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Uses and Limitations of the Tsyganenko Magnetic Field Models

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When exploring a new country, one needs a good map. The Earth's magnetosphere is such a country, and its "maps" are models of the geomagnetic field. In essence they are mathematical prescriptions implemented by computer which, given (x,y,z) of a point and other supporting information, return the point's expected magnetic components (B_x, B_y, B_z) [*Tsyganenko*, 1990].

At present, the empirical magnetic field models by *Tsyganenko* [1987, 1989a,c; denoted henceforth as T87 and T89] are often used in various studies of the Earth's magnetosphere. *Stern* [1990] has devised modifications of T87 with current-free tail lobes and with a plasma sheet that warps in response to dipole tilt [*Peredo and Stern*, 1991].

Features

An ideal model should meet two main requirements. First, it should faithfully represent the effects of the five main sources of the magnetospheric field: the Earth's internal field, magnetopause currents, the ring current, tail currents, and the system of Birkeland currents. This is generally achieved by making it modular, for example, by superposing five contributions, one from each source.

Second, the model should be flexible, taking into account various factors that affect the magnetosphere. One is the tilt angle ψ, the complement of the angle between the dipole axis and the Sun-Earth line. Other factors representing the state of the magnetosphere are gauged by indices such as the auroral electrojet index (AE), Dst, the solar wind pressure (p), the interplanetary magnetic field (IMF), and the area of the polar cap.

The Tsyganenko models satisfy these requirements only in part. They do contain "modules" for two external current systems: the ring current and the tail current. The tail current description in particular is fairly detailed, and this is the main difference between the two models. In T87, the tail is viewed as a superposition of straight filaments, while the T89 tail is built up from modified disk distributions.

However, no specific modules cover the magnetopause and the Birkeland current system, instead the effects of these currents are represented by an all-purpose "polynomial." Their combined contributions to (B_x, B_y, B_z) are given as sums of terms, each a product of powers of (y,z), an x-dependent exponential and sinψ or cosψ, giving the dependence on the tilt angle ψ.

The resulting formulas involve about thirty parameters that specify the model. Coefficients of the polynomial constitute the largest group of parameters, but some others have physical meanings, for example, the distance x_N to the beginning of the plasma sheet (at midnight) or the scale half-thickness D of the plasma sheet current layer. In all cases, the values of these parameters have been obtained by least-squares fitting them to a large set of averaged observations by IMP and HEOS satellites.

The dependence on magnetospheric indices is simplified as well. The models use only one index, the general disturbance index Kp, which is usually only available at 3-hour intervals; a separate set of model parameters is provided for each of several ranges of Kp. This choice is far from ideal, but the Kp index was the only one widely available when the models were derived. For tracking fast variations, one may replace Kp values with roughly equivalent values of the AE index [*Rostoker*, 1991], but users must keep in mind that averaged models do not properly describe fast variations.

Limitations

Any serious user of these models should have at least some familiarity with the articles on which they are based and should, in particular, be aware of the limits of their validity. For instance, an earlier model [*Tsyganenko and Usmanov*, 1982] used a relatively crude "tail module." Its authors warned that it was only valid for $x > -15R_E$ (GSM coordinates), yet many researchers applied it to much more distant points.

T87 and T89 are claimed to be valid up to $x = -60$ to $-70\ R_E$, corresponding to the most distant observations used in deriving them. But even within that limit, there exist at least three other reasons why the predicted B may be quite different from what is observed:

(1) Models only give average values of the field, whereas actual values vary greatly around such averages. Figure 1 gives values of the north-south component B_z in the tail near midnight, observed by ISEE-1 for $Kp < 2+$, $|y| < 10$, distance from the "warped equator" less than 2 R_E and $B_H = (B_x^2 + B_y^2)^{1/2}$ less than 5 nT. Each point represents one average **B** vector observed by ISEE-1, and the values are plotted against x, distance along the Sun-Earth line. Tail values of B_z have a particular physical significance since they determine the onset of nonadiabatic motion and the mapping of the auroral oval. The average (except near Earth) is about 5–6 nT. yet it will be noted that the

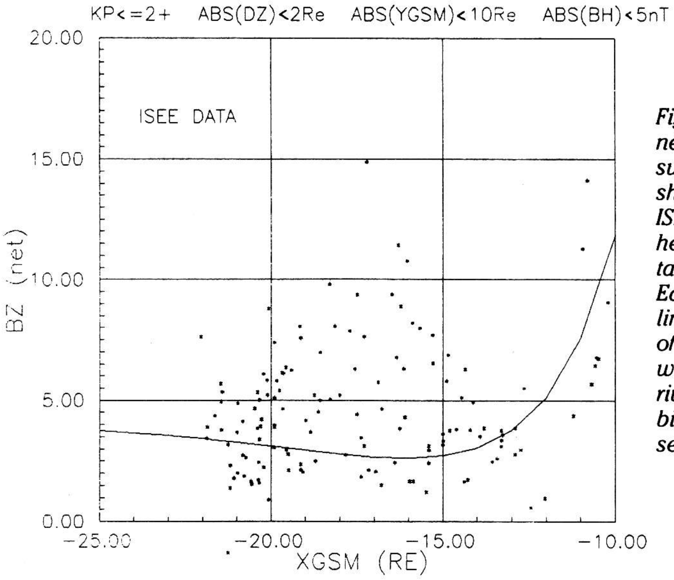

Fig. 1. Values of B_z near the equatorial surface of the plasma sheet, observed by ISEE-1 and plotted here against x, distance along the Sun-Earth axis. The solid line is the prediction of the T89 models, with coefficients derived from the combined IMP-HEOS-ISEE set.

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Availability

FORTRAN codes implementing the models, improved sets of coefficients based on the combined IMP/HEOS/ISEE data set, as well as a full list of Errata for the published versions of the models are available from either author of this article by e-mail until February 1994 over the NSI network, formerly SPAN, at DEC protocol lepvax::ys2nt, or (TCP/IP protocol) at ys2nt@lepvax.gsfc.nasa.gov. T87 codes with Stern's modifications are available at lepvax::u5dps. Coefficients for modified T87 can be requested from Mauricio Peredo at lepvax::xr2mp.

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observed values are spread from 2 nT to 8 nT.

This behavior is typical. Indeed, the published models display an even greater rms scatter, typically 12 nT (see row 3 in tables 1 and 2 in T87, and in Table 1 in T89]. These are contributed mainly by regions of stronger field (for example, synchronous orbit), where the absolute value of the fluctuation is larger than here. To be sure, this variability may be increased because all magnetospheric activity has been lumped into one index Kp; had we taken into account more and/or better indices, the scatter would probably have been smaller. But it would never vanish, for like atmospheric weather, the state of the magnetosphere reflects many small factors that cannot be tracked down in any feasible way.

(2) Models are only as good as the data on which they are based, and one should not expect accuracy in regions of few or no data points. Of the satellites used to construct models, only the two HEOS missions covered high latitudes, and their coverage was limited. The position of the high-latitude magnetopause given by T87 and T89 is therefore mostly an extrapolation, and such features as the north-south asymmetry of the model magnetopause for large values of the tilt angle ψ should not be taken seriously.

A more subtle example is the recently noted discrepancy in the tail's values of B_z between ISEE-1 observations $(-10 > x > -22)$ and values predicted by T87 and T89 [*Huang*, 1990, 1991; *Peredo and Stern*, 1991; *Tsyganenko et al.*, 1992; *Skone and Rostoker*, 1992]. These values turned out to be typically 6 nT, whereas T89 predicted ≈ 3 nT and T87 ≈ 4.5 nT. It turned out that the inclined orbits of the IMP satellites, on whose data the published models were based, produced a relatively sparse coverage of the equatorial region of the plasma sheet. The value of B_z in the tail decreases with increasing $|z|$, ultimately reversing sign to match the flaring of the flanks, and this, together with the sparseness of equatorial data points, might create a bias favoring lower values of B_z. A new set of T87 coefficients extracted from ISEE data (M. Peredo, D. P. Stern, N. A. Tsyganenko, Unpublished Manuscript, 1992) does in fact give B_z values that fit ISEE averages.

(3) If the model is supposed to faithfully represent a certain feature, it is generally essential to "build that feature into the model." Similarly, any factor omitted in constructing a model will have its effects smeared out.

For instance, a general polynomial is not capable of representing the thin sheets of Birkeland currents, so that whatever their signature is in T87 and T89, it is by necessity smeared out. Any good representation of Birkeland currents must be able to represent thin, field-aligned sheets. Similarly, any factor omitted in constructing a model will have its effects smeared out.

The specific representations of current systems by the various "modules" are just approximations, too, and may have built-in inaccuracies. Average tail values of B_z are known to be weaker near midnight than closer to the flanks [*Fairfield*, 1986], and in trying to represent this variation, T89 has underestimated B_z values near midnight. The solid line in Figure 1 is, in fact, the midnight trace of a T89 model, not the one used by C. Y. Huang that was found too low, but one calibrated against combined data from IMP, HEOS, and ISEE-1 [*Tsyganenko et al.*, 1992]. A corresponding trace from y=10 would give higher values, but the one at midnight, which was also used for comparisons by *Skone and Rostoker* [1992], is obviously too low even though the middle of the plasma sheet has been adequately represented in the dataset. Figure 2 shows the y-dependence of B_z at $x = -18\ R_E$, together with ISEE data points taken within $2\ R_E$ of that x, again, with $B_H < 5$ nT. Once more, it is difficult to see the trend because of the wide scatter. Researchers seeking models for field lines near midnight might prefer T87 with Peredo's new coefficients, but they must also be aware of the wide range covered by actual observations in this region, shown in Figure 1.

Outlook

With all their limitations, it is surprising that the models work as well as they do. For instance, the behavior of the near-Earth plasma sheet [*Lopez*, 1990] fits models rather well [*Peredo and Stern*, 1991]. Still, it must be remembered that this is an averaged behavior, not specific observations.

The published versions of both T87 and T89 fail to take into account the fluctuations of the solar wind momentum flux ("pressure") p. The effect of p is most pronounced on the day side and, as expected, values of **B** there fluctuate widely around model predictions [*Fairfield*, 1991]. Future models are expected to represent the magnetopause field by a scalar potential and could accurately reflect effects on the magnetopause due to p and also to IMF B_z, which were recently estimated by [*Sibeck et al.*, 1991; *Roelof and Sibeck*, 1992]. This method not only assures that the magnetopause field is curl-free in the interior, it also remedies the uneven magnetopause boundaries and the spuriously open field lines found in T87 and T89 (especially at large values of ψ), both traceable to the use of a polynomial.

The very idea of the scalar potential representation of the magnetopause field goes back at least to *Mead* [1964]. However, spherical harmonic expansions employed by Mead, later by *Choe and Beard* [1974], and recently by *Schulz and McNab* [1987] are good only within a limited region, $R<10R_E$, and therefore necessarily should be combined with a different representation in the tail (for example, *Voigt* [1981]). In this respect, a much better description can be expected from ellipsoidal harmonic expansions [*Tsyganenko*, 1989b] that provide a single compact representation for the scalar potential, valid from the subsolar region up to at least $50R_E$ down the tail. One convenient feature of the ellipsoidal model is that its parameters can be easily calibrated to represent the results of *Sibeck et al.* [1991], who fitted the average observed magnetopause shape by axisymmetrical ellipsoids. Other ways of representing the magnetopause field also exist, using parabolic harmonic expansions [*Stern*, 1985] or purely numerical schemes [*Spiro et al.*, 1992].

As limited as they are, T87 and T89 are much more detailed and flexible than earlier models, for example that of *Olson and Pfitzer* [1974]. As shown above, many of the shortcomings of these models stem from the use of an "all-purpose" polynomial to represent fields of the magnetopause and the Birkeland current system.

The latter is harder to represent, but a hopeful beginning exists (N. A. Tsyganenko, Unpublished Manuscript, 1992; D. P. Stern, Unpublished Manuscript, 1992). With modules for all four external systems, a realistic representation becomes feasible, and given appropriate supporting data (AE, p, IMF B_z, polar cap size, Dst) it might be possible to substantially reduce the mean error σ. Stay tuned!

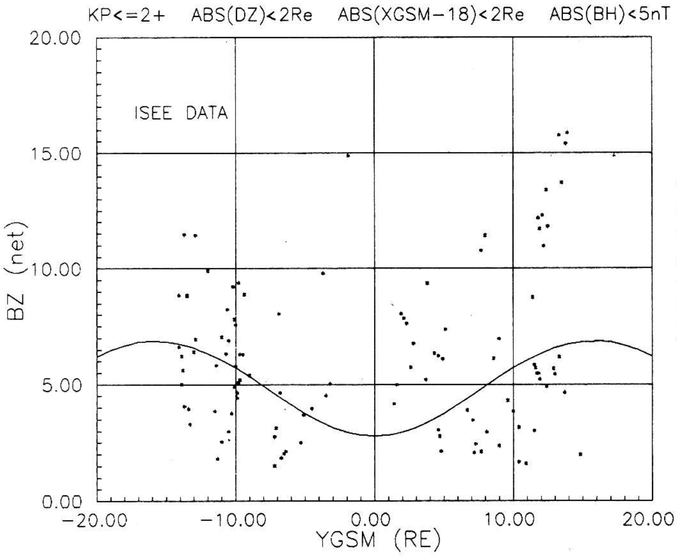

Fig. 2. Values of B_z near $x = -18\ R_E$, observed by ISEE-1 and plotted against the solar-geomagnetic dawn-dusk coordinate y. The solid line is as in Figure 1.

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