Planet. Space Sci., Vol. 30, No. 10, pp. 1007–1012, 1982
Printed in Great Britain.

0032–0633/82/101007–06$03.00/0
Pergamon Press Ltd.

ON THE CONVECTIVE MECHANISM FOR FORMATION OF THE PLASMA SHEET IN THE MAGNETOSPHERIC TAIL

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(*Received in final form* 1 *March* 1982)

Abstract—Calculation of stationary distributions of the most important plasma parameters (particle energy, density, field-aligned and transversal pressure) is performed for a model magnetotail plasma sheet which is formed by convecting plasma mantle particles injected into the closed geomagnetic field line tubes. Computations have been done for two convection models: (i) a model of completely adiabatic particle motion with conservation of the first two invariants and (ii) a model with a strong pitch-angle diffusion which maintains isotropy. It is found that in both cases the heating and compression of the plasma are somewhat more effective than is necessary to account for the observed gradients of magnetic field in the magnetospheric tail. A leakage of accelerated particles through the dawn and dusk edges of the plasma sheet is proposed as a possible mechanism for maintenance of stationary convection in the magnetotail. The question of the dependence of the stationary magnetotail parameters on the solar wind state is discussed briefly.

1. INTRODUCTION

The formation of the magnetospheric tail is one of the unsolved problems of space physics, though some progress has been achieved recently due to improved experimental technique. The most important advances were the discovery of the plasma mantle (Rosenbauer *et al.*, 1975; Sckopke *et al.*, 1976) and the first direct measurements of permanently existing large-scale electric field in the distant magnetospheric tail (McCoy *et al.*, 1975). These enable an explanation to be found for a plasma source which can maintain the observed particle population in the plasma sheet. A detailed study of the mantle plasma injection mechanism was made by Pilipp and Morfill (1978), who have shown that the total particle influx from the mantle can be as large as $\sim 10^{25}$–$10^{26}\,s^{-1}$, in good agreement with an order-of-magnitude estimate for the total loss rate.

The second aspect of the problem concerns further evolution of particles injected into the region of closed magnetotail field lines. Some features of the convecting particle acceleration and precipitation were investigated by Tsyganenko (1975), Tsyganenko and Zaitzeva (1979) and Kropotkin (1977). Erickson and Wolf (1980) in the framework of a simple isotropic compression model have concluded that steady convection is impossible in the magnetotail due to inconsistency between plasma pressure and the model magnetic field distributions.

In this paper we present another treatment of the problem, based on an adiabatic approach to particle dynamics and then compare the results with those obtained in a non-adiabatic limiting case.

2. FORMULATION OF THE PROBLEM

We assume that a major part of the mantle plasma is injected into the plasma sheet at $x_{GSM} \sim -100\,Re$. This estimate corresponds to typical mantle particle energy $w_o \sim 100\,eV$, trajectory-averaged electric and magnetic field magnitudes, respectively, $E \sim 3 \cdot 10^{-6}\,V \cdot cm^{-1}$, $B \sim 10\,\gamma$ and distant tail radius $R_t \sim 25\,Re$. After having been injected into the sheet, the particles perform oscillatory motion between mirror points, on which the earthward electric drift is imposed. Curvature and non-uniformity of the magnetic field in the plasma sheet cause particle energization at a rate which is energy- and pitch-angle dependent. The initial distribution function will thus change as the plasma convects toward the Earth. In particular, a pitch-angle anisotropy may arise, if there is no significant pitch-angle scattering. For an extremely intense wave-particle interaction it is necessary to take spatial diffusion into account and the approximation of convecting field line tubes is invalid. This situation, however, seems to occur only sporadically during the explosive phase of a substorm, because diffusion across the magnetic field in the presence of the electric field would be accompanied by a rapid and strong heating of the plasma with destruction of the stationary flow pattern.

Returning to stationary adiabatic convection, we point out that the net particle energization depends mainly on the large-scale magnetic field gradients, rather than on details of the reversal region structure. If we adopt the magnetic field model without any dependence on x_{GSM}-coordinate, the net drift displacement Δy (and, hence, the net energization) per bounce can be shown (Stern and Palmadesso, 1975) to be zero for any value of particle magnetic moment. Additional "beam" particles, which are necessary in this case for the current sheet (Cowley and Pellat, 1978) require a region with $\partial B/\partial x \neq 0$ to provide their mirroring and to ensure the balance of stresses in the whole configuration.

To investigate the question quantitatively, we formulate the problem as follows. Let us define a tail magnetic field model providing a distribution of **B** close to that observed experimentally. Having specified the initial velocity distribution function in the distant injection region, we shall calculate its evolution along the convection trajectory in the framework of a steady model, assuming the particles move adiabatically. In accordance with the above considerations, the plasma pressure and anisotropy at a given point of space depend on the whole distribution of the magnetic field throughout the tail. By substituting the plasma distribution obtained in the tension balance equations, we shall check the degree of consistency between the model magnetic field and that necessary to maintain the equilibrium.

Two limiting cases will be studied; in the first one we assume conservation of adiabatic invariants

$$\mu = w_\perp / B = \text{const} \quad (1)$$

$$I = \int (B_m - B(s))^{1/2}\, ds = \text{const} \quad (2)$$

In the second case a strong pitch-angle diffusion is implied to destroy anisotropy and a simple polytropic relation is adopted

$$p/p_o = (V_o/V)^\kappa \quad (3)$$

similar to that used by Erickson and Wolf (1980).

3. A MAGNETIC FIELD MODEL

The magnetic field model shown in Fig. 1 is appropriate for our purposes. The model is two-dimensional, since we consider only magnetotail region $x_{GSM} < -10\,Re$ with rather weak y-dependence (effects of finite dimension in the y-direction will be discussed later on). The tail model magnetic field has straight force lines with abrupt kinks at points of intersection with the equatorial plane. This unphysical feature is, however, unimportant for the present analysis, since it significantly alters the values of the second invariant (2) only for a small group of near-equatorial particles. In other words, particles are implied to move along realistic field lines, but for the sake of computational convenience we replace them by broken lines to avoid curvilinearity of the integration path in (2). As Fig. 1 shows, two bunches of magnetic field lines, in projection onto the plane $y = 0$, emerge from a pair of points with $x_0 = -1/A$ and $z_0 = \pm h_m$. A field line passing through a point (x, z) crosses the plane $x = 0$ at the height

$$h = \frac{z + Ah_m x}{1 + Ax}. \quad (4)$$

The quantity h defines uniquely a magnetic field line and hence, the following vector potential representation can be introduced

$$A = \{0, A_y(h), 0\}.$$

By an appropriate choice of the function $A_y(h)$ we obtain the desired profile of the magnetic field variation across the plasma sheet. With the choice

$$A_y(h) = k_1 h \quad (5)$$

the field is almost constant in magnitude and the major part of the current flows in a thin central region. Another choice

$$A_y(h) = k_2 h^2 \quad (6)$$

provides an increase of B with z in the near tail region, corresponding to a somewhat more uniform current distribution. Magnetic field components

$$B_x = \frac{\partial A_y}{\partial z} \quad B_z = -\frac{\partial A_y}{\partial x}$$

in explicit form can be obtained from (4) and (5) or (6). Replacing the field line parameter h by the corresponding equatorial crossing distance x^* (see Fig. 1)

$$h = \frac{Ah_m x^*}{1 + Ax^*},$$

we obtain a simple expression defining the field magnitude variation along a given force line, specified by a value of x^*

$$B(x, x^*) = \frac{C(x^*)}{1 + Ax}. \quad (7)$$

The explicit form of the function $C(x^*)$ is defined by a specific choice of the vector potential dependence on h (for instance, (5) or (6)). The model parameter A is thus an inverse of a characteristic variation scale of the magnetic field along the tail. The cross-section $x = 0$ corresponds in our model to a transition region between a typical tail-like magnetic field ($x > 0$) and a dipolar region ($x < 0$). Hence we may take $x \approx -x_{GSM} - 10\,Re$. Numerical values of the model parameters k_1 (or k_2), A and h_m should be specified basing on observed magnitudes of B, $\partial B/\partial x$ and of the plasma sheet thickness. Adopting (5) for $A_y(h)$ and taking $k_1 = -45\,\gamma$, $h_m = 5\,Re$, $A = 0.1\,Re$, we obtain a satisfactory general agreement with the experimental values of B in the high-latitude tail and B_z in the plasma sheet (Behannon, 1970), as can be seen from Table 1. Model values of B_x pertain to $z = h_m$ and B_z corresponds to $z = 0$; Behannon's results are given in brackets.

4. AN OUTLINE OF CALCULATIONS IN THE ADIABATIC PARTICLE MOTION LIMITING CASE

To obtain the plasma density and its parallel and transversal pressures, it is necessary to calculate at an arbitrary point the particle energy, w and the pitch angle, θ, from their values at a certain point of the initial field line tube delineating the outer boundary of the plasma sheet. The second invariant, with (7), can be rewritten as

$$I = (\cos\alpha)^{-1} \int_{x_m}^{x^*} [B(x_m, x^*) - B(x, x^*)]^{1/2}\, dx = \frac{C(x^*)}{A \cos\alpha B^{*1/2}}\, s(t), \quad (8)$$

where α is the angle between the force line and the x-axis, x_m is the mirror point coordinate, $B^* = B(x^*, x^*)$, $t = B_m/B^*$ and

$$s(t) = (t-1)^{1/2} - t^{-1/2} \ln(t^{1/2} + (t-1)^{1/2}). \quad (9)$$

Conservation of I leads to the equation

$$s(t) = D, \quad (10)$$

where

$$D = \frac{C(x_o^*) \cos\alpha}{C(x^*) \cos\alpha_o} \cdot \left(\frac{B^*}{B_o^*}\right)^{1/2} \cdot s(t_o) \equiv p \cdot s(t_0) \quad (11)$$

(subscripts O refer to a certain point of the initial field line tube). An approximate solution of the equation (10) can be obtained by noting that for small $\delta \equiv t - 1$ a power series expansion in δ for (9) gives

$$s(t) \approx (2/3)\delta^{3/2}$$

and hence,

$$t \approx 1 + (3D/2)^{2/3}.$$

In the limit of large t we have $s(t) \approx (t-1)^{1/2}$, so in this case $t \approx 1 + D^2$. The function

$$q(D) = 1 + (3D/2)^{2/3} + D^2$$

has the proper behaviour in both limits and as a numerical investigation has shown, gives an accurate approximation to the precise solution (maximum error about 10%). We have thus from (10)

$$\frac{w}{w_0} = \frac{B_m}{B_{mo}} = \frac{B^*}{B_o} \cdot \sin^2\theta_o \cdot q(ps(B_{mo}/B_o^*)) \quad (12)$$

and taking the conservation of $\mu = w \cdot \sin^2\theta/B$ into account,

$$\sin^2\theta = \frac{B}{B^*} \frac{1}{q(ps(B_o/B_o^* \sin^2\theta_o))}. \quad (13)$$

The field-aligned plasma pressure can be written as

$$P_\parallel = 2\pi \int_o^\infty \int_o^\pi (2w/m)^{3/2} f(w, \theta) \sin\theta \cos^2\theta\, dw\, d\theta. \quad (14)$$

Using the Liouville's theorem, we rewrite (14) in terms of the variables w_o and θ_o which refer to the

TABLE 1.

x_{GSM}, Re	−25	−35	−45	−55	−65
B_x, γ	18.0(16.0)	12.9(12.5)	10.0(10.5)	8.2(9.0)	6.3(8.5)
B_z, γ	3.60(3.5)	1.84(1.9)	1.11(1.0)	0.74(0.7)	0.53(0.55)

initial field line crossing the equatorial plane at $x = x_o^*$:

$$p_{\parallel} = 2\pi \int_o^\infty (2w(w_o, \theta_o)/m)^{3/2} f(w_o, \theta_o) \sin\theta \cdot \cos^2\theta \frac{D(w, \theta)}{D(w_o, \theta_o)} dw_o \, d\theta_o. \quad (15)$$

Let us assume isotropic pitch angle distribution in the initial injection region, i.e. $f(w_o, \theta_o) = F(w_o)$. This assumption can be substantiated by an argument that counterstreaming beams should exist in the injection region (Eastwood, 1975) leading to a rapid turbulent relaxation to isotropy.

From (12–13) we obtain the Jacobian for transformation between variables (w, θ) and (w_o, θ_o) and then an expression for $p_{\parallel}$, normalized to initial isotropic pressure p_o:

$$\frac{p_{\parallel}}{p_o} = 3 \int_o^{\pi/2} \sin\theta(\theta_o) \cdot \cos^2\theta(\theta_o) \frac{\partial w}{\partial w_o} \frac{\partial \theta}{\partial \theta_o} \cdot (w/w_o)^{3/2} \, d\theta_o. \quad (16)$$

In a similar manner, we have for transversal pressure and density, respectively

$$\frac{p_{\perp}}{p_o} = (3/2) \int_o^{\pi/2} \sin^3\theta(\theta) \frac{\partial w}{\partial w_o} \frac{\partial \theta}{\partial \theta_o} \cdot (w/w_o)^{3/2} \, d\theta_o \quad (17)$$

$$\frac{n}{n_o} = \int_o^{\pi/2} \sin\theta(\theta_o) \frac{\partial w}{\partial w_o} \frac{\partial \theta}{\partial \theta_o} \cdot (w/w_o)^{1/2} \, d\theta_o. \quad (18)$$

5. NUMERICAL RESULTS AND DISCUSSION

Figure 2 shows plots of stationary distribution of $p_{\parallel}/p_o$, $p_{\perp}/p_o$ and n/n_0, computed from (16–18) for small values of z, i.e. just above the central region of the plasma sheet. The degree of pressure anisotropy increases monotonically with decreasing geocentric distance, so that $p_{\parallel}$ more and more prevails over $p_{\perp}$. In the near-tail region ($x_{GSM} \sim -20$ *Re*) we obtain $p_{\parallel}/p_{\perp} \sim 3$–$4$, the result being relatively independent of the value of A defining the rate of the magnetic field variation along the tail, as well as of the current distribution across the sheet defined by a choice of function $A_y(h)$. The Fermi acceleration seems thus to prevail in the magnetotail over the betatron mechanism. This is in agreement with our previous results (Tsyganenko, 1975) concerning the evolution of particle distribution function in the plasma sheet. A similar result has been obtained also by Yamamoto and Tamao (1978). At closer distances ($x_{GSM} > -20$ *Re*) the vertical component B_z grows more and more steeply towards the Earth, which leads eventually to predominance of the betatron acceleration in the region of dipolar magnetic field lines (Cowley and Ashour-Abdalla, 1975) and hence, to the opposite type of pressure anisotropy with $p_{\perp} > p_{\parallel}$.

For estimating the degree of inconsistency between the calculated particle pressure gradients and anisotropy and the assumed magnetic field model distribution, we note that the plasma sheet current flowing in its central region provides a contribution to the magnetic energy density (e.g. Pudovkin and Tsyganenko, 1973)

$$B_{ps}^2/8\pi = (p_{\parallel} - p_{\perp})/2 \quad (19)$$

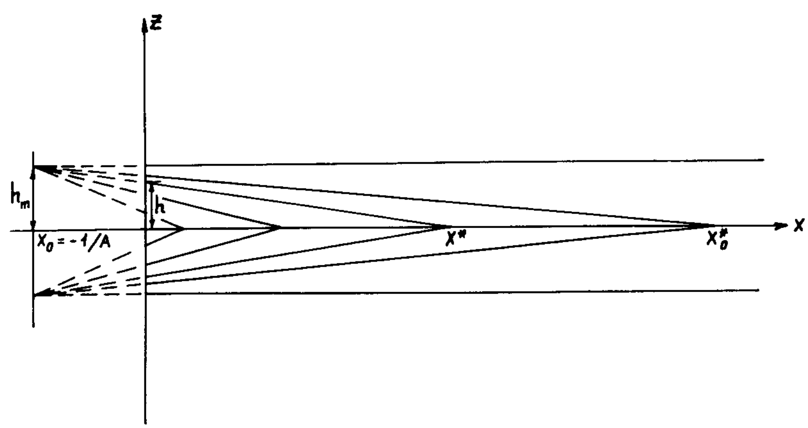

FIG. 1. A SKETCH OF A MODEL MAGNETIC FIELD LINES.
The cross-section $x = 0$ corresponds to $x_{GSM} \sim -10$ *Re*.

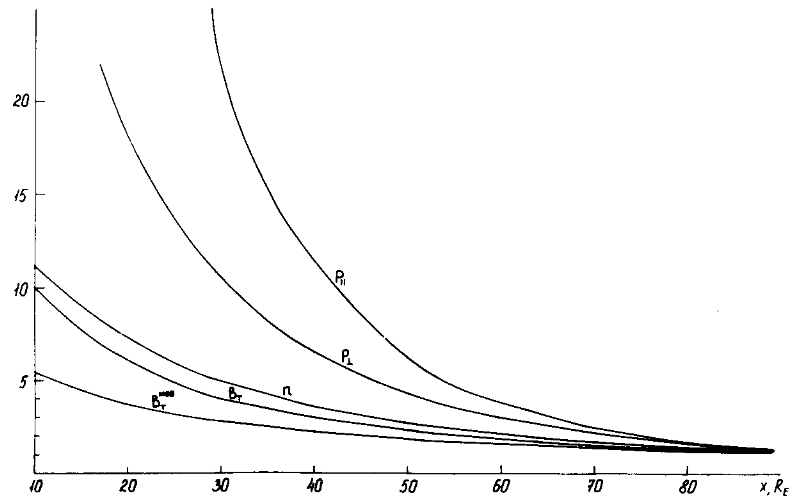

FIG. 2. STATIONARY PLASMA PRESSURE AND DENSITY DISTRIBUTIONS COMPUTED FOR THE CASE OF COMPLETELY ADIABATIC PARTICLE MOTION.

Quantities $p_\parallel$, $p_\perp$ and n are normalized to their values at $x^* = 100$ *Re* and correspond to points adjacent to the central region of the plasma sheet. Two curves below are the model magnetic field (B_t^{mod}) and that required by the tension balance equations (B_t).

Combined with the transverse pressure balance equation, this yields

$$B_t^2/8\pi = B_{ps}^2/8\pi + p_\perp = (p_\parallel + p_\perp)/2$$

The variation of the normalized quantity

$$B_t/B_{to} = [(p_\parallel + p_\perp)/2p_o]^{1/2} \qquad (20)$$

along the tail is shown in Fig. 2 together with the model magnetic field curve. The calculated equilibrium magnetic field increases somewhat more rapidly than the model curve, in agreement with the result of Erickson and Wolf (1980), obtained in the limit of isotropic plasma compression.

It is of interest to investigate this simple limiting case in application to our magnetic field model. Using (3) and an obvious equation for the volume of convecting field line tube

$$\frac{V(x^*)}{V(x_o^*)} = \left(\int \frac{dx}{B(x^*, x)} \bigg/ \int \frac{dx}{B(x_o^*, x)}\right) \frac{\cos\alpha_o}{\cos\alpha} \qquad (21)$$

we have obtained a result shown in Fig. 3 in the same format as in Fig. 2. A close resemblance of the $B_t = (8\pi p)^{1/2}$ curve with a corresponding B_t curve of Fig. 2 shows that the discrepancy mentioned is universal in the sense of being independent on the particle motion model.

Before making a definitive conclusion on the applicability of the steady convection model to the Earth's magnetotail, it is necessary to estimate possible particle losses. In our opinion, losses through lateral boundaries of the plasma sheet are of importance for the maintenance of the magnetotail stationarity. To illustrate this statement quantitatively, let us estimate the net particle displacement in the dawn-dusk direction, Δy, resulting from its azimuthal drifts. Assuming $B \gg B^*$ in (12–13) (as we consider only current-carrying "beam" particles with small pitch angles) and taking into account asymptotical behaviour of functions s and q, we find for the model version (6)

$$w/w_o \approx (B^* x_o^*)/(B_o^* x^*)$$

and thus

$$\Delta y = \Delta w/eE = (w_o/eE)\left(\frac{B^* x_o^*}{B_o^* x^*} - 1\right). \qquad (22)$$

Taking $x_o^* = 100\,Re$, $x^* = 10\,Re$ ($x_{GSM} \sim -20\,Re$), $E = 3.10^{-6}\,V\cdot cm^{-1}$, $w_o = 1\,keV$, we obtain $\Delta y = 20\,Re$. This value is of the order of the tail radius and hence, a considerable part of the mantle plasma with $(w_o/eE) \gtrsim 3\cdot 10^8$ cm will be lost before reaching the near magnetotail region. A steady state can be maintained by means of a self-regulation mechanism which ensures a decrease (or increase) of the plasma injection rate and an increase (or decrease) of losses when an inconsistency between plasma pressure and the magnetic field distribution occurs. An increase of the plasma mantle particle flux should cause an increase of the magnetotail current and hence, an

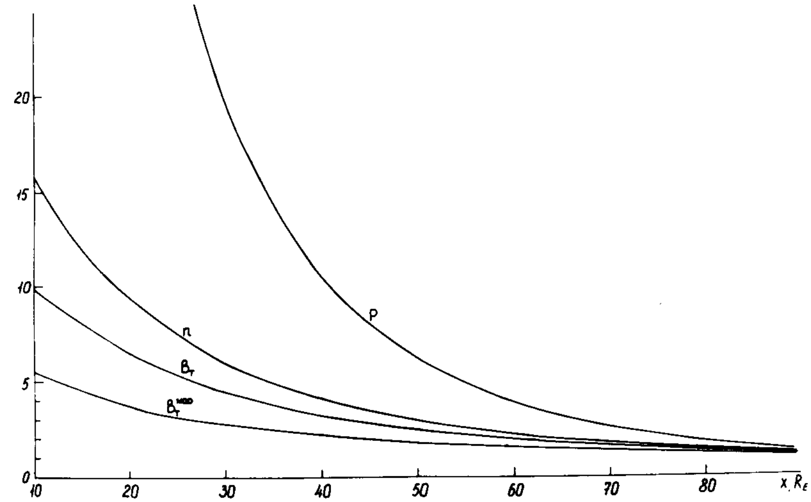

FIG. 3. PRESSURE AND DENSITY DISTRIBUTIONS FOR A COMPLETELY ISOTROPIC COMPRESSION MODEL ($P_{\parallel} = p_{\perp} = p$). Two curves below have the same meaning as those in Fig. 2.

earthward shift of the neutral line separating open and closed magnetotail field lines. This should result in a considerable decrease of the injection rate into the plasma sheet, because more plasma will be ejected tailward beyond the neutral line. Lesser values of the plasma sheet thickness must be observed in this situation. In contrast in the case of a rather weak electric field or a low particle flux from the plasma mantle or both, we should expect a decrease in B_t and a thicker plasma sheet, in agreement with observation. Sergeev *et al.* (1979) have reported a significant poleward shift of auroral activity related to high-speed solar wind streams as well as enormously large values of the plasma sheet thickness. These features can be explained by a tailward shift of the injection region due to enhanced field-aligned velocity of the plasma mantle particles.

Acknowledgement—I thank Dr. V. S. Semenov for stimulating and helpful discussions.

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