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Multiple particle production in p+A and d+A collisions in the framework of the modified Glauber model

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In this article we present the results of further development of the Monte Carlo modified Glauber model [1], [2] for proton-nuclear and deuteron-nuclear collisions. Our model differs from the Standard Glauber model by effectively accounting for energy losses in successive nucleon-nucleon collisions; we also account for the associated decrease in the inelastic nucleon-nucleon interaction cross section. The case of p+A collisions appears to be the most illustrative in terms of number of nucleons-participants defined with the account of energy-momentum conservation in multiple particle production vs. the Standard Glauber approach. We argue that the obtained results demonstrate that widely used application of the Standard Glauber model in the analysis of experimental data of multiple particle production in high energy hadron collisions should be reconsidered.

1. INTRODUCTION

In experiments on p+Pb collisions at LHC energies (ALICE), as well as for p+Au collisions at SPS energies (NA35) [3] and d+Au collisions at RHIC energies (PHOBOS) [4], values of charged-particle pseudorapidity density at midrapidity normalized to the number of participants, calculated with the Glauber model, turned out to be smaller then those for pp ($p\bar{p}$) collisions [5]. And the reason for this lies in the widely used approach of normalizing the experimental data on some the theoretically obtained numbers, calculated under certain assumptions.

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Since the knowledge of the initial conditions of high energy nucleus-nucleus collisions is important for the physics analysis, the events could be ideally characterized by the centrality of the collision and the corresponding impact parameter b. Nuclei are extended objects with a spatial distribution of the nucleon density, and the account of geometrical property of the collision could be easily taken by the value of b, thus providing the value of interaction volume and the initial energy density reached in the event. However, the impact parameter b cannot be measured, and instead, a number of observables are being used by the experimental HEP community as some proxies for collision centrality.

The most widely used observable is the multiplicity of charged particles (N_{ch}) measured in some acceptance of the installation. In this case, a two-component model [6] is used to approximate the number of events by multiplicity, where it is assumed that the number of charged particle sources depends on some factor *f*, which is responsible for the relative contributions of soft and hard processes in particle production.

$$< N_{sources} >\sim (1-f)\cdot N_{part} + f\cdot N_{coll}. \tag{1}$$

The model assumes that with the growing energy of collisions the number of binary nucleon-nucleon collisions N_{coll} will dominate as the main type of charged particle producing sources in «hard»processes of collisions (it is assumed that the factor *f* will have the tendency to approach the value of 1.0 with increase of energy). The remaining fraction of multiplicity $(1-f)$ being from «soft»processes proportional to N_{part}.

The use of the Glauber Monte Carlo model [7] in combination with two-component model [6] to describe the event distribution on N_{ch}, is being used to distinguish classes by multiplicity and to determine for each of them the corresponding average values of $< N_{part} >$, $< N_{coll} >$ and the impact parameter $< b >$ (see [8]).

The correct calculation of these quantities is an important task, since, for example, the nuclear modification factor (R_{AA}), which contains normalization by the number of binary nucleon-nucleon collisions N_{coll} , is widely used to study collective effects in nucleus-nucleus collisions:

$$R_{AA} = \frac{d^2N^{AA}_{ch}/dp_t d\eta}{< N^{AA}_{coll} > d^2N^{pp}_{ch} d^2N^{AA}_{ch}/dp_t d\eta} \tag{2}$$

It also gave rise to the investigation of simpler systems - proton-nuclear and deuteron-nuclear collisions, in which the relation between the number of participating nucleons and the number of binary nucleon-nucleon collisions is trivial.

In case of p+A collisions the factor R_{pA} is introduced

$$R_{pA} = \frac{d^2N_{ch}^{pA}/dp_t d\eta}{< N_{coll}^{pA} > d^2N_{ch}^{pp} d^2N_{ch}^{pA}/dp_t d\eta} \quad (3)$$

It is important to mention that for p+A case the number of binary collisions $< N_{coll}^{pA} >$ and number of N_{part} are related as:
$< N_{coll}^{pA} >=< N_{coll} > = < N_{part} > -1$. However, it should be remembered that the Glauber Monte Carlo and the two-component model [6], both contain a number of assumptions. The most important ones are the following: the value of the inelastic nucleon-nucleon scattering cross section σ_{inel}^{NN} is fixed for all binary nucleon-nucleon collisions; the multiplicity of particles produced in binary collisions also does not change on average; the energy losses of nucleons in particle production processes are not taken into account. In this paper, we give below an extension of the modified Glauber model (MGM) [1], [2] and [9] to explain the N_{part} scaling deviation effect in nucleon-nucleus and proton-nucleon collisions at the LHC energies.

In Chapter 1 we briefly review the Monte Carlo (MC) implementation of the standard Glauber method (SGM) as a base used for our modification, and in Chapter 2 we give a description of the Modified Glauber MC method (MGM). Chapter 3 presents the results obtained using the different approaches and their comparison, as well as and discussion of the observed effect of the deviation of the normalized multiplicity yields from scaling with the number of nucleon-participants N_{part}. We argue in the Conclusions that the application of the Standard Glauber model in the analysis of the experimental data on multiple particle production in high energy hadron collisions should be reconsidered.

2. STANDART GLAUBER MODEL

We start from a brief description of the MC code that was developed in our work for the Standard Glauber model (SGM) as a base for modifications and further analysis of high energy nucleus-nucleus and proton-nucleus collisions. The software implementation of the Standard Glauber Model can be divided into 2 parts: (i) generation of colliding system configuration of nucleons and (ii) modeling of inelastic binary nucleon-nucleon collisions with calculations of multiplicity of charged particles (N_{ch}) produced in the collision.

2.1. Generation of colliding system configuration and modeling of nucleon collisions

In this paper we consider p+Au, p+Pb, d+Au collisions. For each event with some random value of the centrality parameter b in the model, the spatial coordinates of nucleons are generated according to the nuclear density distribution in two colliding nuclei. The Fermi function with two parameters (see in in Table 1) is used for the nuclear density distribution for Au and Pb nuclei :

$$\rho(r) = \frac{1}{1 + exp(\frac{r-R}{a})} \tag{4}$$

The case of deuteron-nucleus collisions is considered in a simplified approach, where the deuteron is represented as two nucleons located at a distance $r = 2$ fm. It is important to note that in such collisions different spatial orientations of deuterons are considered since the system is not spherically symmetric.

2.2. Modeling of binary nucleon-nucleon collisions with calculation of multiplicity

In the standard Glauber model, successive collisions of nucleons are considered as those of some baryon-like structures moving toward each other, where in the events of their inelastic binary interactions the momentum and direction of motion of the nucleons do not change. It is also considered in the SGM that in these successive nucleon-nucleon collisions neither the values of the inelastic cross sections nor the momentum of the nucleons change, and after the collision the nucleons (or baryonic structures) continue to move rectilinearly in the same direction as originally. In the SGM, the cross section of an inelastic nucleon-nucleon collision is initially fixed according to a given energy in the center-of-mass system ($\sqrt{s}$) [10]. Then, for a given impact parameter of the collision of two nuclei, for each nucleon of the first nucleus, the number of nucleons of the second nucleus that fall into the interaction region with the first nucleus is counted.

A nucleon is said to enter the interaction region if the impact parameter for these nucleons does not exceed some value [9] :

$$b < \sqrt{\frac{\sigma_{NN}}{\pi}}. \tag{5}$$

Here and later in the text, σ_{NN} refers to the inelastic nucleon-nucleon cross section. Thus, accounting for all inelastic binary nucleon-nucleon collisions makes it possible to count the number of binary collisions N_{coll}. The number of nucleon-participants N_{part} in the model is defined as the number of nucleons that have experienced at least one collision and it is also counted.

In the SGM he total average value of the number of charged particles ($< N_{ch} >$) produced in a nucleon-nucleon collision at a given impact parameter b and a given collision energy $\sqrt{s_{NN}}$ is the sum of the individual contributions of mean multiplicity of charged particles $< N_{ch}^{pp} >$ produced in each inelastic binary nucleon-nucleon collision. The value of $< N_{ch}^{pp} >$ is calculated for a certain value of $\sqrt{s_{NN}}$ using the formula [11]:

$$< N_{ch}^{pp} >= a + b \cdot ln(s) + c \cdot ln^2(s), \;\; a = 16.65, \;\; b = -3.147, \;\; c = 0.334 \tag{6}$$

Since in this direct approach the final value of the number of produced charged particles $< N_{ch} >$ turns out to be extremely overestimated, a two-component model is used to approximate the shape of the events multiplicity distribution [6].

3. MODIFIED GLAUBER MODEL

The difference of the Modified Monte Carlo Glauber model (MGM) from the SGM is that for successive nucleon-nucleon collisions the energy and momentum conservation laws are considered in the explicit way. In order to take into account the energy losses for particle production, the parameter k, which is unique for all nucleon-nucleon collisions, is introduced in this paper similarly to [1], [2], [9]. This parameter corresponds to the fact that the nucleon in any binary collision loses $(1 - k)$ part of the momentum in the system of the center of mass of each pair of colliding nucleons.

3.1. Calculation of momentum change in binary collisions

The order of collisions of nucleons is determined by the distances between them, i.e., the nearest nucleons collide first. Two nucleons are said to interact if their impact parameter $b < \sqrt{\frac{\sigma_{NN}}{\pi}}$, $\sigma_{NN} = a + b \cdot ln^n(s)$, parameters: $a = 28.84 \pm 0.52$, $b = 0.05 \pm 0.02$, $n = 2.37 \pm 0.12$ are taken from [5].The first collision is calculated as follows:

1. The initial momentum of the nucleons:

$$P_1 = -P_2 = \sqrt{(\frac{\sqrt{s}}{2})^2 - m^2} \tag{7}$$

   where P_1, P_2 are the impulses of nucleons moving in opposite directions;

2. Impulses after the collision:

$$P_1^{,} = k \cdot P_1, \ P_2^{,} = k \cdot P_2 \tag{8}$$

In further collisions the momentum recalculation is carried out as follows when $P_1^{,} > P_2^{,}$ (otherwise the variables are renamed):

1. Nucleons energies:

$$E_1 = \sqrt{(P_1^{,})^2 + m^2}, \ E_2 = \sqrt{(P_2^{,})^2 + m^2} \tag{9}$$

2. Energy per pair of nucleons and energy of each nucleon in the center-of-mass system (CM). of mass (CM):

$$\sqrt{s} = \sqrt{(E_1 + E_2)^2 - (P_1^{,} + P_2^{,})^2}, \ E_{CM} = \frac{\sqrt{s}}{2} \tag{10}$$

3. The momentum modulus of the nucleons in the center-of-mass system:

$$P_{CM} = \sqrt{E_{CM}^2 - m^2} \tag{11}$$

4. Recalculating momentum:

$$\beta = \frac{P_{CM} \cdot E_1 - E_{CM} \cdot P_1}{P_{CM} \cdot P_1 - E_{CM} \cdot E_1}, \ P_{CM}^{new} = k \cdot P_{CM}, \ E_{CM}^{new} = \sqrt{(P_{CM}^{new})^2 + m^2} \tag{12}$$

$$P_1^{new} = \frac{P_{CM}^{new} + \beta \cdot E_{CM}^{new}}{\sqrt{1 - \beta^2}}, \ P_2^{new} = \frac{-P_{CM}^{new} + \beta \cdot E_{CM}^{new}}{\sqrt{1 - \beta^2}} \tag{13}$$

where P_{CM}^{new} is the momentum modulus in CM after the collision, E_{CM}^{new} is the energy of nucleons in CM after the collision, P_1^{CM} collision, P_1^{new}, P_2^{new} are the momenta of the nucleons after the collision.

In our calculations it was assumed that the nucleons whose momentum changes its sign to the opposite one due to the momentum recalculation do not participate in the subsequent inelastic collisions.

3.2. Finding the number of participating nucleons, the number of binary collisions and the number of produced charged particles

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For a given impact parameter, the mean numbers of participating nucleons ($< N_{part} >$) and of binary nucleon-nucleon collisions ($< N_{coll} >$) are determined in the Monte Carlo model by direct counting during successive collisions of each nucleon.

To determine the number of produced charged particles (N_{ch}) in each binary collision, $< N_{ch}^{pp} >$ is determined according to formula (5) for the corresponding energy in the center-of-mass system of the pair of given colliding nucleons. Then all obtained values are summed up, and we obtain the total number of produced charged particles.

4. RESULTS

4.1. p+A collisions

In our calculations with SGM and MGM models for the number of participating nucleons $< N_{part} >$ as a function of centrality, we obtained for p+Au collisions at $\sqrt{s_{NN}} = 200\ GeV$ for p+Pb at $\sqrt{s_{NN}} = 5023\ GeV$ the following results shown in Fig. 1a and Fig. 1b.

We can see a significant difference in the determination of the number of N_{part} by these two models. We have to note that in proton-nucleus collisions the numbers of N_{coll} and N_{part} are closely related ($N_{coll} = N_{part} - 1$). Therefore, the value of R_{pA} (see eq.3) will vary significantly from the SGM to the MGM due to the very different approach to the account of energy-momentum conservation in multiple particle production in collisions.

A similar comparative analysis, including evaluations against alternative MC models such as HIJING and the Non-Glauber Monte-Carlo approach, has been conducted in prior studies [12]. These MC event generators include the proper account of energy-momentum conservation in multiple particle production in A+A and p+A collisions and they are quite successful in describing the multiplicity yields and some other observables measured in high energy experiment. They also demonstrated in proton-nucleus interactions at high energies that the Standard Glauber model tends to considerably overestimate the values of numbers of binary collisions N_{coll}.

4.2. d+A collisions

We also calculated in these two models the number of nucleons-participants N_{part} and number of binary nucleon-nucleon collisions N_{coll} as a function of the impact parameter for deuteron-nucleus collisions. Results of MC calculations in SGM and MGM of dependence of the numbers of N_{part} and N_{coll} on centrality in d+Au collisions at the energy $\sqrt{s_{NN}} = 200\ GeV$ are shown in Fig. 2a and Fig. 2b.

One can see, that the standard Glauber model again gives here in d+Au collisions at $\sqrt{s_{NN}} = 200\ GeV$ the overestimated results both in N_{coll} and in N_{part} in the impact parameter region below 6 fm.

We used the same single parameter k=0.1 in our calculations of numbers of N_{part} and N_{coll} for different types of collisions (p+A, d+A) at RHIC energy. As we see from the results in Fig. 2a , the difference of d+Au with p+Au collisions at $\sqrt{s_{NN}} = 200\ GeV$, shown in Fig. 1a, is in the increased number of N_{part}. This is quite natural in view of weekly bound deuteron represented as 2 nucleons situated at the distance 2 fm.

This value of $k = 0.1$ used in the calculations in Fig. 1a , Fig. 2a, and Fig. 2b, is the one selected earlier in the [9] when fitting the data on the total multiplicity in Au+Au collisions at $\sqrt{s_{NN}} = 200\ GeV$. It is important that we also see here in Fig. 3, that the data obtained by PHOBOS [4] in d+Au collisions at the same $\sqrt{s_{NN}} = 200\ GeV$ on total charged particles multiplicity $< N_{ch} >$ normalized by the number of pairs of participating nucleons $1/2N_{part}$, are well described by the MGM with parameter $k = 0.1$.

5. CONCLUSION

- Judging from the results obtained previously in [1], [2], [9] and in this work, the MGM model, that takes into account the energy-momentum conservation in multiple particle production, gives a good agreement with the experimental data. For our analysis of different data on total charged particles multiplicity in p+Au, d+Au, Au+Au and Pb+Pb collisions, obtained at the same energy $\sqrt{s_{NN}}$, we used in our model one and the same single parameter k. This parameter k was defined previously by fitting the measured in A+A collisions total charged particles multiplicity values $< N_{ch} >$ normalized by the number of pairs of participating nucleons $1/2N_{part}$. It was found that it weakly depends on the $\sqrt{s_{NN}}$, changing from k= 0.2 at LHC energies to $k =$ 0.1 at RHIC.

- The results of applying the Modified Glauber model show smaller values of $< N_{part} >$ and $< N_{coll} >$ for the p+A case in comparison with the Standard Glauber model due to effective accounting of energy loss in the process of nucleon-nucleon collisions, which is confirmed by other MC calculations [12];

- Contrary to Standard Glauber model, the results also show a significant effect of “stopping" of nucleons (i.e losing the possibility of inelastic collision) in p+A and d+A collisions, both at the LHC, at RHIC energies and below;
- Calculations of the nuclear modification factors R_{AA} and R_{pA} in the case of dominance of soft processes require revision.

6. ACKNOWLEGMENTS

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7. CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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MULTIPLE PARTICLE PRODUCTION IN P+A AND D+A COLLISIONS IN THE FRAMEWORK OF THE MODIFIED GLAUBER MODEL

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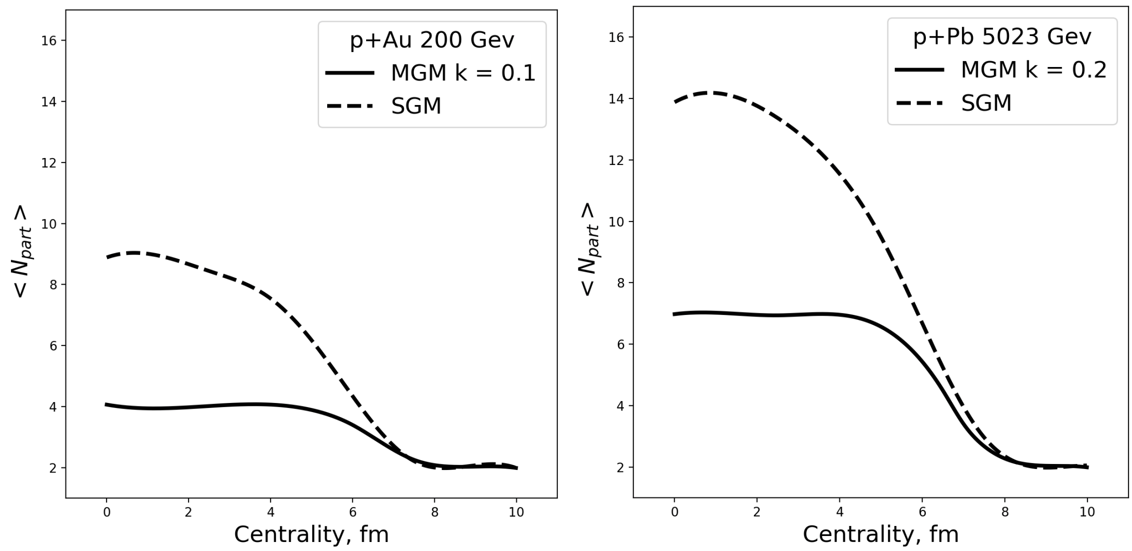
In this article we present the results of further development of the Monte Carlo modified Glauber model [1], [2] for proton-nuclear and deuteron-nuclear collisions. Our model differs from the Standard Glauber model by effectively accounting for energy losses in successive nucleon-nucleon collisions; we also account for the associated decrease in the inelastic nucleon-nucleon interaction cross section. The case of p+A collisions appears to be the most illustrative in terms of number of nucleons-participants defined with the account of energy-momentum conservation in multiple particle production vs. the Standard Glauber approach. We argue that the obtained results demonstrate that widely used application of the Standard Glauber model in the analysis of experimental data of multiple particle production in high energy hadron collisions should be reconsidered.

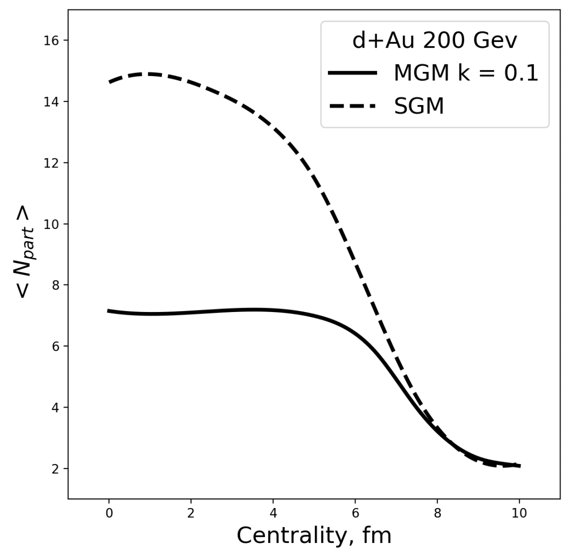
Table 1: Parameters of the Fermi function for Au and Pb nuclei

	Lead (Pb)	Gold (Au)
R, fm	6.62	6.38
a, fm	0.546	0.535
ρ_0, fm^{-3}	0.161	0.1695

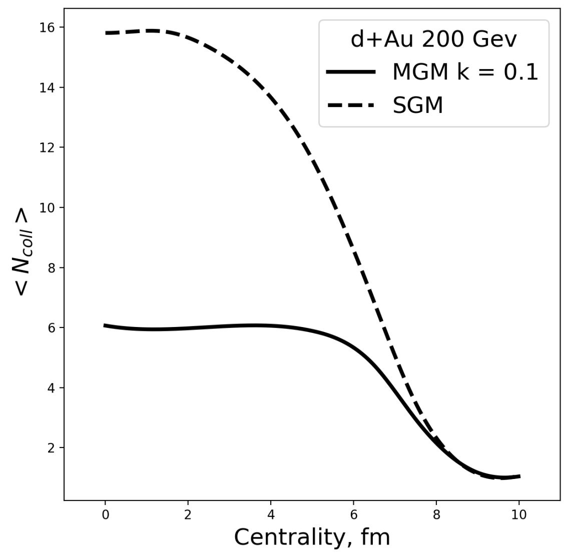


(a) **(b)**

Figure 1: Dependence of the mean number of participating nucleons on centrality in p+Au collisions at $\sqrt{s_{NN}} = 200\ GeV$ (a) and in p+Pb collisions at $\sqrt{s_{NN}} = 5023\ GeV$ (b) in SGM and MGM



(a)



(b)

Figure 2: Dependence of the mean number of participating nucleons (a) and binary nucleon-nucleon collisions (b) on centrality in d+Au collisions at $\sqrt{s_{NN}} = 200\ GeV$ in SGM and MGM

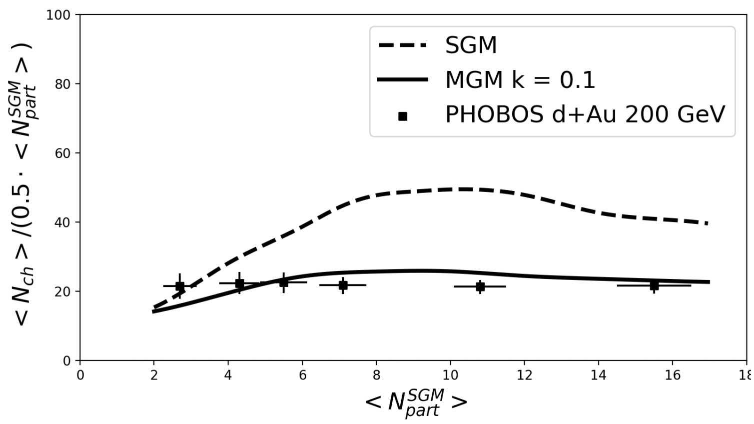


Figure 3: Total integrated charged particle multiplicity per participant pair as a function of number of participants. Experimental data [4] are shown for d+Au collisions at $\sqrt{s_{NN}} = 200\ GeV$. Results of calculations are shown for SGM (dashed line) and MGM ($k = 0.1$, solid line).