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GEOSPACE MAGNETIC FIELD: FROM EARLY CARTOONS TO DATA-BASED MODELS

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ABSTRACT

This paper reviews the data-based approach to modeling the distant magnetospheric magnetic field. Large amount of the spacecraft magnetometer data, accumulated since the beginning of the space era, allowed to develop a quantitative representation of the magnetospheric configurations, based on measurements made within a vast region extending to the Moon's orbit. The task of modeling the distant geomagnetic field is largely complicated both by its dependence on seasonal/diurnal changes of the Earth's dipole tilt angle and a significant variability caused by the solar wind fluctuations and magnetospheric substorms. Though widely used in various experimental and theoretical studies, the existing models provide only average and often too crude approximations for highly variable actual field. This review discusses the fundamentals of the next-generation models, including the problem of their calibration by the solar wind parameters, modeling the field from Birkeland currents, and a new approach to fitting models against data.

I. INTRODUCTION

Studies in the solar-terrestrial physics led to recognizing the role of the geomagnetic field as one of the most important characteristics of our environment. The Earth's magnetic field links the interplanetary medium with the upper atmosphere and the ionosphere, guides the energetic charged particles ejected during solar flares, channels the low-frequency electromagnetic waves and heat flux, confines the radiation belt and auroral plasma particles, and serves as a giant accumulator of the solar wind energy that eventually dissipates during the magnetic storms. Investigations of these phenomena are closely related to the problem of forecasting the "weather" in the near-Earth space which impacts many aspects of modern technologies and human activities.

For this reason, it is important to have reliable tools for computing the structure of the geomagnetic field, capable of taking into account the magnetospheric disturbance and other factors defining variations of the near-Earth electric current systems. In many applications it is often necessary to evaluate the components of the geomagnetic field vector in a wide range of distances, trace the field lines far away from the Earth's surface, calculate the geomagnetically conjugate points, and map a spacecraft position with respect to characteristic magnetospheric/ionospheric boundaries. This requires to use quantitative models of the Earth's distant magnetic field.

This paper overviews the data-based approach to modeling the geomagnetosphere. Its essence consists in collecting large amounts of magnetic field observations inside the magnetosphere, combined with simultaneous data on the state of the solar wind, and using this information to fit the parameters of appropriate mathematical forms representing the model field.

II. FUNDAMENTALS OF THE APPROACH

The first data-based models of the geomagnetic field were created by Gauss [1] still in the first half of the last century. At that time, nothing was known about the ionosphere/magnetosphere and the geomagnetic modeling was limited to the field at the Earth's surface, where $\nabla \times \vec{B} = 0$ and hence $\vec{B} = -\nabla U$ (which is nonetheless a good approximation even out to a few Earth's radii). Since the Earth's surface is very close to a sphere, the natural coordinate system is a spherical one, in which the scalar potential is represented as an expansion in spherical harmonics:

$$U = R_E \sum_{n=1}^{N} [\frac{R_E}{r}]^{n+1} \sum_{m=0}^{n} (g_n^m \cos m\lambda + h_n^m \sin m\lambda) P_n^m(\cos\theta) \qquad (1)$$

This is a mathematical model of the main geomagnetic field, produced by the dynamo currents flowing inside the Earth. The data base used by Gauss included only 84 vector values of $\vec{B}$, so he had to limit the summation at N=4. Nowadays, due to a world-wide and continuous coverage by ground-based, marine, airborne, and low-altitude satellite measurements, the values of the coefficients have been calculated up to $N \simeq 50$ [2], though standard models are usually truncated at N=10.

The main geomagnetic field is rigidly "tied" to the rotating Earth, and its models (i.e. expansion coefficients) are updated every 5 years, known as the IGRF (International Geomagnetic Reference Field) models [3].

If there were no electric currents of $\vec{B}$ outside the Earth, the total geomagnetic field throughout the whole 3D space would be uniquely represented by a scalar potential with coefficients determined from the ground-based data only. The field lines could then be traced from Earth to space without any measurements above the ground, taking the advantage of uniqueness of potential fields satisfying given boundary conditions.

However, due to the presence of hot magnetospheric plasma permeated by strong currents, the approximation $\nabla \times \vec{B} = 0$ breaks down even at relatively low altitudes, so that, beginning from $R \sim 3\text{–}6R_E$, the contribution from the extraterrestrial currents gradually overcomes the Earth's field, which is why we cannot do without space magnetometer data providing information on $\vec{B}$ distribution in three dimensions.

Modeling the magnetospheric field includes thus at least the following four tasks:

(1) Collect data and prepare the modeling data sets,

(2) Devise mathematical methods to represent the field,
(3) Find appropriate criteria of fitting a model to data and develop numerical algorithms for implementing these criteria,
(4) Once the model is fitted to the data, it should be tested for consistency with various physical constraints and checked against independent data (e.g., pressure balance at the boundary, particle precipitation patterns in the ionosphere, etc.)

Although this paper covers the empirical modeling only, a few words should be said about the alternative approach, employing the MHD numerical codes as a tool for simulating the real magnetosphere (e.g., [4]). One should be careful with directly comparing the results of both methods, just because they do quite different things. Data-based models try to be consistent with the observed field; however, they cannot fit the physics exactly, since they use simplified approximations for the distribution of the field sources and are based on data taken by a few spacecraft at different times and, hence, average intrinsically time-varying situations.

On the other hand, the MHD simulations try to be consistent with the physics, but are not always consistent with the nature because of inevitable limitations of the MHD approach, finite grid size, and related mathematical problems.

Hopefully, some day these two approaches will meet somewhere in the middle. As of today, we are still quite far from that goal.

After almost four decades of space research, we know that the distant geomagnetic field is a complex structure, confined within a boundary and gaining contributions from five major sources, which are (1) Earth's internal field, (2) the magnetopause current, (3) the tail current, (4) the ring current, and (5) the Birkeland current systems.

The above current systems largely differ in their properties, and each of them can be best represented by using its own coordinate system and its own set of functions. This is what we call a "modular approach": represent contributions from the principal current systems separately and then sum up the terms:

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_{\mathbf{Earth}} + \vec{\mathbf{B}}_{\mathrm{MP}} + \vec{\mathbf{B}}_{\mathrm{RC}} + \vec{\mathbf{B}}_{\mathbf{Tail}} + \vec{\mathbf{B}}_{\mathbf{Birk}} \tag{2}$$

The problem of quantitative modeling of the magnetospheric field is complicated by the high variability of the external currents due to
(1) external changes in the solar wind pressure, temperature, flow direction, and IMF,
(2) intrinsic rapid changes inside the magnetosphere due to instabilities, including substorms, which, in fact, are initiated by the external factors,
(3) The Earth's rotation and orbital motion, which cause periodic changes in the geodipole tilt angle with respect to the solar wind direction.

With regard to data, it implies that we need a large database with a good coverage not only in XYZ-space, but also with respect to the geodipole tilt angle (which means that both solstice and equinox periods should be represented), solar wind parameters, including the dynamical pressure and the interplanetary magnetic field, as well as geophysical indices quantifying the state of the internal magnetospheric currents (e.g., Dst and AE).

The "raw" magnetometer data cannot be directly used in the modeling studies, because they usually have excessively high resolution in space and time. The closely spaced observations on a given spacecraft pass are in fact highly correlated and cannot be considered as completely independent. Therefore, to avoid redundant and unmanageable multi-gigabyte data sets, one has to somehow consolidate the data [5].

For that reason, the raw data undergo the following procedures:

- crude automatic elimination of the data taken outside the magnetosphere, based on a simple model magnetopause,
- subtraction of the Earth's main field (IGRF),
- accurate visual inspection of the data plots and selection of good data intervals,
- averaging of the measurements over $\leq 0.5\, R_E$ segments,
- tagging the average data records with the simultaneous data on the solar wind (**B**, N, T, V) and geomagnetic indices (Kp, Dst, AE).

The last update of the modeling dataset [6] included 79,745 vector $\vec{B}$ averages taken by 11 spacecraft during 1966–1986. Fig. 1 shows the data point "cloud" corresponding to the database.

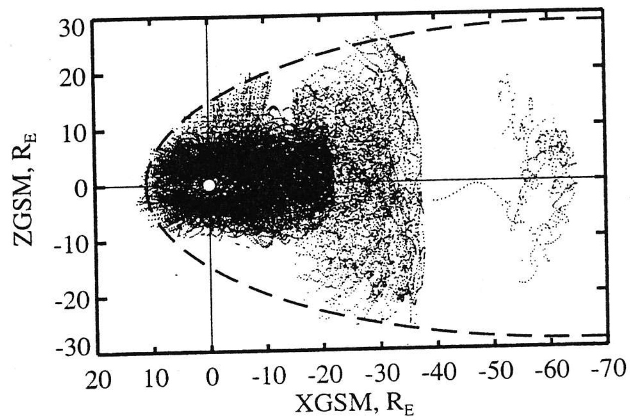

Figure 1. Distribution of the magnetospheric dataset "points", projected on the noon-midnight meridian plane. The average magnetopause position is shown by the broken line.

The data comprised by the observational database contain a lot of information on the average structure of the magnetosphere and its dependence on external conditions. How can that information be extracted ? As already mentioned, our approach is to develop realistic and flexible modules approximating the field from each source in (2) and then fit their parameters to the data. The first term $\vec{B}_{Earth}$ is represented by the IGRF models, and will not be discussed here. This paper deals with the rest four terms in (2), representing the contribution from four major extraterrestrial current systems, which are described below in more detail.

III. MODELING THE MAGNETOPAUSE FIELD

The field $\vec{B}_{MP}$ is produced by currents $\vec{j}_{MP}$ flowing in the thin boundary separating the solar wind plasma from the geomagnetic field. Their presence has a two-fold effect: (a) the corresponding outward electromagnetic force $\vec{j} \times \vec{B}$ balances the inward pressure exerted by the solar wind plasma, and (b) the field $\vec{B}_{MP}$, when added to the rest of the terms in (2), confines the lines of the total field within the boundary and is thus responsible for the very existence of the magnetosphere.

How can the field $\vec{B}_{MP}$ be determined and represented ? One possible approach, developed in [7–10], is not really data-based, but tries to derive both the shape and the field of the magnetopause from a crude model of the pressure-balanced boundary between the solar wind and the geomagnetic field. The resulting field is only given numerically, and this poses another problem, since we need compact analytical expansions, easy to apply to satellite observations.

A more appropriate way is to use direct observations of the magnetopause and approximate its average shape as a function of the pressure and IMF B_z [11,12]. The magnetopause field can be obtained as a solution of a boundary-value shielding problem for the scalar potential U, so that $\vec{B} = -\nabla U$ and

$$\begin{cases} \nabla^2 U = 0 \\ \dfrac{\partial U}{\partial n} = (B^{int} \cdot n)|_S \end{cases} \qquad (3)$$

This approach was used in the past by several authors [13–16]. However, until recently, only a few analytical solutions for this problem were known, limited to the case of shielding the Earth's dipole inside a few analytical boundaries.

In [17], the analytical solutions were extended to the case of the tail current shielding inside the ellipsoidal magnetopause, while in [18] a numerical solution was derived for more general boundary shapes. In our recent paper [19], an analytical model was developed, with the dipole, tail, and ring current fields, confined inside a realistic boundary by means of an approximate "least-squares" shielding method [20].

The latter approach is illustrated in Fig. 2 [19] showing the geodipole field confined inside a given boundary with a realistic shape and size. The configuration in Fig. 2 does not include the effects of the tail and ring currents. Owing to the linearity of the boundary problem (3), these terms can be treated separately, and we will discuss them in the next section.

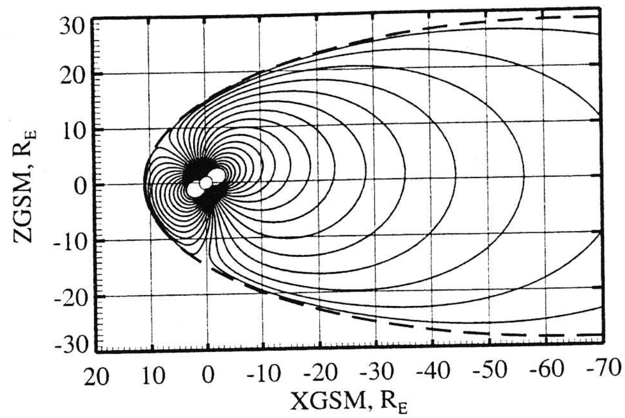

Figure 2. The lines of the Earth's dipole field, confined within a model magnetopause.

The "least-squares" shielding method allows to model the magnetopause field for a wide variety of observed boundary shapes, parametrized by the solar wind pressure and the Earth's dipole tilt angle, and can be extended in a straightforward way to represent open magnetospheric configurations with a controlled reconnection of the magnetic flux across the magnetopause.

IV. MODELING THE TAIL AND RING CURRENT FIELDS

Another problem, successfully solved during recent years [21–24] was the modeling of two major internal magnetospheric current systems: the tail current and the ring current. A principal difficulty was to satisfy two apparently incompatible requirements: the models should provide realistic and flexible current distributions and, at the same time, be described by simple and compact mathematical forms.

According to observations, the tail current is concentrated near the equatorial plane, with the $\vec{J}$ vector directed from dawn to dusk. The current density increases sunward, gradually blending with the ring current contribution and reaching maximal values at $R \simeq 5\text{–}7\,R_E$. The tail current sheet also warps and bends, in response to the Earth's dipole tilt. These are experimental facts which should be somehow matched in a model.

The essence of our approach is first to devise a family of simplest elementary, or "nucleus", solutions and then upgrade and superpose them, gaining more flexibility and realistic features, thus enabling the final model to closely match the observed electric current structures. In the tail field models [23] and [24], the original "nucleus" solutions were based on vector potentials, which ensures that the magnetic field remains divergence-free, whatever modifications of the initial potential were applied. In more detail, the basic "construction block" was defined as a simple axially symmetric vector potential having only one non-zero component in the azimuthal direction

$$\vec{A} = A(\rho, \phi, z)\,\vec{e}_\phi \tag{4}$$

corresponding to an infinitely thin equatorial current disk. A specific form of the function $A(\rho, \phi, z)$ is deduced from a simple solution of the equation $\nabla \times \nabla \times \vec{A} = 0$ for $z > 0$ and $z < 0$.

The following steps are then made to transform this "nucleus" solution into a realistic model for the tail magnetic field:

(1) spread out the initially thin current sheet over a finite thickness $D(x,y)$,

(2) superpose several sheets with different scales of the radial fall-off, in order to obtain a desired profile of the current distribution,

(3) replace Z by $Z' = Z - Z_s(x,y,\Psi)$, which allows one to model the tilt-dependent warping of the sheet in two dimensions,

(4) confine the field of the current sheet inside the model magnetopause by fitting the shielding potential. At this step, assuming $\vec{B} = 0$ outside the boundary interrupts the cross-tail current flow and closes it along the magnetopause, without violating Maxwell's equations. Essentially the same procedure is used for the derivation of a shielded ring current field.

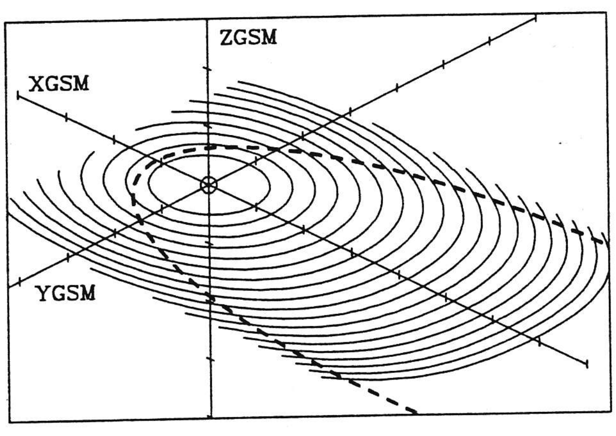

Figure 3. A perspective view of the electric current flow lines of the model tail/ring current [24]. The equatorial cross-section of the magnetopause is shown by broken line.

Fig. 3 shows a 3D view of the electric current flow lines in the model tail/ring current system [24], and Fig. 4 [19] displays the magnetic field lines of the tail current system, confined within the model magnetopause by using the "least squares" shielding method .

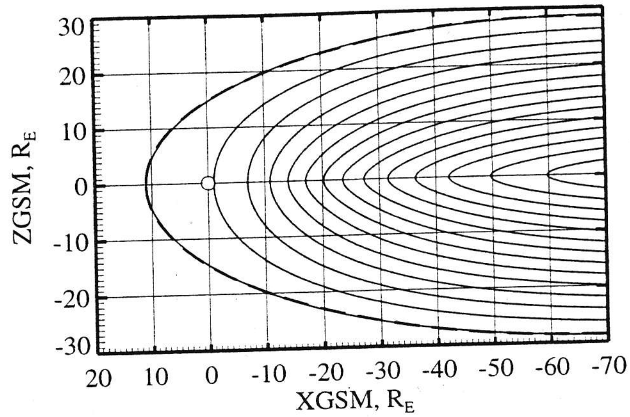

Figure 4. The lines of the field produced by the tail current system, confined within the model magnetopause [19].

V. MODELING THE FIELD OF BIRKELAND CURRENTS

Birkeland currents flow in and out of the auroral oval and link electrodynamically the magnetosphere with the ionosphere. According to [25], the total magnitude of Birkeland currents is comparable with that of other major current systems (a few MAmps), and hence they should significantly modify the magnetic field, at least during southward IMF.

However, there is no clarity as to where the Birkeland currents go in the magnetosphere: almost all of existing statistical data came from low-altitude measurements. In [26], an attempt to trace the Birkeland currents to the distant magnetosphere by using a large spacecraft database. A systematic shear of $\vec{B}$ vectors on crossing the plasma sheet boundary was found, consistent with the Region-1 current. The net field-aligned current in the near tail was estimated to approximately equal 500-700 kA, i.e. about one-third of the total Region 1 current measured at low altitudes, which is consistent with the mapping of the Iijima-Potemra [25] distribution to the magnetosphere along the model field lines.

Another problem in modeling the Birkeland currents is how to represent their magnetic field on a global scale. In principle, the field due to any current system can be computed by integrating the Biot-Savart equation, once the geometry of the current flow is known. However, the Biot-Savart integration is very slow and expensive, not to mention the need to take care of singularities of the integrand. And this is why simple analytical solutions are of great value. Unfortunately, the Birkeland currents have rather complex geometry, and it is difficult to find appropriate generic "nucleus" models for them. One of candidates could be a "cone" model [27], allowing a very simple analytical vector potential. In [28] and [29] analytical models for the field from Region 2 Birkeland currents were developed. Even under simplifying assumptions with regard to the current distribution, the obtained solutions are rather complex.

At present, a significant progress has been made in this area. A feasible way of handling the Birkeland field problem is to start with plausible configurations of thin current sheets and then approximate the field outside the current layer with an appropriate combination of scalar potentials [28].

VI. FITTING AND PARAMETRIZING MODELS

Having developed the mathematical "modules" for the field of the separate magnetospheric current systems, we can represent the net field as the sum:

$$\vec{\mathbf{B}}_{\mathbf{Total}} = \vec{\mathbf{B}}_{\mathbf{Earth}} + \vec{\mathbf{B}}_{\mathbf{MP}} + \vec{\mathbf{B}}_{\mathbf{Tail}} + \vec{\mathbf{B}}_{\mathbf{RC}} + \vec{\mathbf{B}}_{\mathbf{Birk}} \tag{5}$$

where each term on the right is a function of position in space $\vec{\mathbf{R}}$ and also depends on the Earth's dipole orientation, given by the tilt angle $\Psi = \Psi(UT)$. Furthermore, each term but the first one should contain a parametric dependence on the solar wind characteristics and can also be parametrized by routinely observed indices, reflecting the intensity of the magnetospheric currents. For example, the Dst-index can serve (with some caveats in mind) as a measure of the ring current magnitude, while the size of the polar cap should be closely correlated with the tail flux and hence can be used as a measure of the tail current.

The simplest way of parametrizing the model is to bin the data into several intervals of a parameter and then create a sequence of models, corresponding to the sequence of data subsets. That method was used in generating families of the earlier models [22,23], binned by the Kp-index.

In a recent modeling effort [19] that primitive approach has been abandoned: (1) instead of the Kp-index, the models are parametrized by the solar wind pressure, P_{dyn}, Dst-, and AE-indices, and (2) instead of binning the data and computing separate subsets of coefficients, we represent the magnitudes of the electric current in the model magnetospheric sources as continuous analytic functions of P_{dyn}, Dst, and AE, and make the least-squares fitting of the model in the 7-D space $\{X, Y, Z, \Psi, P_{dyn}, Dst, AE\}$.

The next important question is how to choose a criterion for fitting the model to data. The merit function to be minimized should be directly related to the physical quantity we need to define from the model. For example, if a global distribution of the particle gyrofrequency is needed, the only relevant parameter would be the scalar magnitude of the field, regardless of its vector direction. In contrast to this, in many applications of the geomagnetic field models, the most essential requirement is their ability to provide as accurate as possible field-line mapping between the ionosphere and the magnetosphere. In this case, what really counts is the direction of the magnetic field $\vec{\mathbf{b}} = \vec{\mathbf{B}}/B$.

In deriving the old models, the r.m.s. deviation of the full vector of the model field from data was defined as the merit function:

$$\sigma_1 = \sqrt{\sum_{i=1}^{N}[\vec{\mathbf{B}}^{(i)}_{obs} - \vec{\mathbf{B}}^{(i)}_{model}]^2 \Big/ N} \tag{6}$$

That criterion gave preference to the majority of the data points with large values of $|\vec{\mathbf{B}}|$. In particular, a prevalence of relatively large tail lobe fields over weak plasma sheet fields resulted in significant mapping inaccuracies.

The new criterion is focused on minimizing the directional discrepancy

$$\sigma_2 = \sqrt{\sum_{i=1}^{N} \left[\vec{\mathbf{b}}_{obs}^{(i)} - \vec{\mathbf{b}}_{model}^{(i)}\right]^2 / N} \tag{7}$$

which provides much more robust and realistic field configurations on the nightside. Fig. 5 shows a sample of the model field line configuration for a typical set of values of the solar wind pressure, Dst and AE.

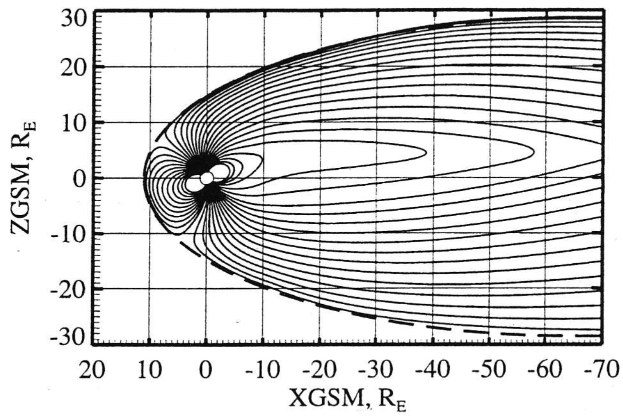

Figure 5. An example of the model magnetospheric field configuration for average solar wind conditions, obtained by using the "directional" fitting criterion [19].

VI. SUMMARY

A significant progress has been made during the last years in the development of new magnetospheric field models:

- A highly flexible, realistic, and simple new model was devised for the tail/ring current.
- Using the "least-squares" approach, the field from the magnetopause currents was constructed, confining the total field within a realistic boundary. This allowed to parametrize the magnetopause field by the solar wind pressure.
- A new "directional" criterion for fitting the model to data was developed, providing better accuracy of the field line mapping.
- First analytical representations for the field of Birkeland currents were developed.

- An extended spacecraft magnetometer data set for the modeling has been compiled, comprising about 80,000 vector averages measured during 1966-86.
- Based on the above achievements, a new magnetospheric field model was devised, parametrized by the solar wind parameters and geophysical indices.

Among the top priorities for future modeling efforts are the following problems:

- Finding more flexible analytical representations for the field of Birkeland currents.
- Incorporation of additional spacecraft data and the polar cap/auroral oval data for parametrizing the tail and field-aligned current modules.
- Devising quantitative models, capable of representing transient events, in particular, the substorm current wedge.

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