \tilde{u} + \theta_2^4(\tilde{u}) - 4\theta_2(\tilde{u})),$$

$$\psi_{36} = -(3\psi_{26}(\tilde{u},\theta_3(\tilde{u}))\tilde{w} + 2(2\tilde{u} - \theta_3(\tilde{u}))(3\theta_3(\tilde{u})\tilde{u}^3 - 6\theta_3^2(\tilde{u})\tilde{u}^2 + 5(\theta_3^3(\tilde{u}) - 1)\tilde{u}$$
$$+ 2\theta_3^4(\tilde{u}) - 11\theta_3(\tilde{u})))((8\theta_3^3(\tilde{u}) - 5\psi_{29}(\tilde{u},\theta_3(\tilde{u})) - 32)\tilde{w})^{-1};$$

$$\hat{a}_1^* = ((\hat{u}^2\hat{v}^6 - 2\hat{v}^3(\hat{v}^3 - 1)\hat{u} + \hat{v}^6 + 10\hat{v}^3 - 2)\psi_5^{1/2} + 2\hat{u}^3\hat{v}^9 - 6\hat{v}^6(\hat{v}^3 - 1)\hat{u}^2$$
$$+ 6\hat{v}^3(\hat{v}^3 - 1)^2\hat{u} - 2(\hat{v}^3 - 1)^3)(9\hat{v}^3\psi_6)^{-1},$$

$$\hat{b}_1^* = ((\hat{v}^3\hat{u}^2 - (10\hat{v}^3 - 4)\hat{u} + 9\hat{v}^3)\psi_5^{1/2} + 2\hat{v}^6\hat{u}^3 + \hat{v}^3(14\hat{v}^3 + 1)\hat{u}^2$$
$$- 2(\hat{v}^3 - 1)(17\hat{v}^3 - 2)\hat{u} + 9\hat{v}^3(2\hat{v}^3 + 1))(6\psi_6)^{-1},$$

$$\hat{a}_2^* = -((4\hat{v}^9\hat{u}^3 - 3\hat{v}^6(4\hat{v}^3 - 3)\hat{u}^2 + 6\hat{v}^6(2\hat{v}^3 + 1)\hat{u} - (4\hat{v}^3 - 1)(\hat{v}^3 + 2)^2)\psi_5^{1/2} + 8\hat{v}^{12}\hat{u}^4 - 2\hat{v}^9(16\hat{v}^3 - 13)\hat{u}^3$$
$$+ 3\hat{v}^6(16\hat{v}^6 - 6\hat{v}^3 + 5)\hat{u}^2 - 2\hat{v}^3(\hat{v}^3 - 1)(16\hat{v}^6 + 37\hat{v}^3 - 8)\hat{u} + (2\hat{v}^3 + 1)(4\hat{v}^3 - 1)(\hat{v}^3 + 2)^2)(54\hat{v}^3\psi_6)^{-1},$$

$$\hat{c}_2^* = ((9\hat{v}^6\hat{u}^2 - 2\hat{v}^3(5\hat{v}^3 - 8)\hat{u} + (\hat{v}^3 + 2)^2)\psi_5^{1/2} - 18\hat{v}^9\hat{u}^3 + \hat{v}^6(34\hat{v}^3 - 49)\hat{u}^2$$
$$- 2\hat{v}^3(7\hat{v}^6 - 5\hat{v}^3 + 16)\hat{u} - (2\hat{v}^3 + 1)(\hat{v}^3 + 2)^2)(6\hat{v}^3\psi_6)^{-1};$$

$$\tilde{a}_1^* = (\tilde{v}^3 - 4)(\tilde{u} + \tilde{v})((2\tilde{v}\tilde{w} - 2\tilde{v}^3 + 3\tilde{u}\tilde{v}^2 + 2)\psi_{27} - 4\tilde{v}^2\tilde{w}^2 - 4\tilde{v}(-2\tilde{v}^3 + 3\tilde{u}\tilde{v}^2 + 2)\tilde{w} - 2\tilde{v}^6$$
$$+ 7\tilde{u}\tilde{v}^5 - 7\tilde{u}^2\tilde{v}^4 + 8\tilde{u}\tilde{v}^2 - 8\tilde{u}^2\tilde{v} - 4)(2\tilde{v})^{-1}\psi_{28}^{-2},$$

$$\tilde{b}_1^* = (\tilde{v}^3 - 4)((\tilde{v}\tilde{w} - \tilde{u}\tilde{v}^2 + \tilde{u}^2\tilde{v} + 1)\psi_{27} - 2\tilde{v}^2\tilde{w}^2 + \tilde{v}(3\tilde{v}^3 - 5\tilde{u}\tilde{v}^2 + 2\tilde{u}^2\tilde{v} - 4)\tilde{w}$$
$$+ (\tilde{v}^3 - 3\tilde{u}\tilde{v}^2 + 2)(\tilde{u}\tilde{v}^2 - \tilde{u}^2\tilde{v} - 1))\tilde{v}^{-2}(2\tilde{v} - 4\tilde{u})^{-1}\psi_{28}^{-1},$$

$$\tilde{a}_2^* = (\tilde{v} - 2\tilde{u})(\tilde{v}^3 - 4)((4\tilde{v}\tilde{w}^2 - (7\tilde{v}^3 - 12\tilde{u}\tilde{v}^2 - 4)\tilde{w} + (2\tilde{v}^3 + 1)(\tilde{v} - 2\tilde{u})^2)\psi_{27} - 8\tilde{v}^2\tilde{w}^3 + 2\tilde{v}(11\tilde{v}^3 - 18\tilde{u}\tilde{v}^2$$
$$- 8)\tilde{w}^2 - (15\tilde{v}^6 - 55\tilde{u}\tilde{v}^5 + 52\tilde{u}^2\tilde{v}^4 - 8\tilde{v}^3 + 4\tilde{u}\tilde{v}^2 + 8\tilde{u}^2\tilde{v} + 8)\tilde{w} + (2\tilde{v}^3 + 1)(\tilde{v}^3 - 3\tilde{u}\tilde{v}^2 + 2)(\tilde{v} - 2\tilde{u})^2)\psi_{28}^{-3}/2,$$

$$\tilde{c}_2^* = \psi_{28}^2(\tilde{v}(\tilde{v} - 2\tilde{u})^2(4 - \tilde{v}^3))^{-1}\tilde{a}_2^*;$$

$$I_* = (-1, \theta_*) \cup (1/2, 2), \quad \text{where} \quad \theta_* \approx -0.17 \quad \text{is zero of } 14\theta^5 - 47\theta^4 + 113\theta^3 - 103\theta^2 + 61\theta + 14;$$

$$I_1^+ = (-7 - \sqrt{37}, -2) \cup (-7 + \sqrt{37}, 0), \quad I_1^- = (0, 7 - \sqrt{37}) \cup (2, 7 + \sqrt{37}), \quad I_2^+ = (0, 1),$$

$$I_2^- = (0, 7^{2/3}13^{-1/3}), \quad I_3 = (-13 \cdot 7^{-1/3}, -7^{-1/3}), \quad I_4 = (-\infty, -1) \cup (5 \cdot 91^{-1/3}, +\infty);$$

$$L1_{34,+1}^{4,2,<,<} = \{r_1, s_2 = 0,\ s_1 = -\tilde{\gamma}(\tilde{\alpha}\tilde{\gamma})^{-3/2}\mu^{-1/2},\ r_2 = (\tilde{\alpha}\tilde{\gamma})^{-1/2}\mu^{-1/2}\};$$

$$L1_{22}^{5,2,<,<} = \{r_1 = 0,\ s_1 = \tilde{\gamma}|\tilde{\beta}|^{-3/2}\mu^{-1/2},\ r_2 = -\tilde{\beta}|\tilde{\beta}|^{-3/2}\mu^{-1/2},\ s_2 = r_2/2\};$$

$$L1_{6}^{6,2,<,<} = L1_{2}^{7,2,<,<} \quad \text{with} \quad \nu = \theta_1\mu;$$

$$L1_{7}^{6,2,<,<} = \{r_1 = 0,\ s_1 = 3^{3/4}\tilde{\gamma}(2\tilde{\xi})^{-3/2}\mu^{-1/2},\ r_2 = \mp 3^{1/4}(2\tilde{\xi})^{-1.2}\mu^{-1.2},\ s_2 = 3^{1/4}(-\sqrt{3}\tilde{\beta} \pm \tilde{\xi})(2\tilde{\xi}\mu)^{-3/2}\mu^{-1/2}\};$$

$$L1_{11,+1}^{6,2,<,<} = \{r_1 = 0,\ s_1 = -\tilde{\gamma}\tilde{\varsigma}^{-3/2}\mu^{-1/2},\ r_2 = \tilde{\varsigma}^{-1/2}\mu^{-1/2},\ s_2 = -\tilde{\beta}\tilde{\gamma}^{-1}s_1\};$$

$$L\hat{1}_{11,+1}^{6,2,<,<} = \{r_1 = (2\hat{v}^3(1+\hat{u}) + 1 - \psi_5^{1/2})(2\hat{v}^3(1-\hat{u}) - 2 - \psi_5^{1/2})(6\hat{v}(4\hat{v}^3 - 1))^{-1}r_2,$$
$$s_1 = (2\hat{v}^3(1-\hat{u}) - 1 + \psi_5^{1/5})(2\hat{v})^{-1}s_2,\ r_2 = (\hat{a}_2^*\hat{c}_2^*)^{-1/4},\ s_2 = (\hat{a}_2^*\hat{c}_2^*)^{1/4}/\hat{c}_2^*\};$$

$$L1_{12}^{6,2,<,<} = \{r_1 = 0,\ s_1 = 2\tilde{\gamma}(|\tilde{\beta}|\mu\psi_{16})^{-1/2},\ r_2 = -(2\tilde{\beta})^{1/3}(|\tilde{\beta}|\mu)^{-1/2}\psi_{16}^{-1/6},\ s_2 = -\tilde{\beta}(|\tilde{\beta}|\mu\psi_{16})^{-1/2}\};$$

$$L1_{2}^{7,2,<,<} = \{r_1 = 0,\ s_1 = 3^{3/2}\tilde{\gamma}\mu(2|\psi_8|\psi_9)^{-1/2},\ r_2 = -3^{1/2}(2|\psi_8|\psi_9)^{-1/6}\operatorname{sgn}\psi_8,\ s_2 = (\tilde{\gamma}\nu - 2\tilde{\beta}\mu)(3\tilde{\gamma}\mu)^{-1}s_1\};$$

$$L2_{34,+1}^{4,2,<,<} = \{r_1 = (\tilde{v} + \psi_{17}^{1/2})r_2/3,\ s_1 = (\tilde{v} - \psi_{17}^{1/2})s_2/3,\ r_2 = 3(\psi_{17}\psi_{18}(3 - \tilde{v} + \psi_{17}^{1/2}))^{-1/4},$$
$$s_2 = \psi_{17}^{-1/4}\psi_{18}^{1/4}(3 - \tilde{v} + \psi_{17}^{1/2})^{-3/4}\};$$

$$L3_{34,+1}^{4,2,<,<} = \{r_1 = -\tilde{v}^{-1/2}r_2,\ s_1 = -\tilde{v}^{1/4}(2 - \tilde{v}^{3/2} - \tilde{v}^3)^{1/4}(2 - 3\tilde{v}^{3/2} + \tilde{v}^3)^{-3/4},$$
$$r_2 = \tilde{v}^{3/4}(2 - \tilde{v}^{3/2} - \tilde{v}^3)^{-1/4}(2 - 3\tilde{v}^{3/2} + \tilde{v}^3)^{-1/4},\ s_2 = \tilde{v}^{1/2}s_1\};$$

$$L4_{34,+1}^{4,2,<,<} = \{r_1 = \tilde{v}(\tilde{v}\tilde{w} - \tilde{v}^3 - 5 - \psi_{22}^{1/2})\psi_{23}^{-1}r_2,\ s_1 = (\tilde{v}\tilde{w} - \tilde{v}^3 + 1 - \psi_{22}^{1/2})\tilde{v}^{-2}s_2/3,$$
$$r_2 = (\hat{a}_2^*(\tilde{\varrho})\hat{c}_2^*(\tilde{\varrho}))^{-1/4},\ s_2 = (\hat{a}_2^*(\tilde{\varrho})\hat{c}_2^*(\tilde{\varrho}))^{1/4}/\hat{c}_2^*(\tilde{\varrho})\}, \quad \text{where} \quad (\tilde{\varrho}) = (-\tilde{v}, \tilde{v}, \tilde{w});$$

$$L2_{22}^{5,2,<,<} = \{r_1 = \mp\tilde{u}s_2,\ s_1 = -(2\tilde{u} \mp \sqrt{3})s_2,\ r_2 = -(2 \mp \sqrt{3}\tilde{u})\tilde{u}^{-1}r_1,\ s_2 = (\tilde{u}^2 \mp \sqrt{3}\tilde{u} + 1)^{-1/2}(\pm 4\tilde{u})^{-1/2}\};$$

$$L3_{22}^{5,2,<,<} = \{r_1 = (\tilde{u} + 2)\tilde{u}^{-1}r_2,\ s_1 = -r_2,\ r_2 = -\tilde{u}((\tilde{u}+1)(\tilde{u}^2 + 2\tilde{u} + 4))^{-1/2},\ s_2 = 2\tilde{u}^{-1}r_2\};$$

$$L4_{22}^{5,2,<,<} = \{r_1 = -s_2,\ s_1 = (\tilde{u} + 3)s_2,\ r_2 = (\tilde{u} + 2)s_2,\ s_2 = ((-\tilde{u} - 1)(\tilde{u}^2 + 5\tilde{u} + 7))^{-1}\};$$

$$L5_{22}^{5,2,<,<} = \{r_1 = -2^{1/3}r_2,\ s_1 = 2^{1/3}((\sqrt{5} \pm 1)/5)^{1/2},\ r_2 = \pm((\sqrt{5} \mp 1)/5)^{1/2},\ s_2 = (2(\sqrt{5} \mp 2)/5)^{1/2}\};$$

$$L6_{22}^{5,2,<,<} = \{r_1 = -7^{-1/3}r_2,\ s_2 = 3r_1,\ r_2 = 7^{1/2}(-7^{1/3}\tilde{u} - 1)^{-1/2}/3,\ s_2 = 0\};$$

$$L7_{22}^{5,2,<,<} = \{r_1 = \tilde{u}r_2,\ s_1 = (\theta_2(\tilde{u}) - \tilde{u})s_2,\ r_2 = \left|\theta_2(\tilde{u})(\tilde{u} + \theta_2(\tilde{u}))^{-1}\psi_{29}^{-1}\right|^{1/2},\ s_2 = r_2\};$$

$$L8_{22}^{5,2,<,<} = \{r_1 = \tilde{v}r_2/2,\ s_1 = \psi_{25}^{-}s_2,\ r_2 = -2s_2,\ s_2 = (12 - 3\tilde{v}^3)^{-1/2}\};$$

$$L9_{22}^{5,2,<,<} = \{r_1 = \psi_{25}^{\mp}r_2,\ s_1 = \psi_{25}^{\pm}s_2,\ r_2 = (4 - \tilde{v}^3)^{-1/2}(\mp(\tilde{v}^{-1}\psi_{25}^{\mp} + 1))^{-1/2},\ s_2 = r_2\};$$

$$L2_{6}^{6,2,<,<} = \{r_1 = -(\tilde{u}^2 + 3)(2\tilde{u})^{-1}r_2,\ s_1 = \tilde{u}s_2,\ r_2 = (2\tilde{u})^{4/3}(\tilde{u}^2 + 9)^{-2/3}s_2,$$
$$s_2 = (-\tilde{u}^2 + 3)^{1/2}(\tilde{u}^2 + 1)^{-1}|2\tilde{u}|^{-1/2}\};$$

$$L3_{6}^{6,2,<,<} = \{r_1 = -(\tilde{u}^2 - 2\tilde{u} + 3)(2\tilde{u} - 1)^{-1}r_2,\ s_1 = \tilde{u}s_2,\ r_2 = (2\tilde{u} - 1)^{4/3}(\tilde{u}^2 - \tilde{u} + 7)^{-2/3}s_2,$$
$$s_2 = |(\tilde{u} - 2)(\tilde{u} + 1)|^{1/2}|2\tilde{u} - 1|^{-1/2}(\tilde{u}^2 - \tilde{u} + 1)^{-1}\};$$

$$L4_{6}^{6,2,<,<} = \{r_1 = -(2\tilde{u}\tilde{v}^2 - \tilde{u}^2\tilde{v} - 3)(\tilde{v}(\tilde{v} - 2\tilde{u}))^{-1}r_2,\ s_1 = \tilde{u}s_2,\ r_2 = \tilde{v}^{2/3}(\tilde{v} - 2\tilde{u})^{4/3}\psi_{26}^{-2/3}s_2,$$
$$s_2 = -\tilde{v}^{1/2}|\psi_{24}|^{1/2}\psi_{29}^{-1}|\tilde{v} - 2\tilde{u}|^{-1/2}\};$$

$$L2_{7}^{6,2,<,<} = \{r_1 = \psi_{25}^{\mp}r_2,\ s_1 = \psi_{25}^{\pm}s_2,\ r_2 = 3^{1/4}\tilde{v}^{1/4}(4 - \tilde{v}^3)^{-1/4}(-\tilde{w})^{-1/2},\ s_2 = r_2\};$$

$$L3_{7}^{6,2,<,<} = \{r_1 = \tilde{u}r_2,\ s_1 = (\theta_2(\tilde{u}) - \tilde{u})s_2,\ r_2 = \left|\theta_2(\tilde{u})(2\tilde{u} - \theta_2(\tilde{u}))\psi_{29}^{-1}(\tilde{u}, \theta_2(\tilde{u}))\tilde{w}^{-1}\right|^{1/2},\ s_2 = r_2\};$$

$$L2^{6,2,<,<}_{11,+1} = \{r_1 = (2\tilde{v}^2\tilde{w} - \tilde{v}^4 + \tilde{u}\tilde{v}^3 - 2\tilde{v} + 8\tilde{u} - \tilde{v}\psi_{27}^{1/2})(2\psi_{28})^{-1}r_2,\ r_2 = (\tilde{a}_2^*\tilde{c}_2^*)^{-1/4},$$
$$s_1 = (2\tilde{v}\tilde{w} - \tilde{v}^3 + \tilde{u}\tilde{v}^2 + 2 - \psi_{27}^{1/2})(2\tilde{v}(\tilde{v} - 2\tilde{u}))^{-1}s_2,\ s_2 = (\tilde{a}_2^*\tilde{c}_2^*)^{1/4}/\tilde{c}_2^*\};$$
$$L3^{6,2,<,<}_{11,+1} = \{r_1 = \tilde{v}r_2/2,\ s_1 = (\tilde{w}r_2)^{-1},\ r_2 = \sqrt{2}\tilde{v}^{1/4}(-\tilde{w})^{-1/2}(4 - \tilde{v}^3)^{-1/4},\ s_2 = 0\};$$
$$L4^{6,2,<,<}_{11,+1} = \{r_1 = \tilde{u}r_2,\ s_1 = (2\theta_3(\tilde{u}) - \tilde{u})s_2/3,\ r_2 = \sqrt{2}\psi_{30}^{1/4}(-\tilde{w})^{-1/2},$$
$$s_2 = -3\sqrt{2}\psi_{30}^{3/4}\psi_{29}(\tilde{u},\theta_3(\tilde{u}))(\theta_3(\tilde{u})(2\tilde{u} - \theta_3(\tilde{u})))^{-1}(-\tilde{w})^{-1/2}\};$$

the constants and substitutions from Collection $(1.1)^3$ will be used as well.

Considering system $(1.1^<)$ with $\gamma - \beta^2 > 0$ at $D < 0$ ($p_2q_1 < 0$, $\nu^2 + \mu^2 = \delta_{pq}$, $\mu > 0$) with substitution J_3^2 results in system $(1.7)^3$ with $\tilde{\alpha}, \tilde{\gamma}, \xi > 0$, according to $(2.18)^1$ and $(2.19)^1$.

Let us divide elements $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, ν and μ of system $(1.7)^3$ into the disjoint sets, wherein $(1.7)^3$ is reduced to a certain form from List 1.1.

Lemma 3.1. *A system* $(1.7)^3$ *is reduced*:

1_1) *at* $\nu = -\tilde{\beta}\tilde{\gamma}^{-1}\mu$, $\tilde{\beta} = 0$ *by substituting* $L1^{4,2,<,<}_{34,+1}$ *to* $CF^{4,2,<,<}_{34,+1}$ *with* $\sigma = 1$, $u = \tilde{\gamma}^2\xi^{-2}$;

1_2) *at* $\nu = -\tilde{\beta}\tilde{\gamma}^{-1}\mu$, $\tilde{\beta} \neq 0$ *by substituting* $L1^{6,2,<,<}_{11,+1}$ *to* $NSF^{6,2,<,<}_{11,+1}$ *with* $\sigma = 1$, $u = -2\tilde{\beta}\xi^{-1}$, $v = -(\tilde{\beta}^2 + \tilde{\gamma}^2)\xi^{-2}$;

2_1) *at* $\nu = \tilde{\beta}(2\tilde{\gamma})^{-1}\mu$, $\tilde{\beta} = 0$ *one has* 1_1;

2_2^a) *at* $\nu = \tilde{\beta}(2\tilde{\gamma})^{-1}\mu$, $\tilde{\beta} \neq 0$, $\tilde{\alpha} = 7\tilde{\beta}^2(4\tilde{\gamma})^{-1}$ *by substituting* $L1^{5,2,<,<}_{22}$ *to* $CF^{5,2,<,<}_{22}$ *with* $\sigma = \operatorname{sgn}\tilde{\beta}$, $u = -(\tilde{\beta}^2 + 4\tilde{\gamma}^2)(2\tilde{\beta})^{-2}$;

2_2^b) *at* $\nu = \tilde{\beta}(2\tilde{\gamma})^{-1}\mu$, $\tilde{\beta} \neq 0$, $\tilde{\alpha} \neq 7\tilde{\beta}^2(4\tilde{\gamma})^{-1}$ *by substituting* $L1^{6,2,<,<}_{12}$ *to* $NSF^{6,2,<,<}_{12}$ *with* $\sigma = \operatorname{sgn}\tilde{\beta}$, $u = (\tilde{\beta}^2 + 4\tilde{\gamma}^2)\psi_{16}^{-1}$, $v = \psi_{16}^{1/3}(2\tilde{\beta})^{-2/3}$;

3) *at* $\nu = \theta_1\mu$ *by substituting* $L1^{6,2,<,<}_{6}$ *to* $NSF^{6,2,<,<}_{6}$ *with* $\sigma = \operatorname{sgn}\psi_8(\cdot)$, *where* $(\cdot) = (\theta_*\mu, \mu)$, $u = 2(2\psi_8(\cdot)\psi_9(\cdot))^{-1/3}\psi_{10}(\cdot)$, $v = -(2\psi_8(\cdot)\psi_{10}(\cdot))^{1/3}\psi_{12}^{-1/3}(\cdot)$;

4_1) *at* $\nu = \psi_{14}^{\pm}\tilde{\gamma}^{-1}\mu$, $\psi_{15}^{\mp} = 0$ $(\tilde{\beta} \neq 0)$ *one has* 2_2^a;

4_2) *at* $\nu = \psi_{14}^{\pm}\tilde{\gamma}^{-1}\mu$, $\psi_{15}^{\mp} \neq 0$ *by substituting* $L1^{6,2,<,<}_{7}$ *to* $NSF^{6,2,<,<}_{7}$ *with* $\sigma = \pm 1$, $u = \psi_{15}^{\mp}\xi^{-1}$, $v = -3(\psi_{14}^{\pm 2} + \tilde{\gamma}^2)(2\xi)^{-2}$;

5) *at* $\nu \neq -\tilde{\beta}\tilde{\gamma}^{-1}\mu$, $\theta_1\mu$, $\psi_{14}^{\pm}\tilde{\gamma}^{-1}\mu$, $\tilde{\beta}(2\tilde{\gamma})^{-1}\mu$ *by substituting* $L1^{7,2,<,<}_{2}$ to $NSF^{7,2,<,<}_{2}$ *with* $\sigma = \operatorname{sgn}\psi_8$, $u = 2(2\psi_8\psi_9)^{-1/3}\psi_{10}$, $v = (2\psi_8)^{2/3}\psi_9^{-1/3}$, $w = -9\tilde{\gamma}^2(2\psi_8\psi_9)^{-2/3}\delta_{pq}$.

Proof. Any substitution $(2.2)^1$ $L = (r_1, s_1; r_2, s_2)$ $(\det L \neq 0)$ with

$$r_1 = 0, \quad s_2 = (\tilde{\gamma}\nu - 2\tilde{\beta}\mu)(3\tilde{\gamma}\mu)^{-1}s_1 \quad (s_1, r_2 \neq 0), \tag{3.1}$$

reduces $(1.7)^3$ to the system:

$$\begin{pmatrix} \hat{a}_1 & \hat{b}_1 & \hat{c}_1 & \hat{d}_1 \\ \hat{a}_2 & 0 & \hat{c}_2 & \hat{d}_2 \end{pmatrix}, \tag{3.2}$$

for which $\hat{a}_2 = -\tilde{\gamma}\mu r_1^3 s_1^{-1} \neq 0$, $\hat{d}_2 = 2(\tilde{\gamma}\mu)^{-2}\psi_8\psi_9 s_1^2/27$.

In $\hat{d}_2$, the homogeneous polynomial satisfies the inequality $\psi_9(\nu,\mu) = \tilde{\gamma}^2\nu^2 + 2\tilde{\beta}\tilde{\gamma}\nu\mu + (9\tilde{\alpha}\tilde{\gamma} - 8\tilde{\beta}^2)\mu^2 > 0$, since it has zeros $\nu_{1,2} = (-\tilde{\beta} \pm 3\varsigma i)\tilde{\gamma}^{-1}\mu$. Thus $\hat{d}_2 = 0 \Leftrightarrow \psi_8 = \tilde{\gamma}\nu + \tilde{\beta}\mu = 0$.

(i) $\psi_8 = 0 \Leftrightarrow \nu = -\tilde{\beta}\tilde{\gamma}^{-1}\mu$. At $s_1 = -\tilde{\gamma}\xi^{-3/2}\mu^{-1/2}$, $r_2 = \xi^{-1/2}\mu^{-1/2}$, $s_2 = -\tilde{\beta}\tilde{\gamma}^{-1}s_1$ substituting $L1^{6,2,<,<}_{11,+1}$ reduced system (3.2) to the form $\begin{pmatrix} -2\tilde{\beta}\xi^{-1} & -(\tilde{\beta}^2 + \tilde{\gamma}^2)\xi^{-2} & -2\tilde{\beta}\xi^{-1} & -(\tilde{\beta}^2 + \tilde{\gamma}^2)\xi^{-2} \\ 1 & 0 & 1 & 0 \end{pmatrix}$.

($\mathrm{i_i}$) $\tilde\beta = 0 \Rightarrow \nu = 0$. At $s_1 = -\tilde\gamma(\tilde\alpha\tilde\gamma)^{-3/2}\mu^{-1/2}$, $r_2 = (\tilde\alpha\tilde\gamma)^{-1/2}\mu^{-1/2}$ substituting $L1^{4,2,<,<}_{34,+1}$ at taking into account (3.1) results in $CF^{4,2,<,<}_{34,+1}$ with $\sigma = 1$, $u = -\tilde\alpha^{-1}\tilde\gamma$ $(= D/4 < 0)$.

($\mathrm{i_{ii}}$) $\tilde\beta \neq 0$, then obtain $NSF^{6,2,<,<}_{11,+1}$ with $\sigma = 1$, $u = -2\tilde\beta\tilde\zeta^{-1}$, $v = -(\tilde\beta^2 + \tilde\gamma^2)\tilde\zeta^{-2} < 0$.

(ii) $\psi_8 = \tilde\beta\mu + \tilde\gamma\nu \neq 0$. At $s_1 = 3^{3/2}\tilde\gamma\mu(2|\psi_8|\psi_9)^{-1/2}$, $r_2 = -3^{1/2}(2|\psi_8|\psi_9)^{-1/6}\operatorname{sgn}\psi_8$ substituting $L^{7,2,<,<}_2$ with considering (3.1) allows system (3.2) to be written as

$$\hat\sigma\begin{pmatrix} 2(2\psi_8\psi_9)^{-1/3}\psi_{10} & -9\tilde\gamma^2(2\psi_8\psi_9)^{-2/3}\delta_{pq} & 3(\psi_8\psi_9)^{-1}\psi_{11} & -(2\psi_8)^{-4/3}\psi_9^{-1/3}\psi_{12} \\ 1 & 0 & -3(2\psi_8\psi_9)^{-2/3}\psi_{13} & 1 \end{pmatrix}, \tag{3.3}$$

where $\hat\sigma = \operatorname{sgn}\psi_8$, homogeneous polynomials $\psi_{10}(\nu,\mu) = 2\tilde\gamma\nu - \tilde\beta\mu$, $\psi_{11}(\nu,\mu) = \tilde\gamma^3\nu^3 + 3\tilde\gamma(\tilde\gamma^2 + 2\tilde\alpha\tilde\gamma - 2\tilde\beta^2)\nu\mu^2 + \tilde\beta(4\tilde\beta^2 - 3\tilde\alpha\tilde\gamma + 3\tilde\gamma^2)\mu^3$, $\psi_{12}(\nu,\mu) = \tilde\gamma^2\nu^2 - 4\tilde\beta\tilde\gamma\nu\mu + (4\tilde\beta^2 + 9\tilde\gamma^2)\mu^2$, $\psi_{13}(\nu,\mu) = \tilde\gamma^2\nu^2 + 2\tilde\beta\tilde\gamma\nu\mu - (3\tilde\alpha\tilde\gamma - 4\tilde\beta^2)\mu^2$. Herewith, one has $\psi'_{11}(\nu,1) = \tilde\gamma(\tilde\gamma^2(\nu^2+1) + 2(\tilde\alpha\tilde\gamma - \tilde\beta^2)) > 0$, thus θ_1 is the unique real zero of $\psi_{11}(\nu,1)$; $\psi_{12}(\nu,\mu) > 0$ owing to its zeros $\nu_{1,2} = (2\beta \pm 3\tilde\gamma i)\tilde\gamma^{-1}\mu$; $\psi_{13}(\nu,\mu)$ possesses zeros $\nu_{1,2} = \psi^{\pm}_{14}\tilde\gamma^{-1}\mu$, $\psi^{\pm}_{14} = -\tilde\beta \pm \sqrt{3}\tilde\zeta$.

($\mathrm{ii_i}$) $\hat a_1 = 0 \Leftrightarrow \psi_{10} = 0 \Leftrightarrow \nu = \tilde\beta(2\tilde\gamma)^{-1}\mu$.

($\mathrm{ii^i_i}$) $\tilde\beta = 0$, then $\psi_{10} = 0 \Leftrightarrow \psi_8 = 0$ $(\nu = 0)$, and one returns to case $\mathrm{i_i}$.

($\mathrm{ii^{ii}_i}$) $\tilde\beta \neq 0$. At $s_1 = 2\tilde\gamma(|\tilde\beta|\mu\psi_{16})^{-1/2}$, $r_2 = -(2\tilde\beta)^{1/3}(|\tilde\beta|\mu)^{-1/2}\psi_{16}^{-1/6}$ $(4\tilde\alpha\tilde\gamma - 3\tilde\beta^2 > 0)$ substituting $L1^{6,2,<,<}_{12}$ on account of (3.1) reduces system (3.3) to the form

$$\operatorname{sgn}\tilde\beta\begin{pmatrix} 0 & -(\tilde\beta^2 + 4\tilde\gamma^2)(2\tilde\beta\psi_{16})^{-2/3} & (\tilde\beta^2 + 4\tilde\gamma^2)\psi_{16}^{-1} & -(\tilde\beta^2 + 4\tilde\gamma^2)(2\tilde\beta)^{-4/3}\psi_{16}^{-1/3} \\ 1 & 0 & (4\tilde\alpha\tilde\gamma - 7\tilde\beta^2)(2\tilde\beta\psi_{16})^{-2/3} & 1 \end{pmatrix}.$$

($\mathrm{ii^{iia}_i}$) $\hat c_2 = 0 \Leftrightarrow \hat\alpha = 7\tilde\beta^2(4\tilde\gamma)^{-1}$. At $s_1 = \tilde\gamma|\tilde\beta|^{-3/2}\mu^{-1/2}$, $r_2 = -\tilde\beta|\tilde\beta|^{-3/2}\mu^{-1/2}$ having the substitution $L1^{5,2,<,<}_{22}$ with considering (3.1) and one obtains $NSF^{5,2,<,<}_{22}$ with $\sigma = \operatorname{sgn}\beta$, $u = -(\tilde\beta^2 + 4\tilde\gamma^2)(2\tilde\beta)^{-2} < -1/4$.

($\mathrm{ii^{iib}_i}$) $4\tilde\alpha\tilde\gamma - 7\tilde\beta^2 \neq 0$, then we have $NSF^{6,2,<,<}_{12}$ with $\sigma = \operatorname{sgn}\tilde\beta$, $u = (\tilde\beta^2 + 4\tilde\gamma^2)\psi_{16}^{-1}$, $v = \psi_{16}^{1/3}(2\tilde\beta)^{-2/3}$.

($\mathrm{ii_{ii}}$) $\hat c_2 = 0 \Leftrightarrow \psi_{11} = 0 \Leftrightarrow \nu = \theta_1\mu$, where $\theta_1 \in \mathbb{R}^1$ is $\forall$ zero of $\psi_{11}(\nu, 1)$. Then system (3.3) at $\nu = \theta_1\mu$ is $NSF^{6,2,<,<}_6$ with $\sigma = \operatorname{sgn}\psi_8(\cdot)$, $u = 2(2\psi_8(\cdot)\psi_9(\cdot))^{-1/3}\psi_{10}(\cdot)$, $v = -(2\psi_8(\cdot)\psi_{10}(\cdot))^{1/3}\psi_{12}(\cdot)^{-1/3}$, $(\cdot) = (\theta_1\mu,\mu)$, because $v \neq 1$ and $\psi_{10}(\cdot)$, $\psi_{12}(\cdot) \neq 0$.

Substituting $L1^{6,2,<,<}_6$ leading to $NSF^{6,2,<,<}_6$ is $L^{7,2,<,<}_2$ with $\nu = \theta_1\mu$.

($\mathrm{ii_{iii}}$) $\hat c_2 = 0 \Leftrightarrow \psi_{13} = 0 \Leftrightarrow \nu = \psi^{\pm}_{14}\tilde\gamma^{-1}\mu$. System (3.3) at $s_1 = 3^{3/4}\tilde\gamma(2\tilde\zeta)^{-3/2}\mu^{-1/2}$, $r_2 = \mp 3^{1/4}(2\tilde\zeta\mu)^{-1/2}$ with substituting $L1^{6,2,<,<}_7$ and taking into account (3.1) is as follows:

$$\pm\begin{pmatrix} \psi^{\mp}_{15}\tilde\zeta^{-1} & -3(\psi^{\pm 2}_{14} + \tilde\gamma^2)(2\tilde\zeta)^{-2} & 3(\psi^{\pm 2}_{14} + \tilde\gamma^2)(2\tilde\zeta)^{-2} & -((\tilde\zeta \mp \sqrt{3}\tilde\beta)^2 + 3\tilde\gamma^2)(2\tilde\zeta)^{-2} \\ 1 & 0 & 0 & 1 \end{pmatrix}.$$

($\mathrm{ii^i_{iii}}$) $\hat a_1 = 0 \Leftrightarrow \psi^{\mp}_{15} = 0$ $(\tilde\beta \neq 0) \Leftrightarrow \{\operatorname{sgn}\tilde\beta = \pm 1, \tilde\alpha = 7\tilde\beta^2(4\tilde\gamma)^{-1}\} \rightarrow$ returning to $\mathrm{ii^{iia}_i}$.

($\mathrm{ii^{ii}_{iii}}$) $\psi^{\mp}_{15} \neq 0 \rightarrow$ one obtains $NSF^{6,2,<,<}_7$ with $\sigma = \pm 1$, $u = \psi^{\mp}_{15}\tilde\zeta^{-1}$, $v = -3(\psi^{\pm 2}_{14} + \tilde\gamma^2)(2\tilde\zeta)^{-2}$.

($\mathrm{ii_{iv}}$) $\hat a_1$, $\hat c_1$, $\hat c_2 \neq 0 \Leftrightarrow \psi_{10}, \psi_{11}, \psi_{13} \neq 0$, then (3.3) is $NSF^{7,2,<,<}_2$ with $\sigma = \operatorname{sgn}\psi_8$, $u = 2(2\psi_8\psi_9)^{-1/3}\psi_{10}$, $w = -9\tilde\gamma^2(2\psi_8\psi_9)^{-2/3}(\nu^2 + \mu^2) < 0$, $v = (2\psi_8)^{2/3}\psi_9^{-1/3} > 0$.

List 1.1 represents 4 $NSF^{m,2,<}$, where D can be negative. Let us show that all they are not $CF^{m,2,<}$ (see [2, definition 1.10]).

Statement 3.1. $NSF^{6,2,<,<}_{12}$, $NSF^{6,2,<,<}_{13}$, $NSF^{6,2,<,<}_{15}$ at all permissible values of parameters substituting (2.2)[1] are reduced to the previous one in accordance with SP $SF^{6,2}_{11}$, and $NSF^{6,2,<,<}_{16}$ is reduced to $SF^{4,2}_{34}$.

Proof. The reasoning below is confirmed by symbolic calculations in [4, App. 3.6.4, p. 155].

(i) Any substitution of $(2.2)^1$ with $\hat{u} = -(\hat{v}s_1 + s_2)(s_1 - 2\hat{v}s_2)(\hat{v}^2(2\hat{v}s_1 - s_2)s_2)^{-1}$ and $r_1 = (s_1 - 2\hat{v}^2 s_2)(2\hat{v}s_1 - s_2)^{-1} r_2$ reduces $NSF_{12}^{6,2,<,<}(\hat{\sigma}, \hat{u}, \hat{v})$, namely, to $SF_{11}^{6,2}$ at $(\hat{u}, \hat{v}) \in ps_{12}^{6,2,<,<} = \{\hat{v} > 4^{-1/3}, \hat{v} \neq 1, 4\hat{u} > \hat{v}^{-3}\}$, herewith $\delta_{rs} = 0$.

The equality for $\hat{u}$ is the quadratic equation relative to s_1 with the roots $s_1^{\pm} = (2\hat{v}^3(1 - \hat{u}) - 1 \pm \psi_5^{1/2})(2\hat{v})^{-1} s_2 \in \mathbb{R}^1$, because the discriminant $\psi_5(\hat{u}) = 4\hat{v}^6\hat{u}^2 - 8\hat{v}^3(\hat{v}^3 - 1)\hat{u} + (2\hat{v}^3 + 1)^2$ is negative. Moreover, $\psi_6 = \hat{v}^3(1 - \hat{u}) - 1 + \psi_5^{1/2} \neq 0$ ($\Leftrightarrow \delta_{pq} \neq 0$).

Finally, the substitution, when $r_1 = (2\hat{v}^3(1 + \hat{u}) + 1 - \psi_5^{1/2})(2\hat{v}^3(1 - \hat{u}) - 2 - \psi_5^{1/2})(6\hat{v}(4\hat{v}^3 - 1))^{-1} r_2$, $s_1 = (2\hat{v}^3(1 - \hat{u}) - 1 + \psi_5^{1/2})(2\hat{v})^{-1} s_2$ reduces $NSF_{12}^{6,2,<,<}$ to $SF_{11}^{6,2} = \begin{pmatrix} \hat{a}_1^* r_2^2 & \hat{b}_1^* r_2 s_2 & \hat{c}_1^* s_2^2 & \hat{d}_1^* r_1^{-1} s_2^3 \\ \hat{a}_2^* r_2^3 s_2^{-1} & 0 & \hat{c}_2^* r_2 s_2 & 0 \end{pmatrix}$. Now at $r_2 = (\hat{a}_2^* \hat{c}_2^*)^{-1/4}$, $s_2 = (\hat{a}_2^* \hat{c}_2^*)^{1/4} / \hat{c}_2^*$ by substituting with $L\hat{1}_{11,+1}^{6,2,<,<}$, we obtain $NSF_{11,+1}^{6,2,<,<}$ with $\sigma = \hat{\sigma}$, $u = \hat{a}_1^*(\hat{a}_2^* \hat{c}_2^*)^{1/2}$, $v = \hat{b}_1^* / \hat{c}_2^*$.

It is worth mentioning that implementation of cumbersome and thus hard-to-verify relationships $\hat{a}_2^* \hat{c}_2^* > 0$, $\hat{c}_1^* = \hat{a}_1^* \hat{c}_2^* / \hat{a}_2^*$, $\hat{d}_1^* = \hat{b}_1^* \hat{c}_2^* / \hat{a}_2^*$ is however expected due to invariance of the degree of the common factor l and signs of discriminants D_0, D ($l = 2$, D_0, $D < 0$).

(ii) Any substitution of $(2.2)^1$ with $s_1 = (v(u + v + 1) + \varrho^{1/2})(2v - 4u)^{-1} s_2$, $r_2 = (3uv + v + \varrho^{1/2})(8u - 3v + 9uv - 3v^2 - \varrho^{1/2})(2uv(3v + 4)(3v + 2)(2u - v))^{-1} r_1$, where $\varrho(u) = 9v^2u^2 - 2v(9v^2 + 15v + 8)u + 9v^2(1 + v)^2$, reduces $NSF_{13}^{6,2,<,<}$ to $SF_{11}^{6,2}$. Herewith, $\varrho(u) > 0$, because $ps_{13}^{6,2,<,<} = \{v < -4/3, 4u > (u + v + 1)^2\}$, and thus the discriminant $27v^3 + 72v^2 + 60v + 16$ of the polynomial ϱ with zeros $-4/3, -2/3, -2/3$ is negative.

(iii) Any substitution of $(2.2)^1$ with $s_1 = (v - v^2 - 2u + \varrho^{1/2})(2v)^{-1} s_2$, $r_2 = (2u - v - \varrho^{1/2})(2u + 3v - 3v^2 + \varrho^{1/2})(2uv^2(3v - 4))^{-1} r_1$, where $\varrho(u) = 4u^2 - 4uv + 9v^2(v - 1)^2$ reduces $NSF_{15}^{6,2,<,<}$ to $SF_{11}^{6,2}$. Herewith, $\varrho(u) > 0$, because $ps_{15}^{6,2,<,<} = \{v \notin [0, 4/3], 4u < -(v - 1)^2\}$, and, therefore, the discriminant $-(9v^2 - 18v + 8)$ of the polynomial ϱ at $v \notin [2/3, 4/3]$ is negative.

(iv) Any substitution $(2.2)^1$ with $s_1 = -(u + v + ((u + v)^2 - u)^{1/2}) s_2$, $r_1 = -(u + v - ((u + v)^2 - u)^{1/2}) r_2$ reduces $NSF_{16}^{6,2,<,<}$ to $SF_{34}^{4,2}$, because $ps_{16}^{6,2,<,<} = \{v > 1/4, u < 0\}$. □

Consider $NSF^{m,2,<,<}$ from List 1.1. A direct verification reveals that $NSF_{22}^{5,2,<,<}$ is $CF_{22}^{5,2,<,<}$, and for other forms $ps^{m,2,<,<} \neq cs^{m,2,<,<}$.

Statement 3.2. The following forms from List 1.1 with the specified values of parameters are reduced to the previous ones in accordance with SP structural forms:

1) $NSF_6^{6,2,<,<}(\tilde{\sigma}, \tilde{u}, \tilde{v})$ with $ps_6^{6,2,<,<} = \{\tilde{v} \in (0, 1), \tilde{u} \in (\psi_7^-(\tilde{v}), \psi_7^+(\tilde{v}))\}$:

(a) at $\tilde{u} = -2^{-5/3}(3 \pm \sqrt{5})$, $\tilde{v} = 2^{-2/3}$ with substitution $L5_{22}^{5,2,<,<}$ is reduced to $CF_{22}^{5,2,<,<}$ with $\sigma = \mp\tilde{\sigma}$, $u = -3$;

(b) at $\tilde{u} = -\tilde{v}$ with substitution $L3_{34,+1}^{4,2,<,<}$ is reduced to $CF_{34,+1}^{4,2,<,<}$ with $\sigma = \tilde{\sigma}$, $u = \psi_{19}(\tilde{v})$;

2) $NSF_7^{6,2,<,<}(\tilde{\sigma}, \tilde{u}, \tilde{v})$ with $ps_7^{6,2,<,<} = \{\tilde{v} \neq -\tilde{u}, 4\tilde{v} < -(\tilde{u} + 1)^2\}$:

(a) at $\tilde{u} = -1$ with substitution $L2_{34,+1}^{4,2,<,<}$ is reduced to $CF_{34,+1}^{4,2,<,<}$ with $\sigma = \tilde{\sigma}$, $u = \psi_{21}(\tilde{v})$;

(b) at $\tilde{v} = [-3(\tilde{u} + 1), \ \tilde{u} \in (-1, 1 1) \vee 3(\tilde{u} + 1)(\tilde{u} + 2)^{-1}, \ \tilde{u} \in (-2, -1)]$ with substitution $[L3_{22}^{5,2,<,<} \vee L4_{22}^{5,2,<,<}]$ is reduced to $CF_{22}^{5,2,<,<}$ with $\sigma = [\tilde{\sigma} \vee -\tilde{\sigma}]$, $u = [-3(\tilde{u} + 1)^{-1} \vee 3((\tilde{u} + 1)(\tilde{u} + 2))^{-1}]$;

(c) at $\tilde{v} = \psi_{32}(\tilde{u})$, $\tilde{u} \in I_*$, with substitution $L3_6^{6,2,<,<}$ is reduced to $NSF_6^{6,2,<,<}$ with $\sigma = \tilde{\sigma}\,\mathrm{sgn}(1 - 2\tilde{u})$, $u = -(\tilde{u}^3 - 3\tilde{u}^2 + 6\tilde{u} + 1)(\tilde{u}^2 - \tilde{u} + 1)^{-1}((\tilde{u}^2 - \tilde{u} + 7)(2\tilde{u} - 1))^{-1/3}$, $v = (2\tilde{u} - 1)^{2/3}(\tilde{u}^2 - \tilde{u} + 7)^{-1/3}$;

3) $NSF_{11,+1}^{6,2,<,<}(\tilde{\sigma},\tilde{u},\tilde{v})$ with $ps_{11,+1}^{6,2,<,<} = \{4\tilde{v} < -\tilde{u}^2\}$:

(a) at $\tilde{v} = \psi_{20}^{\mp}$, $\sqrt{3}\tilde{u} \in I_1^{\mp}$ with substitution $L2_{22}^{5,2,<,<}$ is reduced to $CF_{22}^{5,2,<,<}$ with $\sigma = \pm\tilde{\sigma}$, $u = \psi_{20}^{\mp}\tilde{u}^{-2}$;

(b) at $\tilde{v} = 3(\tilde{u}^2+5)(\tilde{u}^2+1)(2(\tilde{u}^2-3))^{-1}$, $\tilde{u} \in (-\sqrt{3},\sqrt{3})$ with substitution $L2_6^{6,2,<,<}$ is reduced to $NSF_6^{6,2,<,<}$ with $\sigma = -\tilde{\sigma}\operatorname{sgn}\tilde{u}$, $u = -(\tilde{u}^2+5)\tilde{u}^{2/3}(2(\tilde{u}^2+1)(\tilde{u}^2+9))^{-1/3}$, $v = (2\tilde{u})^{2/3}(\tilde{u}^2+9)^{-1/3}$;

4) $NSF_2^{7,2,<,<}(\tilde{\sigma},\tilde{u},\tilde{v},\tilde{w})$ with $ps_2^{7,2,<,<} = \{\tilde{v} \in (0,1) \cup (1,\sqrt[3]{4}),\ \tilde{w} \neq -\tilde{u}(\tilde{v}-\tilde{v}^{-2}),\ 4\tilde{w} < -(\tilde{u}+\tilde{v})^2\}$:

(a) at $\tilde{u} = -\tilde{v}$ with substitution $L4_{34,+1}^{4,2,<,<}$ to $CF_{34,+1}^{4,2,<,<}$ with $\sigma = \tilde{\sigma}$, $u = \tilde{b}_1^*(-\tilde{v},\tilde{v},\tilde{w})/\tilde{c}_2^*(-\tilde{v},\tilde{v},\tilde{w})$;

(b[1]) at $\tilde{v} = [7^{-1/3} \vee \theta_2(\tilde{u})]$, $\tilde{w} = [3(7^{-1/3}\tilde{u} + 7^{-2/3}) \vee (\tilde{u} + \theta_2(\tilde{u}))(\theta_2(\tilde{u}) - 2\tilde{u})]$, $\tilde{u} \in [I_3 \vee I_4]$ with substitution $[L6_{22}^{5,2,<,<} \vee L7_{22}^{5,2,<,<}]$ is reduced to $CF_{22}^{5,2,<,<}$ with $\sigma = [-\tilde{\sigma} \vee \operatorname{sgn}\tilde{u}]$, $u = [3(7^{1/3}\tilde{u}+1)^{-1} \vee (2\tilde{u} - \theta_2(\tilde{u}))(\tilde{u}+\theta_2(\tilde{u}))^{-1}]$;

(b[2]) at $\tilde{u} = [-4\tilde{v} \vee \psi_{25}^{\mp}]$, $\tilde{w} = [3\tilde{v}(4\tilde{v} + \psi_{25}^{-})/2 \vee 3(\tilde{v}\psi_{25}^{\pm} - 2\tilde{v}^{-1})]$, $\tilde{v} \in [0,(2/7)^{2/3} \vee I_2^{\pm}]$ with substitution $[L8_{22}^{5,2,<,<} \vee L9_{22}^{5,2,<,<}]$ is reduced to $CF_{22}^{5,2,<,<}$ with $\sigma = [-\tilde{\sigma} \vee \mp\tilde{\sigma}]$, $u = [(4 + \psi_{25}^{-}\tilde{v}^{-1}/6 \vee (4 - \tilde{v}^3)(\tilde{v}^2\psi_{25}^{\pm} - 2)^{-1}]$;

(c) at $\tilde{w} = \psi_{33}(\tilde{u},\tilde{v})$ with substitution $L4_6^{6,2,<,<}$ is reduced to $NSF_6^{6,2,<,<}$ with $\sigma = \tilde{\sigma}\operatorname{sgn}((2\tilde{u}-\tilde{v})\psi_{24})$, $u = (\tilde{v}(2\tilde{u}-\tilde{v})^{-1}\psi_{26}^{-1})^{1/3}\psi_{34}$, $v = -(\tilde{v}(2\tilde{u}-\tilde{v})^2\psi_{26}^{-1})^{1/3}$;

(d) at $[\tilde{u} = \psi_{25}^{\mp} \vee \tilde{v} = \theta_2(\tilde{u}) \neq 2^{2/3}]$ with substitution $[L2_7^{6,2,<,<} \vee L3_7^{6,2,<,<}]$ is reduced to $NSF_7^{6,2,<,<}$ with $\sigma = [\mp\tilde{\sigma} \vee \tilde{\sigma}\operatorname{sgn}(\tilde{u} - 2^{-1/3})]$, $u = [-(\tilde{v}\tilde{w} - 3\tilde{v}^2\psi_{25}^{\pm} + 6)(\tilde{v}\tilde{w})^{-1} \vee -(\tilde{w} + 2\tilde{u}^2 + \theta_2(\tilde{u})\tilde{u} - \theta_2^2(\tilde{u}))\tilde{w}^{-1}]$, $v = [3(4-\tilde{v}^3)(\tilde{v}\tilde{w})^{-1} \vee -(\psi_{24}(\tilde{u},\theta_2(\tilde{u}))\tilde{w} + \psi_{35})(\psi_{29}(\tilde{u},\theta_2(\tilde{u}))\tilde{w})^{-1}]$;

(e[1]) at $\psi_{27} \geq 0$, $\tilde{u} \neq \tilde{v}/2$ with substitution $L2_{11,+1}^{6,2,<,<}$ is reduced to $NSF_{11,+1}^{6,2,<,<}$ with $\sigma = \tilde{\sigma}$, $u = \tilde{a}_1^*(\tilde{a}_2^*\tilde{c}_2^*)^{-1/2}$, $\tilde{v} = \tilde{b}_1^*/\tilde{c}_2^*$;

(e[2]) at $[\tilde{u} = \tilde{v}/2 \vee \tilde{v} = \theta_3(\tilde{u})]$ with substitution $[L3_{11,+1}^{6,2,<,<} \vee L4_{11,+1}^{6,2,<,<}]$ is reduced to $NSF_{11,+1}^{6,2,<,<}$ with $\sigma = \tilde{\sigma}$, $u = [-3(4\tilde{v} - \tilde{v}^4)^{1/2}(4\tilde{w})^{-1} \vee -\psi_{30}^{1/2}\psi_{31}]$, $v = [(4-\tilde{v}^3)(4\tilde{v}\tilde{w})^{-1} \vee \psi_{36}]$.

Proof. The reasoning below in pp. 1)–3) are subjected to symbolic calculations in [4, App. 3.6.5, p. 161], and in p. 4) are in [4, App. 3.6.6, p. 172].

1) $NSF_6^{6,2,<,<}(\tilde{\sigma},\tilde{u},\tilde{v})$ in case of (a) with substitution with $r_1 = -2^{1/3}r_2$, $s_1 = 2^{-2/3}(3 \pm \sqrt{5})s_2$ is reduced to $SF_{22}^{5,2}$ in (b) with substitution with $r_1 = -\tilde{v}^{-1/2}r_2$, $s_2 = \tilde{v}^{1/2}s_1$ is reduced to $SF_{34}^{4,2}$;

2) $NSF_7^{6,2,<,<}(\tilde{\sigma},\tilde{u},\tilde{v})$ in (a) with substitution with $r_1 = (\tilde{v} \pm \psi_{17}^{1/2})r_2/3$, $s_1 = (\tilde{v} \mp \psi_{17}^{1/2})s_2/3$ is reduced to $SF_{34}^{4,2}$, in (b) with substitution $r_1 = [(\tilde{u}+2)\tilde{u}^{-1}r_2 \vee -(\tilde{u}+2)^{-1}r_2]$, $s_1 = [-\tilde{u}s_2/2 \vee (\tilde{u}+3)s_2]$ is reduced to $SF_{22}^{5,2}$, in (c) with substitution $r_1 = -(\tilde{u}^2 - 2\tilde{u} + 3)(2\tilde{u}-1)^{-1}r_2$, $s_1 = \tilde{u}s_2$ is reduced to $SF_6^{6,2}$;

3) $NSF_{11,+1}^{6,2,<}(\tilde{\sigma},\tilde{u},\tilde{v})$ in (a) with substitution $s_1 = -(2\tilde{u} \mp \sqrt{3})s_2$, $r_2 = -(2 \mp \sqrt{3}\tilde{u})\tilde{u}^{-1}r_1$ is reduced to $SF_{22}^{5,2}$, in (b) with substitution $r_1 = -(\tilde{u}^2+3)(2\tilde{u})^{-1}r_2$, $s_1 = \tilde{u}s_2$ is reduced to $SF_6^{6,2}$.

4) Let us now obtain the previous forms from

$$NSF_2^{7,2,<,<} = \tilde{\sigma}\begin{pmatrix} \tilde{u} & \tilde{w} & \tilde{u}\tilde{v}^{-1} - \tilde{v}(\tilde{u}\tilde{v} + \tilde{w}) & \tilde{u} + \tilde{w}\tilde{v}^{-1} \\ 1 & 0 & \tilde{v}^{-1} - \tilde{v}^2 & 1 \end{pmatrix} \quad (\tilde{u} + \tilde{w}\tilde{v}^{-1} \neq 0).$$

(a) At $\tilde{u} = -\tilde{v}$, $\tilde{w} = (3\tilde{v}^2s_1^2 + 2(\tilde{v}^3-1)s_1s_2 - \tilde{v}(\tilde{v}^3+2)s_2^2)(\tilde{v}s_2(2s_1 - \tilde{v}s_2))^{-1}$, $NSF_2^{7,2,<,<}$ at any substitution of (2.2)[1] with $r_1 = (\tilde{v}^2s_1 - 2s_2)(\tilde{v}(2s_1 - \tilde{v}s_2))^{-1}r_2$ is reduced to $SF_{34}^{4,2}$. The equality for $\tilde{w}$ is the quadratic equation with respect to s_1 and has the real roots $s_1^{\pm} = (\tilde{v}\tilde{w} - \tilde{v}^3 + 1 \pm \psi_{22}^{1/2})\tilde{v}^{-2}s_2/3$, because $\psi_{22} = (\tilde{v}\tilde{w}+1)^2 + \tilde{v}^3(4\tilde{v}^3 - 5\tilde{v}\tilde{w} + 4) > 0$ owing to $\tilde{w} < 0$. Moreover, $\psi_{23} \neq 0 \Leftrightarrow 2s_1 - \tilde{v}s_2 \neq 0$.

Substituting with $r_1 = \tilde{v}(\tilde{w}\tilde{v} - \tilde{v}^3 - 5 - \psi_{22}^{1/2})\psi_{23}^{-1}r_2$, $s_1 = (\tilde{w}\tilde{v} - \tilde{v}^3 + 1 - \psi_{22}^{1/2})\tilde{v}^{-2}s_2/3$ reduces $NSF_2^{7,2,<,<}$ to $SF_{34}^{4,2} = \begin{pmatrix} 0 & \tilde{b}_1^*(-\tilde{v},\tilde{v},\tilde{w})r_2s_2 & 0 & \tilde{d}_1^* r_1^{-1}s_2^3 \\ \tilde{a}_2^*(-\tilde{v},\tilde{v},\tilde{w})r_2^3s_2^{-1} & 0 & \tilde{c}_2^*(-\tilde{v},\tilde{v},\tilde{w})r_2s_2 & 0 \end{pmatrix}$.

A choice of normalization and related issues are given in Statement 3.1, p. i.

In (b[1]) $NSF_2^{7,2,<,<}$ at substitution with $r_1 = [-7^{-1/3}r_2 \vee \tilde{u}r_2]$, $[s_2 = 0 \vee s_1 = (\theta_2(\tilde{u}) - \tilde{u})s_2]$ is reduced to $SF_{22}^{5,2}$.

In (b[2]) $NSF_2^{7,2,<,<}$ at substitution with $r_1 = [\tilde{v}r_2/2 \vee \psi_{25}^{\mp}r_2]$, $s_1 = [\psi_{25}^{-}s_2 \vee \psi_{25}^{\pm}s_2]$ is reduced to $SF_{22}^{5,2}$; the case $\tilde{u} = -4\tilde{v}$, $\tilde{w} = 3\tilde{v}(4\tilde{v} + \psi_{25}^{+})/2$ is impossible because of $D > 0$ therein.

In (c) $NSF_2^{7,2,<,<}$ at substitution with $r_1 = -(2\tilde{u}\tilde{v}^2 - \tilde{u}^2\tilde{v} - 3)(\tilde{v}(\tilde{v} - 2\tilde{u}))^{-1}r_2$, $s_1 = \tilde{u}s_2$ is reduced to $SF_6^{6,2}$. Herewith, $\psi_{26} \neq 0$ as normalization factor.

In (d) $NSF_2^{7,2,<,<}$ at substitution with $r_1 = [\psi_{25}^{\mp}r_2 \vee \tilde{u}r_2]$, $s_1 = [\psi_{25}^{\pm}s_2 \vee (\theta_2(\tilde{u}) - \tilde{u})s_2]$ is reduced to $SF_7^{6,2}$; if $2\tilde{u} = \theta_2(\tilde{u}) \Leftrightarrow \tilde{u} = 2^{-1/3}$, then $\delta_{rs} = 0$, and $\operatorname{sgn}(\tilde{u} - 2^{-1/3}) = \operatorname{sgn}((2\tilde{u} - \theta_2(\tilde{u}))\psi_{29}^{-1}(\tilde{u}, \theta_2(\tilde{u})))$.

(e[1]) At $\tilde{w} = (\tilde{v}(\tilde{v} - 2\tilde{u})s_1^2 + (\tilde{v}^3 - u\tilde{v}^2 - 2)s_1s_2 + \tilde{v}(\tilde{u}\tilde{v}^2 - 2)s_2^2)(\tilde{v}s_2(2s_1 - \tilde{v}s_2))^{-1}$ $NSF_2^{7,2,<,<}$ at any substitution of (2.2)[1] with $r_1 = (\tilde{v}^2s_1 - 2s_2)(\tilde{v}(2s_1 - \tilde{v}s_2))^{-1}r_2$ is reduced to $SF_{11}^{6,2}$. The equality for $\tilde{w}$ is the quadratic equation relative to s_1 and has the real roots $s_1^{\pm} = (2\tilde{v}\tilde{w} - \tilde{v}^3 + \tilde{u}\tilde{v}^2 + 2 \pm \psi_{27}^{1/2})(2\tilde{v}(\tilde{v} - 2\tilde{u}))^{-1}s_2$ at $\psi_{27} \geq 0$ and $\tilde{u} \neq \tilde{v}/2$. In addition, $\psi_{28} \neq 0 \Leftrightarrow 2s_1 - \tilde{v}s_2 \neq 0$. Finally, substituting with $r_1 = (2\tilde{v}^2\tilde{w} - \tilde{v}^4 + \tilde{u}\tilde{v}^3 - 2\tilde{v} + 8\tilde{u} - \tilde{v}\psi_{27}^{1/2})(2\psi_{28})^{-1}r_2$, $s_1 = (2\tilde{v}\tilde{w} - \tilde{v}^3 + \tilde{u}\tilde{v}^2 + 2 - \psi_{27}^{1/2})(2\tilde{v}(\tilde{v} - 2\tilde{u}))^{-1}s_2$ reduces $NSF_2^{7,2,<,<}$ to $SF_{11}^{6,2} = \begin{pmatrix} \tilde{a}_1^* r_2^2 & \tilde{b}_1^* r_2s_2 & \tilde{c}_1^* s_2^2 & \tilde{d}_1^* r_1^{-1}s_2^3 \\ \tilde{a}_2^* r_2^3 s_2^{-1} & 0 & \tilde{c}_2^* r_2s_2 & 0 \end{pmatrix}$, and then similarly with Statement 3.1, p. i.

In (e[2]) $NSF_2^{7,2,<,<}$ at substitution with $r_1 = [\tilde{v}r_2/2 \vee \tilde{u}r_2]$, $[s_2 = 0 \vee s_1 = (2\theta_3(\tilde{u}) - \tilde{u})s_2/3]$ is reduced to $SF_{11}^{6,2}$, and $\psi_{30} > 0$ as being normalization factor.

Finally, $NSF_2^{7,2,<,<}$ can be reduced to the previous $NSF_i^{6,2,<,<}$ $(i = \overline{12\text{–}16})$ from List 1.1, and those are, in turn, reduced to $SF_{11}^{6,2}$ or $SF_{34}^{4,2}$ (see Statement 3.1).

However, all direct replacements to $NSF_{11,+1}^{6,2,<,<}$ and $NSF_{34,+1}^{4,2,<,<}$ were already found above. □

The results for $l = 2$, $D_0 < 0$, $D < 0$ enable one to write out all canonical forms with the appropriate canonical sets.

List 3.1. Six $CF_i^{m,2,<,<}$ and their $cs_i^{m,2,<,<}$ ($\sigma = \pm 1$, u, v, $w \neq 0$).

$$CF_{34,+1}^{4,2,<,<} = \sigma\begin{pmatrix} 0 & u & 0 & u \\ 1 & 0 & +1 & 0 \end{pmatrix}, \quad CF_{22}^{5,2,<,<} = \sigma\begin{pmatrix} 0 & u & -u & u \\ 1 & 0 & 0 & 1 \end{pmatrix},$$

$$CF_6^{6,2,<,<} = \sigma\begin{pmatrix} u & u(v^{-2} - v) & 0 & uv^{-3} \\ 1 & 0 & v^{-1} - v^2 & 1 \end{pmatrix}, \quad CF_7^{6,2,<,<} = \sigma\begin{pmatrix} u & v & -v & u + v \\ 1 & 0 & 0 & 1 \end{pmatrix},$$

$$CF_{11,+1}^{6,2,<,<} == \sigma\begin{pmatrix} u & v & u & v \\ 1 & 0 & 1 & 0 \end{pmatrix}, \quad CF_2^{7,2,<,<} = \sigma\begin{pmatrix} u & w & uv^{-1} - v(uv + w) & u + wv^{-1} \\ 1 & 0 & v^{-1} - v^2 & 1 \end{pmatrix};$$

$$cs_{34,+1}^{4,2,<,<} = \{u < 0\}, \quad cs_{22}^{5,2,<,<} = \{u < -1/4\},$$

$$cs_6^{6,2,<,<} = \{v \in (0,1),\ u \in (\psi_7^-(v), \psi_7^+(v)),\ u \neq -v,\ (u,v) \neq (-2^{-5/3}(3 \pm \sqrt{5}),\ 2^{-2/3})\},$$

$$cs_7^{6,2,<,<} = \{4v < -(u+1)^2,\ u \neq -1,\ v \neq -u,\ -3(u+1),\ 3(u+1)(u+2)^{-1},\ \psi_{32}(u)\},$$

$$cs_{11,+1}^{6,2,<,<} = \{4v < -u^2,\ v \neq \psi_{20}^{\mp}(u),\ 3(u^2+5)(u^2+1)(2(u^2-3))^{-1}\},$$

$$cs_2^{7,2,<,<} = \{v \in (0, \sqrt[3]{4}),\ v \neq 1,\ -u,\ 2u,\ \theta_3(u),\ w \neq -uv,\ -u(v - v^{-2}),\ \psi_{33}(u,v),\ 4w < -(u+v)^2,$$
$$\psi_{27}(u,v,w) < 0,\ (u,w) \neq ([-4v \vee \psi_{25}^{\mp}(v)],\ [3v(4v + \psi_{25}^{-}(v))/2 \vee 3(v\psi_{25}^{\pm}(v) - 2v^{-1})]),$$
$$(v,w) \neq ([7^{-1/3} \vee \theta_2(u)],\ [3(7^{-1/3}u + 7^{-2/3}) \vee (u + \theta_2(u))(\theta_2(u) - 2u)])\}.$$

Theorem 3.1. Any system (2.1)[1] with $l = 2$, written as $(1.1^{<})$ according to (2.15)[1] with $D < 0$, is linearly equivalent to a system generated by some representative of the corresponding canonical form from List 3.1. For each $CF_i^{m,2,<,<}$ one has below: (a) conditions to the coefficients of system $(1.1^{<})$; (b) substitutions (2.2)[1] that transform the right-hand part of system $(1.1^{<})$ at the above conditions into the chosen form; (c) thus-obtained values of σ and parameters from $cs_i^{m,2,<,<}$:

A. $CF_{34,+1}^{4,3,<,<}$: (1_a) $\nu = 0,\ \tilde\beta = 0$, (1_b) J_3^2, $L1_{34,+1}^{4,2,<,<}$, (1_c) $\sigma = 1,\ u = \tilde\gamma^2\tilde\xi^{-2}$;

(2_a) $\nu = \theta_1\mu,\ \psi_8^2(\cdot)\psi_9(\cdot) = 2\psi_{10}^2(\cdot)\psi_{12}(\cdot)$, where $(\cdot) = (\theta_1\mu, \mu)$, (2_b) J_3^2, $L1_6^{6,2,<,<}$, $L3_{34,+1}^{4,2,<,<}$, (2_c) $\nu = \operatorname{sgn}\psi_8(\cdot)$, $u = \psi_{19}(\tilde v)$, where $\tilde v = -(2\psi_8(\cdot)\psi_{10}(\cdot))^{1/3}\psi_{12}^{-1/3}(\cdot)$;

(3_a) $\nu = \psi_{14}^{\pm}\tilde\gamma^{-1}\mu,\ \psi_{15}^{\mp} = -\tilde\xi$, (3_b) J_3^2, $L1_7^{6,2,<,<}$, $L2_{34,+1}^{4,2,<,<}$, (3_c) $\sigma = \pm 1,\ u = \psi_{21}(\tilde v)$, where $\tilde v = -3(\psi_{14}^{\pm^2} + \tilde\gamma^2)(2\tilde\xi)^{-2}$;

(4_a) $\nu = 0$, $\tilde\beta,\ 4\tilde\beta^2 - 3\tilde\alpha\tilde\gamma,\ 4\tilde\beta^2 - 3\tilde\alpha\tilde\gamma + 3\tilde\gamma^2 \neq 0$, (4_b) J_3^2, $L1_2^{7,2,<,<}$, $L4_{34,+1}^{4,2,<,<}$, (4_c) $\sigma = \operatorname{sgn}\tilde\beta$, $u = \tilde b_1^*(-\tilde v, \tilde v, \tilde w)/\tilde c_2^*(-\tilde v, \tilde v, \tilde w)$, where $\tilde v = (2\psi_8)^{2/3}\psi_9^{-1/3}$, $\tilde w = -9\tilde\gamma^2(2\psi_8\psi_9)^{-2/3}(\nu^2 + \mu^2)$;

B. $CF_{22}^{5,2,<,<}$: (1_a) $\nu = \tilde\beta(2\tilde\gamma)^{-1}\mu,\ \tilde\beta \neq 0,\ \tilde\alpha = 7\tilde\beta^2(4\tilde\gamma)^{-1}$, (1_b) J_3^2, $L1_{22}^{5,2,<,<}$, (1_c) $\sigma = \operatorname{sgn}\tilde\beta,\ u = -(\tilde\beta^2 + 4\tilde\gamma^2)(2\tilde\beta)^{-2}$;

(2_a) $\nu = \theta_1\mu,\ 2^{7/3}\psi_{10}(\cdot) = -(3 \pm \sqrt5)(\psi_8(\cdot)\psi_9(\cdot))^{1/3},\ \psi_{12}(\cdot) = -8\psi_8(\cdot)\psi_{10}(\cdot)$ $((\cdot) = (\theta_1\mu, \mu))$, (2_b) J_3^2, $L1_6^{6,2,<,<}$, $L5_{22}^{5,2,<,<}$, (2_c) $\sigma = \mp\operatorname{sgn}\psi_8(\cdot),\ u = -3$;

(3_a) $\nu = \psi_{14}^{\pm}\tilde\gamma^{-1}\mu,\ \psi_{15}^{\mp}\tilde\xi^{-1} = [-3(\tilde u + 1),\ \tilde u \in (-1, 11) \vee 3(\tilde u + 1)(\tilde u + 2)^{-1},\ \tilde u \in (-2, -1)]$, where $\tilde v = -3(\psi_{14}^{\pm^2} + \tilde\gamma^2)(2\tilde\xi)^{-2}$, (3_b) J_3^2, $L1_7^{6,2,<,<}$, $[L3_{22}^{5,2,<,<} \vee L4_{22}^{5,2,<,<}]$, (3_c) $\sigma = [\pm 1 \vee \mp 1]$, $u = [-3(\tilde u + 1)^{-1} \vee 3((\tilde u + 1)(\tilde u + 2))^{-1}]$;

(4_a) $\nu = -\tilde\beta\tilde\gamma^{-1}\mu,\ \tilde\beta \neq 0,\ -(\tilde\beta^2 + \tilde\gamma^2)\tilde\xi^{-2} = \psi_{20}^{\mp}(\tilde u),\ \tilde u = -2\tilde\beta\tilde\xi^{-1},\ \sqrt3\tilde u \in I_1^{\mp}$, (4_b) J_3^2, $L1_{11,+1}^{6,2,<,<}$, $L2_{22}^{5,2,<,<}$, (4_c) $\sigma = \pm 1,\ u = -2\tilde\beta\tilde\xi^{-1}$;

(5_a) $\nu = \tilde\beta(2\tilde\gamma)^{-1}\mu,\ \tilde\beta \neq 0,\ \tilde\alpha \neq 7\tilde\beta^2(4\tilde\gamma)^{-1},\ \hat b_1^*/\hat c_2^* = \psi_{20}^{\mp}(\tilde u),\ \sqrt3\tilde u \in I_1^{\mp},\ \tilde u = \hat a_1^*(\hat a_2^*\hat c_2^*)^{1/2},\ \hat u = \hat a_1^*(\hat a_2^*\hat c_2^*)^{1/2}$, $\hat v = \hat b_1^*/\hat c_2^*$, (5_b) J_3^2, $L1_{12}^{6,2,<,<}$, $L\hat1_{11,+1}^{6,2,<,<}$, $L2_{22}^{5,2,<,<}$, (5_c) $\sigma = \pm\operatorname{sgn}\tilde\beta,\ u = \psi_{20}^{\mp}(\tilde u)\tilde u^{-2}$;

(6_a) $\nu \neq -\tilde\beta\tilde\gamma^{-1}\mu,\ \theta_1\mu,\ \psi_{14}^{\pm}\tilde\gamma^{-1}\mu,\ \tilde\beta(2\tilde\gamma)^{-1}\mu,\ [7^{-1/3} \vee \theta_2(\tilde u)] = (2\psi_8)^{2/3}\psi_9^{-1/3},\ [3(7^{-1/3}\tilde u + 7^{-2/3}) \vee (\theta_2(\tilde u) - 2\tilde u)(\tilde u + \theta_2(\tilde u))] = -9\tilde\gamma^2(2\psi_8\psi_9)^{-2/3}(\nu^2 + \mu^2),\ \tilde u = 2(2\psi_8\psi_9)^{-1/3}\psi_{10} \in [I_3 \vee I_4]$, (6_b) J_3^2, $L1_2^{7,2,<,<}$, $[L6_{22}^{5,2,<,<} \vee L7_{22}^{5,2,<,<}]$, (6_c) $\sigma = [-\operatorname{sgn}\psi_8 \vee \operatorname{sgn}\tilde u]$, $u = [3(7^{1/3}\tilde u + 1)^{-1} \vee (2\tilde u - \theta_2(\tilde u))(\tilde u + \theta_2(\tilde u))^{-1}]$;

(7_a) $\nu \neq -\tilde\beta\tilde\gamma^{-1}\mu,\ \theta_1\mu,\ \psi_{14}^{\pm}\tilde\gamma^{-1}\mu,\ \tilde\beta(2\tilde\gamma)^{-1}\mu,\ [-4\tilde v \vee \psi_{25}^{\mp}] = 2(2\psi_8\psi_9)^{-1/3}\psi_{10}$, $[3\tilde v(4\tilde v + \psi_{25}^{-})/2 \vee 3(\tilde v\psi_{25}^{\pm} - 2\tilde v^{-1})] = -9\tilde\gamma^2(2\psi_8\psi_9)^{-2/3}(\nu^2 + \mu^2),\ \tilde v = (2\psi_8)^{2/3}\psi_9^{-1/3} \in [(0, (2/7)^{2/3}) \vee I_2^{\pm}]$, (7_b) J_3^2, $L1_2^{7,2,<,<}$, $[L8_{22}^{5,2,<,<} \vee L9_{22}^{5,2,<,<}]$, (7_c) $\sigma = [-\operatorname{sgn}\psi_8 \vee \mp\operatorname{sgn}\psi_8]$, $u = [(4 + \psi_{25}^{-}\tilde v^{-1})/6 \vee (4 - \tilde v^3)(\tilde v^2\psi_{25}^{\pm} - 2)^{-1}]$;

C. $CF_6^{6,2,<,<}$: (1_a) $\nu = \theta_1\mu,\ \neg(A2_a, B2_a)$, (1_b) J_3^2, $L1_6^{6,2,<,<}$, (1_c) $\sigma = \operatorname{sgn}\psi_8(\cdot),\ u = 2(2\psi_8(\cdot)\psi_9(\cdot))^{-1/3}\psi_{10}(\cdot)$, $v = -(2\psi_8(\cdot)\psi_{10}(\cdot))^{1/3}\psi_{12}^{-1/3}(\cdot),\ (\cdot) = (\theta_1\mu, \mu)$;

(2_a) $\nu = \psi_{14}^{\pm}\tilde\gamma^{-1}\mu$, $-3(\psi_{14}^{\pm 2} + \tilde\gamma^2)(2\tilde\xi)^{-2} = \psi_{32}(\tilde u)$, $\tilde u = \psi_{15}^{\mp}\tilde\xi^{-1} \in I_*$, $\neg(B3_a)$, (2_b) J_3^2, $L1_7^{6,2,<,<}$, $L3_6^{6,2,<,<}$, (2_c) $\sigma = \pm\operatorname{sgn}(1-2\tilde u)$, $u = -(\tilde u^3 - 3\tilde u^2 + 6\tilde u + 1)(\tilde u^2 - \tilde u + 1)^{-1}((\tilde u^2 - \tilde u + 7)(2\tilde u - 1))^{-1/3}$, $v = (2\tilde u - 1)^{2/3}(\tilde u^2 - \tilde u + 7)^{-1/3}$;

(3_a) $\nu = -\tilde\beta\tilde\gamma^{-1}\mu$, $\tilde\beta \ne 0$, $-2(\tilde\beta^2 + \tilde\gamma^2)\tilde\xi^{-2} = 3(\tilde u + 5)(\tilde u^2 + 1)(\tilde u^2 - 3)^{-1}$, $\tilde u = -2\tilde\beta\tilde\xi^{-1} \in (-\sqrt3, \sqrt3)$, $\neg(B4_a)$, (3_b) J_3^2, $L1_{11,+1}^{6,2,<,<}$, $L2_6^{6,2,<,<}$, (3_c) $\sigma = -\operatorname{sgn}\tilde u$, $2u = -(\tilde u^2 + 5)(\tilde u^2 + 1)^{-1/3}v$, $v = (2\tilde u)^{2/3}(u^2 + 9)^{-1/3}$;

(4_a) $\nu = \tilde\beta(2\tilde\gamma)^{-1}\mu$, $\tilde\beta \ne 0$, $\tilde\alpha \ne 7\tilde\beta^2(4\tilde\gamma)^{-1}$, $\hat b_1^*/\hat c_2^* = 3(\tilde u^2 + 5)(\tilde u^2 + 1)(2(\tilde u^2 - 3))^{-1}$, $\tilde u = \hat a_1^*(\hat a_2^*\hat c_2^*)^{1/2} \in (-\sqrt3, \sqrt3)$, $\hat u = (\tilde\beta^2 + 4\tilde\gamma^2)\psi_{16}^{-1}$, $\hat v = \psi_{16}^{1/3}(2\tilde\beta)^{-2/3}$, $\neg(B5_a)$, (4_b), J_3^2, $L1_{12}^{6,2,<,<}$, $L\hat 1_{11,+1}^{6,2,<,<}$, $L2_6^{6,2,<,<}$, (4_c) $\sigma = -\operatorname{sgn}(\tilde\beta\tilde u)$, $2u = -(\tilde u^2 + 5)(\tilde u^2 + 1)^{-1/3}v$, $v = (2\tilde u)^{2/3}(\tilde u^2 + 9)^{-1/3}$;

(5_a) $\nu \ne -\tilde\beta\tilde\gamma^{-1}\mu, \theta_1\mu, \psi_{14}^{\pm}\tilde\gamma^{-1}\mu, \tilde\beta(2\tilde\gamma)^{-1}\mu$, $-9\tilde\gamma^2(2\psi_8\psi_9)^{-2/3}(\nu^2 + \mu^2) = \psi_{33}(\tilde u, \tilde v)$, $\tilde u = 2(2\psi_8\psi_9)^{-1/3}\psi_{10}$, $\tilde v = (2\psi_8)^{2/3}\psi_9^{-1/3}$, $\neg(A4_a, B6_a, B7_a)$, (5_b) J_3^2, $L1_2^{7,2,<,<}$, $L4_6^{6,2,<,<}$, (5_c) $\sigma = \operatorname{sgn}((2\tilde u - \tilde v)\psi_8\psi_{24})$, $u = (\tilde v(2\tilde u - \tilde v)^{-1}\psi_{26}^{-1})^{1/3}\psi_{34}$, $v = -(\tilde v(2\tilde u - \tilde v)^2\psi_{26}^{-1})^{1/3}$;

D. $CF_7^{6,2,<,<}$: (1_a) $\nu = \psi_{14}^{\pm}\tilde\gamma^{-1}\mu$, $\psi_{15}^{\mp} \ne 0, -\tilde\xi$, $\neg(B3_a, C2_a)$, (1_b) J_3^2, $L1_7^{6,2,<,<}$, (1_c) $\sigma = \pm 1$, $u = \psi_{15}^{\mp}\tilde\xi^{-1}$, $v = -3(\psi_{14}^{\pm 2} + \tilde\gamma^2)(2\tilde\xi)^{-2}$;

(2_a) $\nu \ne -\tilde\beta\tilde\gamma^{-1}\mu$, $\theta_1\mu, \psi_{14}^{\pm}\tilde\gamma^{-1}\mu$, $\tilde\beta(2\tilde\gamma)^{-1}\mu$, $[\tilde u = \psi_{25}^{\mp} \vee \tilde v = \theta_2(\tilde u) \ne 2^{2/3}]$, $\tilde u = 2(2\psi_8\psi_9)^{-1/3}\psi_{10}$, $\tilde v = (2\psi_8)^{2/3}\psi_9^{-1/3}$, $\tilde w = -9\tilde\gamma^2(2\psi_8\psi_9)^{-2/3}(\nu^2 + \mu^2)$, $\neg(A4_a, B6_a, B7_a, C5_a)$, (2_b) J_3^2, $L1_2^{7,2,<,<}$, $[L2_7^{6,2,<,<} \vee L3_7^{6,2,<,<}]$, (2_c) $\sigma = [\mp\operatorname{sgn}\psi_8 \vee \operatorname{sgn}((\tilde u - 2^{-1/3})\psi_8)]$, $u = [-(\tilde v\tilde w - 3\tilde v^2\psi_{25}^{\pm} + 6)(\tilde v\tilde w)^{-1} \vee -(\tilde w + 2\tilde u^2 + \theta_2(\tilde u)\tilde u - \theta_2^2(\tilde u))\tilde w^{-1}]$, $v = [3(4 - \tilde v^3)(\tilde v\tilde w)^{-1} \vee (\psi_{24}(\tilde u, \theta_2(\tilde u))\tilde w + \psi_{35}]$;

E. $CF_{11,+1}^{6,2,<,<}$: (1_a) $\nu = -\tilde\beta\tilde\gamma^{-1}\mu$, $\tilde\beta \ne 0$, $\neg(B4_a, C3_a)$, (1_b) J_3^2, $L1_{11,+1}^{6,2,<,<}$, (1_c) $\sigma = 1$, $u = -2\tilde\beta\tilde\xi^{-1}$, $v = -(\tilde\beta^2 + \tilde\gamma^2)\tilde\xi^{-2}$;

(2_a) $\nu = \tilde\beta(2\tilde\gamma)^{-1}\mu$, $\tilde\beta \ne 0$, $\alpha \ne 7\tilde\beta^2(4\tilde\gamma)^{-1}$, $\neg(B5_a, C4_a)$, (2_b) J_3^2, $L1_{12}^{6,2,<,<}$, $L\hat 1_{11,+1}^{6,2,<,<}$, (2_c) $\sigma = \operatorname{sgn}\tilde\beta$, $u = \hat a_1^*(\hat a_2^*\hat c_2^*)^{1/2}$, $v = \hat b_1^*/\hat c_2^*$, where $\hat u = (\tilde\beta^2 + 4\tilde\gamma^2)\psi_{16}^{-1}$, $\hat v = \psi_{16}^{1/3}(2\tilde\beta)^{-2/3}$;

(3_a) $\nu \ne -\tilde\beta\tilde\gamma^{-1}\mu$, $\theta_1\mu$, $\psi_{14}^{\pm}\tilde\gamma^{-1}\mu$, $\tilde\beta(2\tilde\gamma)^{-1}\mu$, $[\tilde u \ne \tilde v/2,\ \psi_{27} \ge 0 \vee \tilde u = \tilde v/2 \vee \tilde v = \theta_3(\tilde u)]$, where $\tilde u = 2(2\psi_8\psi_9)^{-1/3}\psi_{10}$, $\tilde v = (2\psi_8)^{2/3}\psi_9^{-1/3}$, $\tilde w = -9\tilde\gamma^2(2\psi_8\psi_9)^{-2/3}(\nu^2 + \mu^2)$, $\neg(A4_a, B6_a, B7_a, C5_a, D2_a)$, (3_b) J_3^2, $L1_2^{7,2,<,<}$, $[L2_{11,+1}^{6,2,<,<} \vee L3_{11,+1}^{6,2,<,<} \vee L4_{11,+1}^{6,2,<,<}]$, (3_c) $\sigma = \operatorname{sgn}\psi_8$, $u = [\tilde a_1^*(\tilde a_2^*\tilde c_2^*)^{-1/2} \vee -3(4\tilde v - \tilde v^4)^{1/2}(4\tilde w)^{-1} \vee -\psi_{30}^{1/2}\psi_{31}]$, $v = [\tilde b_1^*/\tilde c_2^* \vee (4 - \tilde v^3)(4\tilde v\tilde w)^{-1} \vee \psi_{36}]$;

F. $CF_2^{7,2,<,<}$: (a) $\nu \ne -\tilde\beta\tilde\gamma^{-1}\mu, \theta_1\mu, \psi_{14}^{\pm}\tilde\gamma^{-1}\mu, \tilde\beta(2\tilde\gamma)^{-1}\mu$, $\neg(A4_a, B6_a, B7_a, C5_a, D2_a, E3_a)$, (b) J_3^2, $L1_2^{7,2,<,<}$, (c) $\sigma = \operatorname{sgn}\psi_8$, $u = 2(2\psi_8\psi_9)^{-1/3}\psi_{10}$, $v = (2\psi_8)^{2/3}\psi_9^{-1/3}$, $w = -9\tilde\gamma^2(\nu^2 + \mu^2)(2\psi_8\psi_9)^{-2/3}$.

Here, the notation $\neg(\ldots)$ means that none of conditions in parentheses are implemented.

The proof of the theorem follows from Lemma 3.1 and Statements 3.1 and 3.2.

4. Extraction of $mcs^{m,2,<}$. Let us now demonstrate the linear nonsingular replacements, which allow distinction of minimum canonical sets for CF from List 1.1 (see [2, Definition 1.11]).

Statement 4.1. The values of parameters in $cs^{m,2,<}$ can be limited only for the following $CF^{m,2,<}$ from Lists 2.1 and 3.1:

1) in $CF_{8,+1}^{4,2,<}$ at $\tilde\sigma = \sigma$, $\tilde u = u$ substitution with $r_1, s_2 = 0$, $s_1, r_2 = |\tilde u|^{-1/2}$ gives $\sigma = \tilde\sigma\operatorname{sgn}\tilde u$, $u = \tilde u^{-1}$;

2) in $CF_{34,+1}^{4,2,<}$ normalization $(2.6)^1$ with $r_1, -s_2 = 1$ changes sign σ; at $\tilde u = u$substitution with $s_1, r_2 = |\tilde u|^{-1/2}$, $r_1, s_2 = 0$ gives $u = \tilde u^{-1}$ without changing σ;

3) in $CF_1^{6,2,<,>}$ at $\tilde\sigma = \sigma$, $\tilde u = u$ substitution with r_1, $s_2 = 0$, $s_1 = v^{1/2}|\tilde u|^{-1/2}$, $r_2 = v^{-1/2}|\tilde u|^{-1/2}$ gives $\sigma = \tilde\sigma\operatorname{sgn}\tilde u$, $u = \tilde u^{-1}$ without changing v;

4) in $CF_3^{6,2,<,=}$ $(u = 1)$ at $\tilde{v} = v > 1/2$ substitution with $r_1 = 1$, $s_1 = (2\tilde{v} - 1)s_2$, $r_2 = 0$, $s_2 = (4\tilde{v} - 1)^{-1}$ gives $v = \tilde{v}(4\tilde{v} - 1)^{-1} < 1/2$ $(v > 1/4)$;

5) in $CF_{4,+1}^{6,2,<,=}$ $(u = 1)$ at $\tilde{v} = v < 0$ normalization with r_1, $-s_2 = 1$ gives $v = -\tilde{v}$;

6) in $CF_6^{6,2,<,<}$ $(u < 0)$ at $\tilde{\sigma} = \sigma$, $\tilde{u} = u \le -1$ substitution with r_1, $s_2 = 0$, $s_1 = (-\tilde{u})^{-1/2}$, $r_2 = \tilde{v}s_1$ gives $\sigma = -\tilde{\sigma}$, $u = \tilde{u}^{-1}v^2 > -1$;

7) in $CF_7^{6,2,<,<}$ $(v < 0)$ at $\tilde{\sigma} = \sigma$, $\tilde{u} = u$, $\tilde{v} = v < -3$ substitution with $r_1 = -(2\tilde{v} + 3\tilde{u})\varrho$, $s_1 = (\tilde{v} + 3)\varrho$, $r_2 = -s_1$, $s_2 = -(\tilde{v} + 3\tilde{u} - 3)\varrho$, where $\varrho = (-\tilde{v}(\tilde{v}^2 + 3\tilde{u}\tilde{v} + 3(\tilde{u}^2 - \tilde{u} + 1)))^{-1/2}$, gives $u = -(3\tilde{u} + \tilde{v} + 3)\tilde{v}^{-1}$, $v = 9\tilde{v}^{-1} > -3$ without changing σ;

8) in $CF_{11,+1}^{6,2,<,<}$ $(v < 0)$ at $\tilde{\sigma} = \sigma$, $\tilde{u} = u < 0$, $\tilde{v} = v$ normalization with r_1, $-s_2 = 1$ gives $\sigma = -\tilde{\sigma}$, $u = -\tilde{u} > 0$ without changing v;

at $\tilde{\sigma} = \sigma$, $\tilde{u} = u$, $\tilde{v} = v < -1$ substitution with r_1, $s_2 = \tilde{u}\varrho$, $s_1 = -r_2$, $r_2 = (\tilde{v} + 1)\varrho$, where $\varrho = (-\tilde{v}(\tilde{u}^2 + (\tilde{v} + 1)^2))^{-1/2}$, gives $u = -\tilde{u}\tilde{v}^{-1}$, $v = \tilde{v}^{-1} > -1$ without changing σ.

Corollary 4.1. In accordance with Definition 1.12 from [2] we have:

$$acs_{8,+1}^{4,2,<,>} = \{|u| > 1\},\quad acs_{34,+1}^{4,2,<,>} = \{\sigma = -1,\ u > 1\},\quad acs_{34,+1}^{4,2,<,<} = \{\sigma = -1.\ u < -1\},$$

$$acs_1^{6,2,<,>} = \{|u| > 1\},\quad acs_3^{6,2,<,=} = \{v > 1/2\},\quad acs_{4,+1}^{6,2,<,=} = \{v < 0\},$$

$$acs_6^{6,2,<,<} = \{u \le -1\},\quad acs_7^{6,2,<,<} = \{v < -3\},\quad acs_{11,+1}^{6,2,<,<} = \{u < 0,\ v < -1\};$$

for other canonical forms from List 2.1 $mcs^{m,2,<,*} = cs^{m,2,<*}$.

5. Canonical forms and canonical sets at $l = 2$. Let us now give the unique list of canonical forms and canonical sets for $l = 2$, which are obtained in [3] and in the present work.

List 5.1. Twenty two $CF_i^{m,2}$ and their $cs_i^{m,2}$ of System (2.1)[1] at $l = 2$ $(\sigma, \kappa = \pm 1)$.

$$CF_3^{2,2,=,\ge} = \sigma\begin{pmatrix} u & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},\quad CF_4^{2,2,>,\ge} = \sigma\begin{pmatrix} 0 & u & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},\quad CF_{7,\kappa}^{2,2,=,\gtreqless} = \sigma\begin{pmatrix} 0 & \kappa & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$CF_{8,\kappa}^{2,2,>,\gtreqless} = \sigma\begin{pmatrix} 0 & 0 & \kappa & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};\quad CF_7^{3,2,=,\ge} = \sigma\begin{pmatrix} u & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},\quad CF_{10}^{3,2,>,\ge} = \sigma\begin{pmatrix} 0 & u & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$CF_{12}^{3,2,=,<} = \sigma\begin{pmatrix} 1 & u & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},\quad CF_{13}^{3,2,=,=} = \sigma\begin{pmatrix} 0 & 0 & u & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix},\quad CF_{16}^{3,2,>} = \sigma\begin{pmatrix} 0 & 1 & u & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};$$

$$CF_{8,\kappa}^{4,2,\gtreqless,>} = \sigma\begin{pmatrix} u & 0 & \kappa u & 0 \\ 0 & 1 & 0 & \kappa \end{pmatrix},\quad CF_{14,-1}^{4,2,>,>} = \sigma\begin{pmatrix} 0 & u & u & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix},\quad CF_{23}^{4,2,>} = \sigma\begin{pmatrix} 0 & u & v & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix},$$

$$CF_{34,+1}^{4,2,<,\gtreqless} = \sigma\begin{pmatrix} 0 & u & 0 & u \\ 1 & 0 & +1 & 0 \end{pmatrix};\quad CF_7^{5,2,<,\ge} = \sigma\begin{pmatrix} u & -u & u & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix},\quad CF_{22}^{5,2,<} = \sigma\begin{pmatrix} 0 & u & -u & u \\ 1 & 0 & 0 & 1 \end{pmatrix};$$

$$CF_1^{6,2,<,>} = \sigma\begin{pmatrix} u & u & uv & 0 \\ 0 & 1 & 1 & v \end{pmatrix},\quad CF_3^{6,2,<,=} = \sigma\begin{pmatrix} u & u(1-v) & 0 & -uv^2 \\ 0 & 1 & 1 & v \end{pmatrix},$$

$$CF_{4,+1}^{6,2,<,=} = \sigma\begin{pmatrix} u & v & u & v \\ 0 & 1 & 0 & +1 \end{pmatrix},\quad CF_6^{6,2,<,<} = \sigma\begin{pmatrix} u & u(v^{-2} - v) & 0 & uv^{-3} \\ 0 & 1 & v^{-1} - v^2 & 1 \end{pmatrix},$$

$$CF_7^{6,2,<,<} = \sigma\begin{pmatrix} u & v & -v & u+v \\ 1 & 0 & 0 & 1 \end{pmatrix},\quad CF_{11,+1}^{6,2,<,<} = \sigma\begin{pmatrix} u & v & u & v \\ 1 & 0 & +1 & 0 \end{pmatrix},$$

$$CF_2^{7,2,<,<} = \sigma\begin{pmatrix} u & w & uv^{-1} - v(uv + w) & u + wv^{-1} \\ 1 & 0 & v^{-1} - v^2 & 1 \end{pmatrix};$$

$$cs_3^{2,2,=,>} = \{u \neq 1\}, \quad cs_3^{2,2,=,=} = \{u = 1\}; \quad cs_4^{2,2,>,>} = \{u \neq 1\}, \quad cs_4^{2,2,>,=} = \{u = 1\};$$

$$cs_{7,\kappa}^{2,2,=,>} = \{\kappa = 1\}, \quad cs_{7,\kappa}^{2,2,=,<} = \{\kappa = -1\}; \quad cs_{8,\kappa}^{2,2,>,>} = \{\kappa = 1\}, \quad cs_{8,\kappa}^{2,2,>,<} = \{\kappa = -1\};$$

$$cs_7^{3,2,=,>} = \{u \neq \pm 1\}, \quad cs_7^{3,2,=,=} = \{u = 1\}; \quad cs_{10}^{3,2,>,>} = \{u \neq 1\}, \quad cs_{10}^{3,2,>,=} = \{u = 1\};$$

$$cs_{12}^{3,2,=,<} = \{u < -1/4\}, \quad cs_{13}^{3,2,=,=} = \{u = 1\};$$

$$cs_{16}^{3,2,>,>} = \{u > -1/4\}, \quad cs_{16}^{3,2,>,=} = \{u = -1/4\}, \quad cs_{16}^{3,2,>.<} = \{u < -1/4\};$$

$$cs_{8,-1}^{4,2,>,>} = \{u \neq \pm 1\}, \quad cs_{8,+1}^{4,2,<,>} = \{u \neq 1\}, \quad cs_{8,+1}^{4,2,<,=} = \{u = 1\};$$

$$cs_{14,-1}^{4,2,>,>} = \{u \neq -1, -2, -3\};$$

$$cs_{23}^{4,2,>,>} = \{u \neq 1, v > -(1-u)^2/4, v \neq u, (2u-1)/4, u(2-u)/4\},$$

$$cs_{23}^{4,2,>,=} = \{u \neq -1, v = -(1-u)^2/4\}, \quad cs_{23}^{4,2,>,<} = \{v < -(1-u)^2/4\};$$

$$cs_{34,+1}^{4,2,<,>} = \{u > 0, u \neq 1\}, \quad cs_{34,+1}^{4,2,<,<} = \{u < 0\}; \quad cs_7^{5,2,<,>} = \{u \neq \pm 1, 3\}, \quad cs_7^{5,2,<,=} = \{u = 1\};$$

$$cs_{22}^{5,2,<,>} = \{u > -1/4, u \neq 3/2, 6, 4 \pm \sqrt{13}\}, \quad cs_{22}^{5,2,<,=} = \{u = -1/4\},$$

$$cs_{22}^{5,2,<,<} = \{u < -1/4\};$$

$$cs_1^{6,2,<,>} = \{u \neq \pm 1, v > 1/4, v \neq \psi_1^2(u), \psi_2(u), \psi_3(u)\};$$

$$cs_3^{6,2,<,=} = \{u = 1, v > 1/4, v \neq 1/3, 1, (49 \mp 7\sqrt{46})/6\}; \quad cs_{4,+1}^{6,2,<,=} = \{u = 1, |v| < 1\};$$

$$cs_6^{6,2,<,<} = \{v \in (0,1), u \in (\psi_7^-(v), \psi_7^+(v)), u \neq -v, (u,v) \neq (-2^{-5/3}(3 \pm \sqrt{5}), 2^{-2/3})\};$$

$$cs_7^{6,2,<,<} = \{4v < -(u+1)^2, u \neq -1, v \neq -u, -3(u+1), 3(u+1)(u+2)^{-1}, \psi_{32}(u)\};$$

$$cs_{11,+1}^{6,2,<,<} = \{4v < -u^2, v \neq \psi_{20}^{\mp}(u), 3(u^2+5)(u^2+1)(2(u^2-3))^{-1}\};$$

$$cs_2^{7,2,<,<} = \{v \in (0, \sqrt[3]{4}), v \neq 1, -u, 2u, \theta_3(u), w \neq -uv, -u(v - v^{-2}), \psi_{33}^{\mp}(u,v), 4w$$
$$< -(u+v)^2, \psi_{27}(u,v,w) < 0, (u,w) \neq ([-4v \vee \psi_{25}^{\mp}(v)], [3v(4v + \psi_{25}^-(v))/2 \vee 3(v\psi_{25}^{\pm}(v)$$
$$- 2v^{-1})]), (v,w) \neq ([7^{-1/3} \vee \theta_2(u)], [3(7^{-1/3}u + 7^{-2/3}) \vee (u + \theta_2(u))(\theta_2(u) - 2u)])\}.$$

Addition. Continuing the discussion begun in [1, Section 1.5], we are going to focus on the approaches in choosing the unperturbed part of two-dimensional systems, which is, first of all, subjected to classification and corresponding normalization. As is evident in the cycle of works proposed, classification and normalization have to be applied to homogeneous cubic polynomials, whose canonical forms are then used as unperturbed parts for normalization of perturbations.

The need for classification and normalization of homogeneous quadratic polynomials in two-dimensional systems whose right-hand part decomposition begins from the second order is also obvious. This classification was established by K.S. Sibirskii [5] and then newly developed by V.V. Basov et al. (see Refs. in [1]) based on the other ordering principle.

Normalization of two-dimensional systems $\dot{x} = Ax + X(x)$ with the nilpotent matrix A in the unperturbed part was attempted by F. Takens [6], where the *GNF* (generalized normal form) was obtained as $\dot{y}_1 = y_2$, $\dot{y}_2 = y_1 f(y_1) + y_2 g(y_1)$ and it was equivalent to *GNF*

$$\dot{y}_1 = y_2 + y_1 g(y_1), \quad \dot{y}_2 = y_1 f(y_1) + y_2 g(y_1), \tag{*}$$

where $f = \sum_{k=\mu}^{\infty} \alpha_k y_1^k$, $g = \sum_{k=\nu}^{\infty} \beta_k y_1^k$, $(\alpha_\mu, \beta_\nu \neq 0, \mu, \nu \geq 1)$. This system is one of the particular cases of incomplete Belitskii's *NF* and was established by G.R. Belitskii [7].

A. Baider and J. Sanders used this *GNF* [8] for creation of the full formal classification of germs of analytical vector fields in $\mathbb{R}^2$ with the nilpotent linear part based on the correlation between μ and ν values. Calling (*) normal first-order form in cases when $\mu \neq 2\nu$, they established and obtained the *NF*s of the second, third, and further down to infinite orders, terminating this process when simplification stops and obtaining in a certain sense a single *NF*. Further, Kokubu, Oka, and Wang [9] found a single second-order *NF* in the form $\dot{y}_1 = y_2$, $\dot{y}_2 = \alpha_2 y_1^3 + \beta_1 y_1 y_2 + y_1 \sum_{k=3}^{\infty} y_1^k$ for the unstudied case $\mu = 2$, $\nu = 1$ provided that the α_2/β_1^2 is not algebraic value.

It is worth mentioning that uniqueness of *NF* in fact testifies to distinction of the simplest *NF* on a certain basis. Therefore, the techniques proposed in [8, 9] do not allow extraction of all structures of normal forms, as takes place while establishing *GNF* via the method combining the resonance equations and sets and reported in [1, Section 1.3].

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