Semiclassical analysis of Klein tunneling in graphene placed in a non-uniform magnetic field

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Abstract: The effect of an external non-uniform magnetic field on the tunneling of charge carriers through a smooth electrostatic barrier in graphene was studied. The wave function of charge carrier was found as a semiclassical solution of the Dirac equation. The dependence of transmission on the angle of incidence was investigated.

The study of the possibilities to control the transport of the charge carriers in graphene by external electrostatic and magnetic fields is important for construction of electronic devices.

At low energies, the wave function of a charge carrier (Dirac fermion) in a graphene monolayer satisfies the Dirac equation. Unlike nonrelativistic electrons, Dirac fermions completely penetrate an electrostatic barrier at normal incidence, even if the barrier height is greater than their energy. This is the Klein paradox [1]. Reflection occurs if the angle of incidence is different from normal. Among the many studies on Klein tunneling through barriers describing $n-p-n$ or $p-n-p$ junctions without an external magnetic field, we mention papers [5]- [7], in which a semiclassical approach was developed. The effect of a constant magnetic field on Klein tunneling was studied primarily for model potentials [2]– [4]; the case of a linear potential is considered in [2], the case of a parabolic potential in [3]. We cannot provide a complete list of references.

We present here a semiclassical approach to solving the Dirac equation in the presence of external static inhomogeneous electric and magnetic fields that smoothly vary only in one spatial variable x. The wave function of charge carrier is taken in the form

$$\boldsymbol{\Upsilon}(\mathbf{r},t) = e^{i(p_y y-\mathcal{E}t)/\hbar}\,\boldsymbol{\Psi}(x), \quad \mathbf{r}=(x,y), \tag{1}$$

where $\mathcal{E}$ is the carrier energy. Let the carrier momentum have components $(\beta(x), p_y)$. If there are no external fields at $x=0$, then $\beta(0)=v_F^{-1}\mathcal{E}\cos\theta$, $\quad p_y = v_F^{-1}\mathcal{E}\sin\theta$, where v_F is the Fermi velocity, θ is the incidence angle of the carrier. The function $\boldsymbol{\Psi}$ in (1) satisfies the system

$$-i\hbar\hat{\sigma}_1\frac{d\boldsymbol{\Psi}(x)}{dx} = \hat{\mathcal{K}}(x)\boldsymbol{\Psi}(x), \quad \hat{\mathcal{K}}(x) = \begin{pmatrix} v_F^{-1}(\mathcal{E}-U(x)) & i(p_y+eA_y(x)) \\ -i(p_y+eA_y(x)) & v_F^{-1}(\mathcal{E}-U(x)) \end{pmatrix}, \tag{2}$$

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad A_y(x) = \int\limits_0^x B(\tilde{x})\,d\,\tilde{x},$$

where e is the electron charge, $U(x)$ and $A_y(x)$ are the electrostatic potential and the y-component of the vector potential, respectively; $B(x)$ is the magnetic field. If $\hbar \ll 1$, then two solutions of (2) can be found in the adiabatic approximation

$$\boldsymbol{\Psi}_{\pm}(x) \approx exp\left(\pm\frac{i}{\hbar}\int^x \beta(s)\mathrm{d}s\right)\left(\boldsymbol{\Phi}_{\pm}^{(0)}(x)+\dots\right). \tag{3}$$

If $\beta(x)$ is real (complex), then x belongs to the classically allowed (forbidden) zone. We introduce such $x_{\mathcal{E}}$ that $\mathcal{E} = Q(x_{\mathcal{E}})$. Let the angle of incidence θ be restricted by the estimate

$$p_y + A_y(x_{\mathcal{E}}) = \sqrt{\hbar}\gamma(x_{\mathcal{E}}), \quad \gamma \sim 1. \tag{4}$$

Then the matrix $\hat{\mathcal{K}}$ admits the representation

$$\hat{\mathcal{K}}(x) = \mathcal{K}(x) + \sqrt{\hbar}\mathcal{B}, \quad \mathcal{B} = -\gamma(x_{\mathcal{E}})\sigma_2, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{5}$$

near $x_{\mathcal{E}}$. The point $x = x_{\mathcal{E}}$ is the intersection point of the curves $\pm\beta(x)$ if $\gamma(x_{\mathcal{E}}) = 0$. It is near this point that the classically forbidden zone appears if $\gamma(x_{\mathcal{E}}) \neq 0$, and the approximation (3) fails near it. To calculate the coefficient of transition through this zone, we use the general results from [10] obtained for the equation (2) with the operator $\hat{\mathcal{K}}$ in the form (5) without specifying the type of the self-adjoint operators $\mathcal{K}$, $\mathcal{B}$ and $\hat{\mathcal{K}}$. The article [10] presents an analogue of the Landau-Zener formula obtained in [8], [9] for nonadiabatic transitions in a diatomic molecule.

As a result, we found formulas for the reflection and transmission coefficients in the case of $n-p$ and $p-n$ junctions, as well as in the case of a barrier describing the $n-p-n$ junction, and numerically studied the dependence of the transmission coefficient on the angle of incidence. The calculation results show Fabry-Perot resonances caused by multiple reflections inside the barrier.

References

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