HYPERBOREUS

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ναυσὶ δ᾽ οὔτε πεζὸς ἰὼν κεν εὕροις
ἐς Ὑπερβορέων ἀγῶνα θαυμαστὰν ὁδόν

(Pind. *Pyth.* 10. 29–30)

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Summary in Russian and English

ZENO'S DEBT TO HIPPASUS*

1

The way of argumentation employed by the Eleatic philosophers, Parmenides and Zeno, was repeatedly compared with methods of demonstration characteristic for Greek mathematics. Some scholars considered the mathematics as the source of inspiration for the Eleatic mode of reasoning,[1] the others took the opposite view[2] and yet others adopted a more flexible approach.[3] The entire problem will be not discussed in this paper. I will just address a particular type of argument found in both Zeno's antinomies and an ancient demonstration of the incommensurability of the side and diagonal of a square. I will argue that in this particular case the debtor was Zeno.

In his *Parmenides*, Plato makes Zeno read his book before an Athenian audience that includes young Socrates whose reaction to what he just heard implies that he was quite impressed by the very first of Zeno's propositions: "If the things that are are many, then they must be both like and unlike, which is impossible" (ἐστι τὰ ὄντα, ὡς ἄρα δεῖ αὐτὰ ὅμοιά τε εἶναι καὶ ἀνόμοια, τοῦτο δὲ δὴ ἀδύνατον, 127 e).

Simplicius cites Zeno for a similarly constructed argument: "if there are many things, the same things are both limited and unlimited" (in Phys. 140, 28; 29 B 3 DK). Although Zeno's own words cited by Simplicius are not framed in such a concise formulation, they unambiguously

* I am grateful to Ivan Mikirtumov, Livio Rossetti and *Hyperboreus*' anonymous reviewer for the comments on the previous version of this paper.

[1] Zaicev 2003; Zaicev 1993, 172 f.; Zhmud 2012, 251–254, and especially 252, n. 47 for a list of earlier works.

[2] Most notably in the works by Árpád Szabó; see also Burkert 1972, 425 f.

[3] Mueller 1997, 279: "I make no more than my guess in saying that I don't believe that mathematical argument was influenced by either one of them and that, whereas Parmenides' argumentation looks to be autonomous and satisfactorily explained without invoking mathematical precedent, Zeno's considerations of infinite divisions seem likely to reflect mathematical preoccupations".

agree with Simplicius' interpretation. Another time both the same type of argument and its concise formulation emerge in the direct quotation from Zeno: "if there are many things, it is necessary that they are both small and large" (εἰ πολλά ἐστιν, ἀνάγκη αὐτὰ μικρά τε εἶ ναι καὶ μεγάλα, Simpl. in Phys. 140, 34; 29 B 1 DK).[4]

Now, Aristotle who repeatedly mentions the incommensurability of the side and diagonal of a square makes us also aware of the reason why this is so: "They prove that the diagonal of a square is incommensurable with its side by showing that, if it is assumed to be commensurable, odd numbers will be equal to even" (ἀσύμμετρος ἡ διάμετρος διὰ τὸ γίνεσθαι τὰ περιττὰ ἴσα τοῖς ἀρτίοις συμμέτρου τεθείσης).[5] Now, a proof that applies the dichotomy of odd and even is found in the Tenth Book of the *Elements*. "Considering further that the inclusion of this proof in the *Elements* could be justified only for the historical interest of the proof, since its placement obviously has no bearing on the development of the propositions of the Tenth Book, it has been generally argued that one should accept this version as the original form by which the incommensurability was discovered and proved".[6] It is appropriate to specify that the wording used in the received proof is much closer to that of Zeno's argument than it is the case with the formulation used by Aristotle. In words of our geometer, "it will follow that one and the same number is both even and odd" (τὸν αὐτὸν ἀριθμὸν ἄρτιον εἶναι καὶ περισσόν).

We are dealing thus with a very similar and quite a remarkable type of argument.

The evidence concerning the discovery of the incommensurability of the side and the diagonal (*diameter*, in the ancient terms) of a square is in no way clear and abundant. Scholars typically assume that the discovery was due to Hippasus, but they are less explicit in attributing to him the demonstration cited by Aristotle, and an alternative method for Hippasus' demonstration was proposed by Kurt von Fritz. Further, the date of the discovery caused significant disagreement. I will try to remove all possible doubts. I will start with the question of attribution, proceed to chronological matters and discuss the plausibility of the intellectual interchange that would involve both Zeno and Hippasus.

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4 Unfortunately, it is not immediately clear what precisely Zeno means here, but this is of minor importance for our argument.

5 *APr* 1. 23. 41 a 23–27; transl. after Heath 1949, 22; cf. 1. 44. 50 a 35–38.

6 Knorr 1975, 23.

2

The discovery of the incommensurability or irrationality belong to the glories of Greek mathematics, but the evidence concerning it is peculiar.

We are told that the person who first disclosed the Pythagorean teaching of irrationals perished in a shipwreck (Pappus. *Comm*. p. 63–64 Junge-Thomson; *Schol. Eucl*. 417. 12 sqq.; Elias. *In Aristot*. *Cat*. 125. 12; Iamb. *VP* 247), and Iamblichus (*VP* 88) introduces the name of Hippasus in this connection:

> Hippasus who was a Pythagorean but, owing to his being the first to publish and write down the (construction of the) sphere with the twelve pentagons, perished by shipwreck for his impiety, but received credit for the discovery, whereas it really belonged to HIM, for it is thus that they refer to Pythagoras, and they do not call him by name.[7]

Iamblichus specifies that some people say that the impious person who perished at sea disclosed the discovery of the dodecahedron inscribed in a sphere, while others maintain that he disclosed the teaching of irrationality and incommensurability (*VP* 247).

We also hear from Iamblichus that the one who divulged knowledge of the commensurability and incommensurability was expelled from Pythagorean community, and that the Pythagoreans erected a tomb for him as if he were dead (*VP* 88; 18 A 4 DK). Clemens relates this story (substituting a stele for a tomb) about Hippasus, who is mistakenly called 'Hipparchus' (*Strom*. 5. 58; 18 A 4 DK); he does not specify what was the disclosed "teaching of Pythagoras".

The emerging picture is clear: Hippasus published the discovery of both incommensurability / irrationality and of the dodecahedron inscribed in a sphere, and he perished at sea.[8] One may speculate that the version with erected grave monument derives from the habit to construct a cenotaph to a person lost at sea. The legend, then, has preserved sad truth about the last moment in the life of Hippasus.

It is worth emphasizing that the ancient tradition has no candidate for the discovery of incommensurability / irrationality other than Hippasus and Pythagoras, and the latter can be safely discounted. We also know the names of the mathematicians who made further important contributions to the theory of irrationality. The next who came after Hippasus was

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[7] Transl. Heath 1981, 160.

[8] Cf. Burkert 1972, 457 f. and especially Zhmud 2012, 275.

Theodorus of Cyrene. He appears as a mathematician demonstrating the irrationality of the square roots of 3, 5, etc. up to 17 in Plato's *Theaetetus* (147 d), the dramatic date of which is 399 BC. Kurt von Fritz most properly remarks: "Even if we assume that Theodorus' demonstrations had been worked out for the first time not very long before, Plato's dialogue would still indicate that the irrationality of the square root of 2, or the incommensurability of the side and diameter of a square, had been discovered by someone else. For it is difficult to see why he should have made Theodorus start with the square root of 3, unless he wished to give a historical hint that this was the point where Theodorus' own contribution to mathematical theory began".[9]

Sir Thomas Heath arrives at a similar conclusion: "The actual method by which the Pythagoreans proved the incommensurability of $\sqrt{2}$ with unity was no doubt that referred to by Aristotle (*APr* I. 23, 41 a 26–27), a *reductio ad absurdum* by which it is proved that, if the diagonal is commensurable with the side, it will follow that the same number is both odd and even. The proof formerly appeared in the texts of Euclid as X. 117, but it is undoubtedly an interpolation, and August and Heiberg accordingly relegate it to an Appendix".[10]

In the paper cited above, von Fritz gives the full translation of the proof and accompanies it by appropriate comments:

> One glance at this demonstration shows that it does not presuppose any geometrical knowledge beyond the Pythagorean theorem in its special application to the isosceles right-angled triangle, which, as is well-known, can be 'proved' simply by drawing the figure in such a way that the truth of the theorem in that particular case is immediately visible. Apart from this the demonstration remains in purely arithmetical field; and since the early Pythagoreans speculated a good deal about odd and even numbers the demonstration itself cannot have been beyond their reach.[11]

Nevertheless, von Fritz makes a sudden move and expresses his doubts that the demonstration is early:

> Most significant is the fact that the whole proof, as presented, uses the terms commensurable and incommensurable, just as Theodorus did in

[9] Von Fritz 1945, 244.

[10] Heath 1908, 2. Heath has not included this Appendix in his translation of Euclid.

[11] Von Fritz 1945, 254 f.

> Platos's *Theaetetus*, as something already known. This seems to presuppose that incommensurability was already known when the demonstration was elaborated.[12]

But how can one start to *demonstrate* even a particular case of incommensurability without having already the notion of incommensurability? Imagine the mathematician who made the discovery and decided to publish it. How could he describe what he had done without introducing the corresponding terms?

Von Fritz proposes that Hippasus worked with the side and diameter of a pentagon and used the process of mutual subtraction that goes on infinitely, which indicated that the given magnitudes were not in numerical ratio. The method itself is ingenious, but our sources invariably speak of the side and the diameter (that is, the diagonal) of a square and not of a pentagon. One may also doubt that the demonstration based on an infinite regress would have been accepted as truly convincing in early age of geometry, since such a demonstration shows that a common measure cannot be found rather than it does not exist.

Some other objections to the reconstruction proposed by von Fritz were advanced by Wilbur Richard Knorr.[13] He, moreover, emphasized that the demonstration attached to the Tenth Book of the *Elements* is not only in accord with abbreviated reference to it by Aristotle, but also contains an antiquated, early Pythagorean notion: the unit is not considered there as odd.[14] It is true, Knorr believes that the demonstration as we have it has reached us in an edited version. He even ventures to propose a reconstruction of the original one.[15] However, this does not affect our argument, for Knorr admits the crucial elements – that the demonstration involved the side and the diameter of a square, on the one hand, and the impossible conclusion that the same number would be both odd and even, on the other hand.

To sum up, there is no candidate in the ancient tradition other than Hippasus to claim the discovery of the incommensurability of the diagonal and the side of a square and there is no early demonstration of it, known to the ancient tradition, other than that involving odd and even numbers.

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12 Von Fritz 1945, 256.
13 Knorr 1975, 29–31.
14 Knorr 1975, 23 f.
15 Knorr 1975, 26 f.

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Kurt von Fritz dates Hippasus to the middle of the fifth century. Wilbur Richard Knorr assigns to him (or rather to the discovery of the incommensurability) even a later date, c. 430–410 BC. According to Walter Burkert, "the evidence seems to point toward the first half of the fifth century",[16] while a few scholars, most recently Leonid Zhmud, conclude that Hippasus was active c. 500 BC. Let us consider the evidence.

Iamblichus says that Theodorus of Cyrene and Hippocrates of Chios worked after Hippasus (*Comm. math. sc.* 77, 18 sqq. Festa). Von Fritz properly notes that Theodorus of Cyrene and Hippocrates of Chios appear together in Proclus' commentary on Euclid (p. 66 Friedlein), in a passage that is likely to be an excerpt from the history of mathematics by Eudemus of Rhodes. However, he draws from this a very problematic conclusion: "according to this work Hippasus belonged to the generation preceding that of Theodorus".[17] While Hippasus does not figure in Proclus, it is true that the context of Iamblichus' passage would not imply an immense distance in time between Hippasus and the pair of Theodorus and Hippocrates. Nevertheless, the passage contains no hint to whether this would amount to one generation, or rather a half, or two.

Theodorus of Cyrene is presented as a highly reputed mathematician in Plato's *Theaetetus*, and the dramatic date of this dialogue is a short time before Socrates' death (142 c), in 399 BC. Hippocrates of Chios, who is named before Theodorus in Proclus' catalogue, was somewhat older. Walter Burkert is right to observe that Hippocrates' theory of comets must have been published before 427/426 BC.[18] For the comet observed in winter of 427/426 BC is referred to by Aristotle as displaying characteristics incompatible with the theory advanced by "Hippocrates and his disciple Aeschylus" (*Mete*. 342 b 29; 42 A 5 DK). One may further think that Aeschylus published the theory when his teacher was no longer alive. A shared claim implies in any case that Aeschylus, a disciple, was not just a beginner, while Hippocrates had to get prominence before start teaching. Thus Hippocrates must have become prominent a good number of years before 427/426 BC.

For his other argument, von Fritz also uses Iamblichus, though this time his *De vita Pythagorica*, 257. Von Fritz maintains that "Hippasus had an important part in the political disturbances in which the Pythagorean

[16] Burkert 1972, 206.
[17] Von Fritz 1945, 245.
[18] Burkert 1972, 314 n. 77.

order became involved in the second quarter of the 5th century, and which ended in the revolt of ca 445, which put an end to Pythagorean domination in southern Italy".[19] We need not discuss here the general issue of anti-Pythagorean movement in southern Italy and the epoch of its final phase. The point is that it is not clear who was that Hippasus who "had an important part in the political disturbances" in Croton. In Iamblichus, Hippasus, Diodorus and Theages appear as members of Crotonian *Thousand* who advocate democratic principles and are opposed by the Pythagoreans. Hippasus is thus included in a group that is distinct from the Pythagoreans. And how Hippasus of Metapontum (as Aristotle refers to our Hippasus) took an office in Croton? Some of the ancients searched a solution in the assumption that Hippasus was in fact a citizen of Croton rather than of Metapontum (Iam. *VP* 81 = *Comm. math. sc*. p. 76. 23 Festa). In the catalogue of the Pythagoreans, Hippasus figures, however, among the citizens of Sybaris (Iam. *VP* 267)! As if this were not enough, a scholium to Plato's *Phaedo* 61 e tells us that Hippasus and Philolaus were the only Pythagoreans who survived the massacre in Croton, while on another version, the two survivors were Archippus and Lysis (Porph. *VP* 55; Iambl. *VP* 250). Thus, no safe inference concerning Hippasus, the mathematician and philosopher, is possible from the Iamblichus' passage.[20]

Knorr observes that the discovery of the incommensurability never figures in the sources before the time of Theodorus of Cyrene, c. 430–410, and he wonders of "what could have hindered its dissemination".[21] But he is too quick to leave the question he raises without an answer. The only option he mentions just to dismiss it ironically is the "Pythagorean jealousy". Yet it is by no means surprising if the line of inquiry started by Hippasus was advanced further only fifty or hundred years later. The discovery of the incommensurability was of no use in astronomy or in solving the problems that catch imagination (such as, say, determining the height of a pyramid). It is the history of mathematics is to be written on the evidence about the epoch of Hippasus and not the epoch of Hippasus is to be established on a priory assumptions what this history should be like.[22]

If, however, we are to involve general considerations, I venture to suggest a hypothesis that may illuminate the circumstances of the discovery of the incommensurability and its effects. Let us recall the frequently

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[19] Von Fritz 1945, 245.

[20] However, Zhmud 2012, 98 employs it to support his belief in Hippasus' "very real political rivalry with Pythagoras".

[21] Knorr 1975, 38.

[22] Zhmud 2012, 124 also finds such efforts misleading.

stated view according to which the proof of incommensurability must have destroyed the whole Pythagorean philosophy of numbers. It was rightly emphasized in several recent studies that the alleged crisis is not verified in our sources, but this does not eliminate the problem. Leonid Zhmud's contention that Pythagorean philosophy of numbers is a later fabrication[23] would do that, but I am not prepared to follow his approach though his criticism of the standard view includes many valuable points. Let us assume that Pythagoras made (or helped to) circulate the idea that the ratios expressed in numbers were essential constituents of the world fabric. I propose that this was the background of the emergence of the notion of the incommensurability. On empirical level, the incommensurability had vaguely been known as long as a right-angled triangle with the sides 3, 4, 5, that is, from time immemorial (more precisely, since the carpenter's square had come in use). Anyone who was aware of such a triangle and was endowed with curiosity should have wondered what the length of the hypotenuse in a triangle with both other sides of 3 or 4, etc., units would be. In other words, one had to face the difficulty of expressing the length of the hypotenuse in integer numbers, or the other sides in integers, if one had assigned an integer number to the length of the hypotenuse. It was the same problem, in a sense, as that of the commensurability of a side and diagonal of a square. Now, imagine a person who is aware of such a problem and who hears about an idea according to which all the major phenomena of the world are in a correspondence with numerical ratios. This person, Hippasus, puts the idea to test and demonstrates that its radical version cannot be true. However, his rival, Pythagoras, overcomes the difficulty by proving the theorem that bears his name, that is, by proving that the *squares* of otherwise incommensurable elements nevertheless have a ratio ("rational only in the square", later Greek mathematicians would say). One understands, then, the pride and joy of Pythagoras, echoed in our sources, and why the proof of the incommensurability of the diagonal and a side of a square did not destroy the Pythagorean philosophy of numbers even though it caused some uneasiness reflected in the tradition about an impious person who divulged Pythagorean teaching. One can also understand better why the idea of incommensurability was formulated from the outset as a general one and why there could be for long no strong stimulus for addressing various particular cases of incommensurability.[24]

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23 In his various publications over many years, most recently Zhmud 2012, 394 ff.

24 Scholars are accustomed to call Hippias a Pythagorean, but one should not ignore the ancient tradition according to which this was rather the other way round and the allegedly Pythagorean μαθηματικοί were known as the followers of Hippasus (see below).

If this hypothesis hits the mark, it is immediately clear that the discovery made when Pythagoras was still active is older than Zeno's philosophy. Yet this hypothesis is a matter for a further discussion, and we will not make our argument depend on it.

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In the extant tradition, there are several cases of direct or indirect synchronization of Hippasus with other thinkers. Iamblichus includes him in the list of those who were the disciples of Pythagoras when he was old man (*VP* 104); yet the list does not seem to derive from a good authority.

There was an ancient assertion (known to Neanthes and denied by him) according to which "Empedocles was the pupil of both Hippasus and Brontinus" (D. L. 8. 55). Such a report indirectly makes Hippasus contemporary with Brontinus, who is one of the addresses of Alcmaeon' treatise (24 B 1 DK). According to a passage in Aristotle, Alcmaeon was young when Pythagoras was old (*Metaph*. 986 a 22; 24 A 3 DK). Yet Brontinus was not young when Pythagoras was old, if we are to believe that Pythagoras married his daughter Theano (D. L. 8. 42); it seems that already some ancient scholars tried to avoid the difficulty by assuming that Theano "was Brontinus's wife and Pythagoras's pupil" (*ibid.*). Hippasus appears thus to be a (younger?) contemporary of Pythagoras. It is hardly in doubt that the activity of Pythagoras in southern Italy dates to later decades of the sixth century.

Aristotle pairs Hippasus with Heraclitus for the view that the ἀρχή is fire (*Metaph*. 984 a 7; 18 A 7 DK). Hippasus is named first. Simplicius and Theodoretus in similar contexts have the same order; this is, however, reverse in Pseudo-Plutarch and Stobaeus. Clemens and Tertullian pair Hippasus' and Heraclitus' views on god and soul respectively (18 A 8–9 DK); both name Hippasus first. We are told after all that Heraclitus heard Xenophanes and Hippasus (the *Suda*, s. v. Heraclitus; 18 A 1a DK). Zhmud reasonably suggests that the similarity of the views would have disposed to making Hippasus a disciple of famous Heraclitus unless there was a strong tradition that Hippasus was older.[25] Several links between the views of Hippasus and Heraclitus can indeed be suggested. Hippasus seems to have been the first to hold the doctrine that "there is a definite time which the changes in the universe take to complete" (D. L. 8. 84;

[25] Zhmud 2012, 125. It can be replied, however, that one could not resist temptation to find a teacher for Heraclitus and that Hippasus with his fire=ἀρχή was the best candidate.

18 A 1 DK; R. D. Hicks' transl.), while Heraclitus seems to have been the first to indicate the amount of time, 10800 years, for that (fr. 65 Marcovich; fr. 63 Lebedev).[26] Diogenes Laertius (9. 1) and the *Suda* date Heraclitus' acme to 69th Olympiad (504/503–501/500 BC). The value of such information is not clear, yet it is unlikely to be very wrong. On the one hand, the Ephesians were hardly in a position to expel Hermodorus (for which they are blamed by Heraclitus) before the outbreak of the Ionian revolt (c. 499 BC). On the other hand, no echo of great victories of 480–479 BC is discernible in Heraclitus.

Hippasus was an outstanding geometer, but he does not figure in Proclus' catalogue of geometers. Zhmud provides a convincing explanation of what we are dealing with. Proclus, he notes, attributes to Pythagoras precisely those achievements that the rival tradition attributes to Hippasus. Namely, Proclus speaks of constructing the so-called cosmic bodies and the study of irrationality. One may suppose that Hippasus figured in the original catalogue, composed apparently by Eudemus. In any case, we can see the historical position of the achievements ascribed to Hippasus. They appear in Proclus' text between general remarks on the importance of Pythagoras (he turned the philosophy of geometry into a form of liberal education, etc.), on the one hand, and the names of Anaxagoras and Oenopides, on the other hand (pp. 65 sq. Friedlein). In spite of largely accepted date that goes back to Apollodorus, Anaxagoras was born earlier than 500 BC (probably in 519/518 BC);[27] his ideas were well known in Athens in 460-s.[28]

The picture emerging from dispersed pieces of evidence is remarkably consistent. Hippasus appears as a person active in the late sixth and, perhaps, early fifth centuries.[29]

The ancient dates for Zeno range from Ol. 78 = 468/467–465/464 BC to Ol. 81 = 456/455–453/452 BC (29 A 1. 29; A 2; A 3 DK). The Greek chronographic tradition incorporated, however, too many conjectures and too much confusion to use it without examination. In the case of Zeno, we have Plato's *Parmenides* according to which Parmenides was about sixty five years old and Zeno about forty when Socrates was very young (127 a–b). Since Socrates died in 399 BC and he was almost certainly at the age of seventy, it is easy to calculate that the conversation described

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26 The alleged Italian provenance of Hippasus is not an obstacle. We know that Pythagoras and Xenophanes came to Italy as Ionian émigrés and that Herodotus of Thurii was born in Halicarnassus.

27 Panchenko 2000, 45 n. 29.

28 "Anaxagoras writes ca. 470–65" (Sider 2005, 11); Schofield 1980, 33–35.

29 Zhmud 2012, 125 arrives at the same result.

in the dialogue should be dated to c. 450 at the very latest and that Zeno was, then, born c. 490 BC or slightly earlier. However, the conversation is fictional. The indications that Parmenides was very old and Socrates very young seem to signalize that Plato's idea of the distance in time between the two was in fact hardly compatible with the possibility of their meeting. It is likely, then, that Parmenides was a somewhat earlier figure than it follows from the dialogue. His *floruit* in the chronographic tradition falls indeed in Ol. 69 = 504/503–501/500 BC (D. L. 9. 23; 28 A 1 DK) and not in c. 475. We cannot say what was the reason for such dating, but its deviation from what is implied in the *Parmenides* suggests that it was a bit more than just a guess.

Let us turn again to synchronisms. In Eusebius' *Chronicle*, Zeno appears synchronized with Heraclitus, under Ol. 81 = 456/455–454/453 BC. This is to be compared with the tradition according to which Hippasus was a teacher of Heraclitus.

While there was tradition that made Hippasus a teacher of Empedocles, Zeno appears coeval with Empedocles in Diogenes Laertius (8. 56) who cites an author of the early fourth century BC:

> Alcidamas tells us in his *Physics* that Zeno and Empedocles were pupils of Parmenides at the same time, that afterwards they left him, and that, while Zeno practiced philosophy on his own, Empedocles became the pupil of Anaxagoras and Pythagoras.[30]

While geometrical discoveries by Hippasus antedate geometrical contribution by Anaxagoras (see above), Empedocles, coeval with Zeno, became the pupil of Anaxagoras, and Aristotle confirms that Anaxagoras was older than Empedocles (*Metaph*. 984 a 11).

Thus, the preserved dates for both Parmenides and Zeno indicate that Zeno was younger than Hippasus. Relative chronology of the Pre-Socratics yields the same conclusion.

5

Now the distance in time between Hippasus and Zeno cannot be great and they both belonged to the same region, southern Italy. In view of the facts like that Xenophanes mentioned Thales and criticized Pythagoras and was criticized himself by Heraclitus who also criticized Pythagoras and many others as well, it is highly unlikely that Zeno could have been unaware of Hippasus' achievements. Moreover, it is possible

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[30] R. D. Hicks' transl., modified.

to indicate a case of intellectual contact between Zeno's teacher and Hippasus, whose name in the related testimony was distorted in the manuscript tradition:

> Hipparchus says that rays from each of the eyes reach out with their ends, fasten around external bodies as if touching them with hands, and thus render them apprehensible by vision. Some associate this view with Pythagoras also, because he is an authority in mathematics, and beside him with Parmenides, who expounds it in his poems.[31]

The uncommon name 'Hippasus' was difficult for the scribes. In the works of two authors, Clemens of Alexandria and Tertullianus, it appears as 'Hipparchus' (18 A 4; 9 DK). The renowned astronomer of the second century BC otherwise never figures in 'Aëtius', while Hippasus does. It would have been very strange if Hipparchus, famous for his brilliance in mathematical astronomy, risked his reputation through insecure speculation concerning vision and, in so doing, repeated the view found in old text by Parmenides. Moreover, the views of Parmenides and Hippasus are paired in the doxographic tradition (18 A 9 DK, from Stobaeus, while a parallel testimony of Tertullianus reads 'Hipparchus' instead of 'Hippasus'). The conclusion seems certain: we have to read 'Hippasus'.[32] We cannot decide was it Parmenides who repeated the view of Hippasus or the other way round, but it is clear that one of them was aware of the view of another.

We can also maintain, though, perhaps, with lesser confidence, that the practice of taking into account the opinions of experts in mathematical and astronomical matters and even of citing such opinions was already inaugurated by the time of Zeno. According to unduly neglected testimony by Lactantius, "Xenophanes most foolishly believed mathematicians who said that the circle of the moon was eighteen times larger than the earth".[33] Now it was Anaximander who expressed the dimension of the circle of the moon through the size of the earth and the ratio cited is very close to his; Anaximander's recorded value is nineteen (12 A 22 DK). Further, we are told that Alcmaeon "agrees with certain μαθηματικοί" that "the planets move from west to east contrary to the movement of the fixed stars" (*Dox. Gr.* 345; 24 A 4 DK). There is one more striking

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[31] *Dox. Gr.* 404, addition concerning competing claims appears only in Stobaeus; 28 A 48 DK. Tr. D. Gallop, modified.

[32] Burkert 1972, 42 f. n. 76 and 408 does not see it. For the subject matter cf. Barbero 2014.

[33] *Div. Inst.* 3. 23: *Xenophanes dicentibus mathematicis orbem lunae doudeviginti partibus maiorem esse quam terram stultissime credidit.*

testimony that is hardly irrelevant: "There are two varieties of Italian philosophy which is called Pythagorean. For those who practiced it were also of two sorts, the ἀκουσματικοί and the μαθηματικοί. Of these, the ἀκουσματικοί were accepted as Pythagoreans by the other party, but they did not allow that the μαθηματικοί were Pythagoreans, holding that their intellectual pursuit derived not from Pythagoras but from Hippasus. But those of the Pythagoreans who concerned themselves with science agreed that the ἀκουσματικοί were Pythagoreans, and claimed that they themselves were so to a still greater degree, and that what they themselves stated was the truth" (Iamblichus, *Comm. math. sc*. pp. 76, 16 – 77, 2 Festa).[34] Given the fame of Pythagoras, it is unlikely that in the later tradition Hippasus could have replaced Pythagoras as the progenitor of the μαθηματικοί. In all probability, an early tradition already had Hippasus in this place. Moreover, Pythagoras is paired with the μαθηματικοί in Stobaeus' version of 'Aëtius' 4. 14 rather than presented as his head, and Iamblichus speaks of "Pythagoras and the μαθηματικοί of his time" (18 A 15 DK). The whole issue of early μαθηματικοί and their relation to Hippasus was recently discussed elsewhere.[35] For the present purpose, it is enough to see that in the epoch in question it was rather common for those philosophers who lacked sufficient training in mathematics and astronomy to address the achievements of experts.

To sum up, Zeno must have been aware of the work of Hippasus on all counts.[36]

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Unfortunately, we do not know what was Zeno's way to show that if the things that are are many, then they must be both like and unlike.[37] One has yet to bear in mind that in an early usage the Greek word ὅμοιος could mean that two (or more) things are like in all respects so that there would be no difference between them. We know, for instance, that equal triangles we called ὅμοιοι at an early stage of Greek geometry, and only later this word came to designate similar triangles while for the equal

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[34] Transl. after Kirk, Raven, Schofield 1983, 234.

[35] Panchenko 2016 [Д. В. Панченко, *На восточном склоне Олимпа. Роль греческих идей в формировании китайской космологии*], 178–189.

[36] If Plato confesses that he became acquainted with the fact of incommensurability only late in his life (*Leg*. 819 d sqq.), this is a separate case that belongs with a different epoch and different intellectual milieu.

[37] McKirahan 2010, 177 f. For general discussion of Zeno's fragments B 1–3 DK see now Köhler 2014.

ones was reserved the term ἴσοι.[38] Such a usage of the word ὅμοιος made it difficult to say that the same things can be like in some respects and unlike in another respects. Zeno's formulation was thus strong. But was it truly impressive? Socrates' reaction in the *Parmenides* implies that it was difficult to controvert. Nevertheless, it was just a puzzle, a paradox (compare: when Parmenides speaks of being that undergoes no change he reveals, at least formally, new knowledge, however strange). Right or wrong, one would suspect a kind of trick or manipulation behind Zeno's reasoning. Again, one is puzzled rather than feels endowed with new knowledge when one is told that if there are many things, things are both unlimited and limited in number. The demonstration of the incommensurability is a different case. It appeals to what is immediately clear and convincing: there are even numbers that are divisible into equal parts and there are odd numbers that are not; and no number can be both even and odd. The chronological priority of the demonstration conceived by Hippasus seems to conform its psychological superiority over Zeno's antinomy. This adds to the probability that Zeno, the philosopher, imitated Hippasus, the mathematician.[39]

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[38] Panchenko 1994.

[39] My thesis was partly anticipated by Kneale 1967, 10: "when Zeno used *reductio ad absurdum* in his queer arguments against the possibility of motion, his reason may have been that this pattern of argument had already had a great success of a negative kind". Yet Kneale offers no elaborate support for his assertion and he compares the discovery of the incommensurability with Zeno's *aporiai* and not with his antinomies for which the similarity with Hippasus' demonstration is much more obvious.

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The way of argumentation employed by the Eleatic philosophers was repeatedly compared with methods of demonstration characteristic for Greek mathematics. The paper addresses a particular type of argument, *argumentum ad impossibile*, found in both Zeno's antinomies and an ancient demonstration of the incommensurability of the side and diagonal of a square. It is argued that in this particular case the debtor was Zeno.

Рассуждения Парменида и Зенона обнаруживают сходство с доказательствами, которыми оперируют математики. Статья касается старого спора о том, кто на кого повлиял, но при этом в ней рассматривается лишь один частный случай: в антиномиях Зенона используется тот же весьма специфический ход, *argumentum ad impossibile*, на котором зиждется и древнее доказательство несоизмеримости стороны и диагонали квадрата. Свидетельства, хотя и косвенные, не оставляют сомнения в том, что несоизмеримость стороны и диагонали квадрата доказал Гиппас, однако в науке нет согласия относительно того, когда жил Гиппас и каким образом он выстроил свое доказательство. В статье подтверждаются ранняя датировка Гиппаса и использование им *argumentum ad impossibile*. Общий вывод: все указывает на то, что в своих антиномиях Зенон использовал прием, разработанный Гиппасом.