NET-PROTON HIGH-ORDER CUMULANTS IN EVENT-BY-EVENT STUDIES IN HIGH ENERGY A+A COLLISIONS AT NICA

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Abstract

The non-Gaussian fluctuations of conserved quantities, like net baryon number, were proposed recently as a tool for search of the critical endpoint of strongly interacting matter. However, studies of the energy dependence of higher-order cumulants of net-protons in A+A collisiins at RHIC and SPS have not revealed any convincing evidence of criticality. We argue that in the current analysis the non-dynamical contribution of trivial volume fluctuations are large, and we propose to use the reduced cumulants with the volume of interaction defined for each event .

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Introduction

The region of nucleus-nucleus collision energies $\sqrt{s_{NN}}$ $3-10GeV$ is very attractive for the detailed study of the phase diagram of strongly interacting matter, where the first-order phase transition and the existence of the critical endpoint (CEP) are predicted by the QCD. It is expected in the experimental energy scan, that the critical point should manifest itself by an anomaly in fluctuations of some observables. In heavy-ion collisions, the non-Gaussian fluctuations of conserved quantities, like net electric charge Q, net strangeness S, net baryon number B, could be very sensitive to the proximity of the critical point [1]. The higher moments of the distributions of conserved quantum numbers like the net-protons (as a proxy of net baryon number) are related to a higher power of the increased correlation length that is expected to diverge at the vicinity of CEP. So it was suggested in [1] to study the non-monotonic behavior with collision energy of the higher moments of the net-baryon number distribution that could be considered, if it is observed, as a signature of CEP.

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Fluctuations of some conserved quantity (Q_i of type i) relate directly to the susceptibilities χ_i, which are quantities that can be calculated for thermodynamic systems, e.g. in lattice QCD. Susceptibilities are defined as derivatives of the pressure with respect to the chemical potential.

In any event of nucleus-nucleus collision we can define the deviations δQ_i of the give measured quantity Q_i from some mean value $< Q_i >$. The last one is relevant to the given class of selected events assumed to be a certain class of centrality of collisions, usually characterized by some proxy (like multiplicity or energy of spectator nucleons).

In theoretical thermal model approach, the deviations $\delta Q_i = N_i - < Q_i >$ are related to the susceptibilities $\chi_n Q_i$ by [2]:

$$< (\delta Q_i)^n >= T^n \frac{\delta^n}{\delta \mu_i^n} lnZ(T, \mu_i) = VT^3 \chi_n^{Q_i} \quad (1)$$

Here – Q_i is a conserved charge of interest, T –the temperature, μ_i – the corresponding chemical potential and Z –the partition function [2].

Therefore the experimental values (in the Left of equation (1)) could be used to define the thermodynamic model parameters (in the Right).

Net-protons are considered as a proxy for the net baryon number. Studies of the energy dependence of higher-order cumulants of net-protons were started in BES-I and in BES-II programs at RHIC in Berkley National Laboratory [4], [5]. The search for the critical point is also ongoing by the NA61/SHINE [6] at the Super Proton Synchrotron (SPS)at CERN. A systematic search for a non-monotonic dependence of various correlation and fluctuation observables on collision energy and size of colliding nuclei was performed by the NA61/SHINE in a two-dimensional scan at SPS in beam momentum (13A-150A GeV/c) and in system size (p+p, p+Pb, Be+Be, Ar+Sc, Xe+La, Pb+Pb). A search for the existence and location of the critical point of strongly interacting matter in A+A collisions is also planned at the future Facility for Antiproton and Ion Research (FAIR) at GSI and by the MPD at NICA at JINR [7], [8].

Recent precision measurement for the net-proton cumulant ratios vs. Au+Au collision energies at RHIC showed some hints of the non-monotonic energy dependence [4], [5]. The first evidence of a nonmonotonic variation in kurtosis times variance of the net-proton number as a function of $\sqrt{s_{NN}}$ with 3.1 σ significance, was reported by STAR for head-on (central) gold-on-gold (Au+Au) collisions at RHIC [4]. Event-by-event net-proton number distributions for head-on (0-5% central) Au+Au collisions for nine of collision energies values measured by STAR demonstrated the noticeable deviation from the Gauss distribution.

However, these experimental investigations, carried out so far at SPS and at RHIC, have shown no convenient evidence for the critical phenomena. One of the reasons could be in the well-known problem of the non-dynamical contributions of trivial volume fluctuations of number of protons and net-protons, that could be rather large in relativistic nuclear collisions. The effect of volume fluctuations on cumulants of the net baryon number was pointed at in [9], [10]. A general expression was derived for the cumulants, under the assumption that the fluctuations of baryon number and volume are independent. It was shown that the critical behavior of higher-order cumulants may be modified significantly. However, we have to note that there is a certain correlation of baryon number and the volume defined by the initial geometry of the collision event. It brings a challenging task to the experimental attempts of precise event-by-event estimates of the initial conditions in A+A collisions.

As it is mentioned in [11], the effect of volume fluctuations on cumulants of the net baryon number arises from the impact parameter fluctuations and from the requirement of overall net-baryon number or net-charge conservation, and it may mask the dynamical fluctuations of interest. These trivial volume fluctuations are to be carefully taken into account in experimental data analysis before comparison to theory. Methods were developed in [11] to deal with these non-dynamical contributions to event-by-event fluctuation measurements of net-particle numbers. In particular, formulae were derived in [11] for net-particle fluctuations and a rigorous approach was developed to take into account contributions from participant nucleon fluctuations in realistic experimental environments and at any cumulant order.

However, we have to note that the participant nucleon ($Npart$) fluctuations, mentioned in [11], cannot be eliminated completely even in the case of the fixed values of the impact parameter used in any theoretical approach due to the effects of the quantum mechanics in the inelastic nucleon-nucleon interactions. Role of the impact parameter fluctuations and the impact on the multiplicity and transverse-momentum correlations in collisions of ultra-relativistic ions was studied in detail in [18] and in [19] where the correlations between the event-averaged transverse momenta of particles produced.

The width of the event centrality class selected for the analysis in relativistic A+A collisions, could be also among the major factors in the appearance of these trivial volume fluctuations [18], [20].

In this report we consider and discuss these two problem of volume fluctuations and of the width of the event centrality class. We argue also, that the currently applied experimental approaches, based in the most cases on the multiplicity classes of events as a proxy for centrality of A+A collisions, are still lacking the proper estimate of the values of high-order cumulants due to the assumptions of the fixed values of interaction volume (V) in the class of events used in the analysis. Thus, the values of the high-order net-proton cumulants and their ratios are biased because of the inevitable mixture of the events with different impact parameters, selected using the multiplicity data.

We propose in this work to use the reduced cumulants, where both the cumulants and the volume of interaction are defined for each event and then averaged over the given narrow class$^{c}haracterisedbytheinteractionvolume.Thelastprocedurerequirestheevent-by-eventestimatesofthemostprobablevalueofnumberofparticipatingnucleons(\mathrm{N}_{part})$ in each event of A+A collision. The aim to minimize the trivial volume fluctuation effects could be achieved by using such approaches as [21] or by application of ML technique [22], [24] that are currently in the progress of development.

In this paper, in Chapter 1 we briefly review the main ovservables and in Chapter 2 we describe our proposals and present the discussion of the problems that might be handled in A+A collisions at the MPD experiment at NICA energies.

1 Event-by-event fluctuations of conserved quantities

Net electric charge Q,Net strangeness S, Net baryon number B, ... are sensitive to the degrees of freedom that are active in the system, and the moments (Variance (σ^2), Skewness(S), Kurtosis(k)) of distributions of Q,S,B, are predicted to be sensitive to the correlation length of hot dense matter created in high energy A+A collisions [1]. This sensitivity of high order cumulants to the QCD critical endpoint of the first order phase transition between quark-gluon plasma and hadron gas has motivated the experimental search of the non-monotonic behavior of cumulants and their ratios with the collision energy.

Below we consider the cumulants of the conserved charge of net-protons (denoted here as N and used as a proxy for the net baryon number B. We follow the definitions of the central moments and cumulants for N as presented in [15]. The average of any variable over all events in some selected class is denoted below as $< ... >$.

The moments of the net-protons distribution can be measured experimentally [15] by standard statistical quantities: mean, variance, skewness and kurtosis, see in Table 1 . Following eq.(1), where we have to substitute instead of Q the net-proton charge N, these moments $< (\delta N_i)^n >$ of distribution of the conserved net-protons charge N correspond to certain susceptibilities $\chi_n^{N_i}$. In order to get rid of the volume dependence in eq.(1), and considering the fixed values of temperature and baryon chemical potential (T, μ_B), one may assume the possibility to use the ratios of cumulants, as proposed in [16]. The last procedure was widely used in the data analysis. However, we have to note also that this assumption is not valid: the trivial volume fluctuations are too large to consider the volume V in the equation (1) to be the same in all events in the class of events selected by the charged particles multiplicity.

Table 1: Definition of cumulants of conserved charge N (Net-protons)

Net-proton multiplicity in a given event	N
the average over all events	$< ... >$
Mean	$M =< N >= C_1$
Standard deviation	$\delta N = N- < N >$
Variance	$\sigma^2 = \delta N^2 = C_2$
Skewness	$S = \delta N^3/\sigma^3 = C_3/C_2^{3/2}$
Kurtosis	$k = \delta N^4/\sigma^4 - 3 = C_4/C_2^2$

2 Problems of centrality class selection

As it was mentioned, the event-by-event fluctuations of the impact parameter b in the given centrality class, selected for analysis, will produce trivial volume fluctuations. Therefore, in the event-by-event analysis of distribution of net-proton numbers, the deviation $\delta N = N- < N >$ from the mean value in the given class of events, brings to the higher cumulants an added weight to the outliers.

Naturally, in any class of selected events, these outliers will be with the dominating events relevant to the larger impact parameter values. Therefore, the trivial volume fluctuation effect will be amplified by the second, third, or fourth power of δN – and that brings the dominant contribution to the relevant values of the net proton number susceptibilities $\chi_n^{N_i}$.

We must note that the shape of the distribution of the net-protons will also be biased due to the increased number of events with a larger impact parameter in any class, the smaller overlap areas of colliding nuclei and therefore the smaller volumes will be dominating [12].

In order to treat the problem of the wide centrality class defined form the multiplicity of charged particles distribution, a technique called Centrality Bin Width Correction (CBWC), was developed in [13] to calculate the various moments of particle number N) distributions. In this approach, first of all, the wide distribution of events in the given multiplicity class is split in narrow multiplicity bins. Then the quantities of interest (cumulants) are defined in these narrow bins and averaged [13].

This procedure was used by STAR experiment at RHIC to calculate the various moments of particle number distributions in very narrow multiplicity bins belonging to the centrality class of a certain width in studies of net-proton number fluctuations using ratios of higher-order cumulants like $C_4/C_2, C_5/C_1$, and C_6/C_2 [14], [15], [17]. Application of CBWC has shown the noticeable decrease in the values of cumulants for central classes of collisions.

However, in spite of using narrow classes of multiplicity, this CBWC procedure is also introducing the bias into the values of net-proton cumulant ratios. This is due to the presence, even for the case of selected events with the fixed multiplicity, of the inevitable mixture of the collisions with different impact parameters (see Fig.44 in [7] where the impact parameter distribution is shown to be fluctuating from 0 up to 4 fm for 0-5% of collision multiplicity class).

Besides, we have to note, that the assumption to remove the volume dependence in equations, like in eq.(1), by considering the ratios of the high-order cumulants of net-protons $< (\delta N_i)^n >$, is not justified. This is due to the fact that the mean value of the interaction volume V in eq.(1) is also a quantity strongly fluctuating from event to event. Therefore, the ratios of interest, i.e. of susceptibilities $\chi_n^{N_i}$ will be still affected by the presence of the trivial volume fluctuations.

We have to mention again, that the narrow class in multiplicity, e.g. of 1% width, does not mean narrow distribution neither in the impact parameter nor in the volume V. Any narrow r_{th} multiplicity bin " selects" rather wide class of impact parameters [20], [7] and still contains trivial volume fluctuations. The same is true even for the case of hypothetical class with the fixed impact parameter [18]. Therefore, our proposal is to try to mitigate further the effects of trivial volume fluctuations and of the width of the events centrality classes selected for the analysis, by considering the data not in the narrow multiplicity bins, but with a more precisely defined initial conditions.

3 Reduced cumulants

We propose to use the reduced cumulants C_n^r values estimated with the most probable value of volume V_r defined in each narrow class r of events selected with $< N_{part}^r >$.

First of all, the most probable value N_{part}^r of number of nucleons-participants is to be estimated in each event. Secondly, depending on the final resolution of $< N_{part}^r >$ values, the relevant narrow classes r of events are defined. We assume that in each narrow bin in N_{part}^r class of events, the value of the interaction volume V_r is proportional in each collision to the relevant number of pairs of nucleons-participants: $Vr = V_0 \cdot N_{part}^r/2$. Here a volume factor is $V_0 = 2.83 fm^3$, which we fix to be equal to the volume of the proton,(see in [10]).

Thus we may obtain for each event in this narrow class the standard deviation of the net-proton density N_r/V_r:

$$\delta(N_r/V_r) = N_r/V_r - < N_r/V_r > \tag{2}$$

Here $< N_r/V_r >$ is the mean net-proton density calculated for the given narrow class of events selected with $< N_{part}^r >$. Cumulants of net-proton density are presented in Table 2.

Table 2: Reduced cumulants of net-proton density defined in the narrow class of N_{part}^r

	Net-proton density in a given event	N_r/V_r
	The average over all events in the class	$< ... >$
	Mean	$M =< N_r/V_r >= {C_1}^r$
	Standard deviation	$\delta(N_r/V_r) = N_r/V_r - < N_r/V_r >$
	Variance	${\sigma_r}^2 =< \delta(N_r/V_r)^2 >= C_2^r$
	Skewness	$S_r =< \delta(N_r/V_r)^3/{\sigma_r}^3 >= C_3^r/(C_2^r)^{3/2}$
	Kurtosis	$k_r =< \delta(N_r/V_r)^4/{\sigma_r}^4 > -3 = C_4^r/(C_2^r)^2$

The quantities of interest (high order cumulants of net protons density distributions), defined in these narrow bins of N_{part}^r, could be combined into a wider class of events and averaged similar to the procedure used in [13]:

$$\sigma = \frac{\sum_r (N_r/V_r)\sigma_r}{\sum_r (N_r/V_r)} = \sum_r \omega_r \sigma_r \tag{3}$$

$$S = \frac{\sum_r (N_r/V_r)S_r}{\sum_r (N_r/V_r)} = \sum_r \omega_r S_r \tag{4}$$

$$k = \frac{\sum_r (N_r/V_r)k_r}{\sum_r (N_r/V_r)} = \sum_r \omega_r k_r \tag{5}$$

Here σ, S and k are representing the standard deviation,skewness and kurtosis for the value (N_r/V_r) defined for the centrality class of events selected for the analysis. N_r is the number of events in the r-th narrow bin. σ_r, S_r and k_r represent the standard deviation, skewness and kurtosis of net-proton number density distributions in the r-th bin. $\omega_r = (N_r/V_r)/\sum_r (N_r/V_r)$ - is the corresponding weight for the r_{th} narrow bin of the $N_{part}r$ class.

The values of N_{part} should be estimated in each event with the highest accuracy to select the classes of events in narrow bins of N^r_{part}. Work is in progress for the event-by-event determination of the number of participant nucleons N_{part} in nucleus-nucleus collisions in the MPD experiment on Au+Au collisions planned at NICA. The estimates of the values of N_{part} for each event and the relevant errors could be obtained with the number of spectator nucleons (N_s) from the energy measured in the hadron calorimeter by using the Bayes' approach, as described in [21]. The estimates of the accuracy of N_s reconstruction was studied both within the analytical approach and through simulation modeling. In case of central collisions, both approaches give similar results: for central collisions, when the number of spectators is more than 10, the relative error in determining their number does not exceed 10% [21]. The capability of neural networks to directly estimate the impact parameter in the experiment is being investigated at present in [22] using the time-of-flight information and charged particles distribution that might be obtained event-by-event from the fast microchannel plate detector. The developed algorithms can correctly select more than 98% of events with an impact parameter smaller than 1 fm [23]. It was also shown recently, that application of "Deep reconstruction neural network", trained on mixed data of QGSM and EPOS event-generator, is capable to select some class of very central events in the data from another generator (PHQMD) with the accuracy of $< N_{part} >$ determination at the level of $\sim 1\%$ [24].

4 Conclusions

We propose to use the reduced cumulants, where both the cumulants and the volume of interaction are defined for each event. The last procedure requires the event-by-event estimates of the most probable value of number of participation nucleons (N_{part}) by using such approaches as [21] or the ML technique [22], [23], [24] that are currently in progress.

The reduced cumulants, defined within the class of events with selected certain mean volume, defined by the relevant mean value of N_{part}, have the benefits of a more direct and more precise unfolding of physical information. If compared to the multiplicity based classes of events, the classes of N_{part} are relevant to the initial conditions of the collision events, while the multiplicity variable is characterizing the final mean result of collisions.

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Conflict of interest

Author declairs no any conflicts of interest.

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