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Mathematical Modeling of Nonequilibrium Processes at the Department of Physical Mechanics, St. Petersburg State University. Part 3. Modeling of the Dynamics of Natural and Man-Made Microparticles in Space Plasma

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Abstract—This work provides an overview of the research of employees of the Department and Laboratory of Physical Mechanics of St. Petersburg University that are devoted to modeling the dynamics of natural and man-made microparticles in space plasma. Special attention is given to several major results: (i) the existence of three possible regimes of vertical oscillations of a dust particle over the Moon's surface; (ii) the identification of possible mechanisms for the long-term retention of man-made microparticles in near-Earth space; (iii) the application of methods of Kolmogorov–Arnold–Moser (KAM) theory for the proof of conservation of orbital parameters for a microparticle with constant electric charge; (iv) determination of the conditions for applicability of the canonical formulation of the problem regarding the motion of a particle with locally equilibrium electric charge in near-Earth space; and (v) generalization of results of analytical modeling for a case of variable electric charge. Other results are given short enough.

Keywords: modeling, space, atmosphereless celestial body, locally equilibrium electric charge, plasma, canonical equations, geomagnetic field, field of corotation, KAM theory

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This work is a natural continuation of paper [1] because it describes the studies of employees of the Department and Laboratory of Physical Mechanics of St. Petersburg University aimed at solving issues arising in the study of the charging of bodies and beam transport in space plasma.

1. MODELING THE DYNAMICS OF DUST PARTICLES NEAR THE SURFACE OF ATMOSPHERELESS CELESTIAL BODIES

The problem described in Section 3 of paper [1] concerning the remote elemental analysis of surface rocks of atmosphereless celestial bodies, in particular, the Moon, required theoretical study of the processes and phenomena near their surface, which can sophisticate this procedure. One such phenomenon can be the existence of an exosphere partially consisting of finely dispersed levitating particles of lunar regolith, which was discovered by the astronauts of "Apollo 17" observing scattering of solar light near the lunar terminator causing illuminance of the horizon [2]. The theoretical study included the following problems: (i) construction of the model of charging of a planet's surface; (ii) determination of the distributions of electric field and plasma particles in its vicinity; (iii) construction of the criterion for the lift-off of particles from a planet's surface; and (iv) description of the dynamics of charged particles of soil in the exosphere. The first two items were partially solved long before the discovery of levitating soil microparticles [3–6]; it was shown that the distribution of the electrostatic potential has a nonmonotonic character. In works [7–9], unlike other researchers, Kolesnikov and Manuilov considered the case of a surface covered by a single layer of hydrogen-containing compounds. Only in recent years has this approach been regarded as the most perspective one for explaining the observed phenomena. Another choice of boundary conditions for this problem was considered in [10–12]. It was demonstrated that such choice of

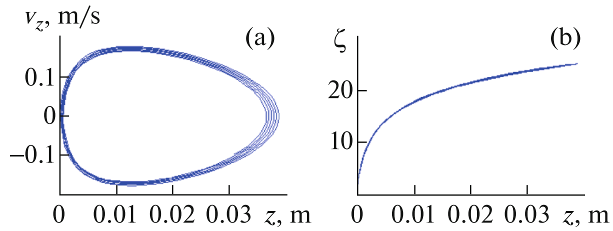

Fig. 1. Calculation results of the time evolution of a microparticle with a radius of 0.23 μm for $\theta_0 = 80°$: (a) the phase trajectory and (b) the dependence of the dimensionless charge ζ on the altitude z.

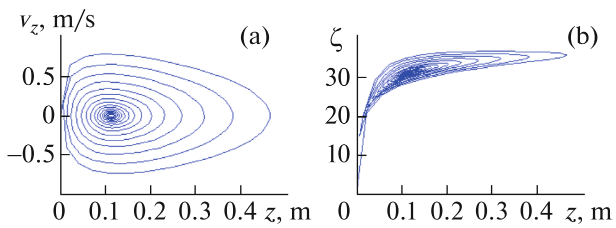

Fig. 2. Calculation results of the time evolution of a microparticle with a radius of 0.1 μm for $\theta_0 = 80°$: (a) the phase trajectory and (b) the dependence of the dimensionless charge ζ on the altitude z.

boundary conditions is more advantageous, because it eliminates fictitious solutions for the parameters of the nonmonotonic electric potential.

In [13], assuming that a single layer of hydrogen-containing compounds covers both the lunar surface and the surface of a spherical soil particle, we provided a levitation condition for a lunar regolith particle near the terminator:

$$8\pi\varepsilon_0\left(\frac{k_BT}{e}\right)^2 R\ln\left[\frac{(1+G\sqrt{\cos\theta_0}z)^2}{4\cos\theta_0}\right]\frac{G\sqrt{\cos\theta_0}}{1+G\sqrt{\cos\theta_0}z}-\frac{16}{9}\pi^2R^3\rho^2\gamma R_m > 0. \quad (1)$$

Here, R is the radius of the microparticle, $G = \sqrt{2\pi n/k_BT\varepsilon_0}$, n is the linear concentration of photoelectrons on the body surface, θ_0 is the angle between the local normal and the direction towards the Sun, ρ is the density of the surface material (lunar regolith), k_B is the Boltzmann constant, and R_m is the radius of the body (the Moon). Finding condition (1) allowed considering the vertical dynamics and transport along the lunar surface for the above-mentioned particles in [11, 14–19].

In particular, in [15, 16] it was proved that there exist three possible modes of vertical oscillations of a dust particle, which occur at certain values of its radius. The first of the mentioned modes takes place if the dust particle radius is close to the critical radius of levitating microparticles, which is determined by relation (1). The peculiarities of the vertical motion of dust particles in this case is illustrated by the data in Fig. 1, in which we present the calculation results of the temporal dynamics of a microparticle 0.23 μm in radius at $\theta_0 = 80°$. In this case, the vertical oscillations of a dust particle occur at a low altitude over a planet's surface, and their qualitative pattern is close to that described in [14] due to the proximity of the charge of the microparticle to a locally equilibrium value (see Fig. 1b). The effect of small deviations of the dust particle's charge from equilibrium manifests itself, in particular, in a nonclosed phase trajectory (see Fig. 1a).

The second mode of vertical motion takes place for dust particles with radii in the range of (0.01–0.1) μm. Figure 2 presents example calculation results for the vertical motion of a microparticle with a radius of 0.1 μm at $\theta_0 = 80°$. The effect of nonequilibrium electric charge of the microparticle leads in this case to rapid decay of its vertical oscillations, and the dust particle levitates motionless at a fixed altitude determined by its radius (Fig. 2a). The dimensionless charge of microparticle ζ equal to the ratio of its current value to the equilibrium one on the planet's surface also tends asymptotically to the equilibrium value at the hovering altitude (see Fig. 2b).

Finally, the third mode of the vertical motion of dust particles takes place in the case of ultrafine particles with radii of less than 0.01 μm. In this case, when determining the charge of a microparticle, we

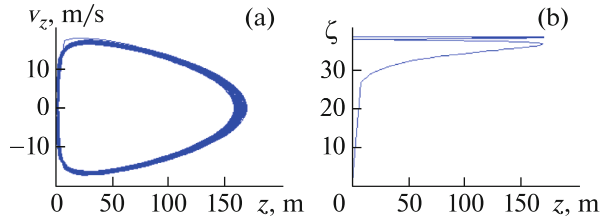

Fig. 3. Calculation results of the time evolution of a microparticle with a radius of 0.007 μm for $\theta_0 = 80°$: (a) the phase trajectory and (b) the dependence of the dimensionless charge ζ on the altitude z.

should also take into account the currents from electrons and ions of the solar wind. The qualitative pattern of the dynamics of ultrafine dust particles is given in Fig. 3 for a dust particle with a size of 0.007 μm at $\theta_0 = 80°$. We see in Fig. 3b that the electric charge of a particle asymptotically approaches a certain constant value Q_a with time (Fig. 3b). The kinematic consequence of this effect is the asymptotic approach of the vertical motion of the dust to the periodic oscillatory motion with a constant amplitude (Fig. 3a).

The majority of the obtained solutions are numerical; however, within the proposed method of generalized charge-field potentials, we also obtained analytical solutions (in quadratures) to the problems of the vertical dynamics of a lunar regolith particle in the near-surface photoelectronic layer [11, 18]. In work [14], unlike works [20, 21] published almost simultaneously, we considered the vertical dynamics of charged microparticles of lunar soil without initial velocity. In addition, in [16] we proposed a mechanism of the horizontal transfer of lunar soil particles and derived the dependences of its maximum length on the radius of microparticles and the latitude of the Moon's surface. We presented the analysis of the physical causes of the existence of different types of vertical motion in [17].

2. MODELING THE BEHAVIOR OF DUST CLOUDS IN NEAR-EARTH SPACE

More than half a century of human activity in near-Earth space has led to several negative consequences, the primary of which is ever increasing pollution of near-Earth space (NES) by products of anthropogenic activity (so called space debris (SD)). The sources of man-made particles in NES are fine-disperse emissions of the solid-propellant rocket (SPR) of upper stages of spacecraft, degradation of the surface materials of orbital spacecraft under the action of physical factors of space, as well as large objects of SD and their fragments. The results of field measurements show that the dimensions of man-made particles are in a broad range from a few nanometers to hundreds of microns. Collisions of spacecraft with man-made particles at collision speeds on the order of 10 km/s can lead to the destruction of materials used in different elements of the spacecraft structure, as well as erosion processes, cratering, spallation phenomena, and the ejection of secondary particles in these materials. Today, the number of man-made particles in NES is growing rapidly, and, therefore, the danger they represent for orbital spacecraft is also increasing, especially upon the passage of spacecraft through clouds of man-made particles (so called astrosols). Man-made particles pose a danger not only for spacecraft, but also create serious problems for Earth's ecology [22]. In particular, particles made of radioactive materials and microbiological objects (MBOs) reentering the Earth from space become extremely dangerous for the Earth's biosphere. MBOs can be transferred from the Earth to space both on the surfaces of spacecraft and launch vehicles and due to the action of ionospheric uplift, which transports tropospheric aerosols from the Earth's surface to the upper ionosphere.

Due to the rapidly growing pollution of near space from the middle of the 1980s, researchers began intense theoretical studies of the dynamics in NES of man-made particles. These studies, first of all, aimed to determine the characteristics of man-made particles (MPs) with long-term orbital existence. At the first stage, special attention was drawn to determining the conditions of the long-term orbital existence in NES of relatively large man-made particles with dimensions in the range of 1–100 μm, because MPs of these dimensions have the greatest negative effect on orbital spacecraft and modeling of their motion in the NES did not require taking into account the effect of the dynamics of MPs of electrodynamic forces, which essentially simplified the solution to the corresponding problems. The calculation results of the lifetime of large MPs in NES showed the possibility of the long-term orbital existence of large MPs injected into NES at both high and low orbits. At the same time, the principal possibility of submicron MPs remaining in the vicinity of Earth for a long time was met with doubt.

However, numerical studies [23–34] of the dynamics in NES of ultrafine submicron particles with dimensions less than 0.1 μm performed by employees of the Department and Laboratory of Physical Mechanics showed that, under certain conditions, man-made particles with dimensions in the range of 5–100 nm moving in the Earth's plasmasphere can remain in NES for a long time and can therefore also be an important factor of anthropogenic pollution of NES. It was shown that the main physical mechanism leading to the long-term retention of ultrafine MPs in NES is the action of the Lorentz force on the motion of these particles, which is caused by the interaction of the geomagnetic field and the induced MP charge. For several model cases, the results of computational experiments coincide with data on the analytical studies presented in works [35–42].

2.1. Numerical Modeling of Orbital Motion of Submicron Man-Made Particles Injected into Near-Earth Space in Different Structural Regions of Earth's Plasma Sheath

The numerical modeling of motion of submicron MPs in the vicinity of Earth is a problem that must be solved taking into account the action on MP motion in the NES of the following complex of forces: the gravity force acting upon the MP from the central gravitational field of the Earth and its perturbations; the solar pressure force; the drag force of the upper atmosphere; and the electrodynamic forces caused by interaction of the electric charged induced on the MP with the magnetic and electric fields of the NES. An especially complicated problem is the need to take into account the action of electrodynamic forces on MP motion. First of all, this is associated with the difficulty of determining the electric charge of a particle. Because of the spatial heterogeneity of the near-Earth plasma parameter and illumination conditions of a microparticle in NES, the electric charge of a MP changes along its trajectory, and the charge value at a current time instance, due to the finiteness of the characteristic time of MP charging, depends on the physical conditions not only at the point of the current position of the microparticle, but also at previous time instances along the trajectory. Therefore, in the general case, when we solve ballistic problems of submicron particles in NES, we must solve equations describing the process of their charging in space plasma together with the equations of motion of a submicron particle in NES.

Thus, in the geocentric equatorial reference frame (not rotating together with the Earth), the motion of a submicron particle in NES is described by the equation

$$m\frac{d^2\mathbf{r}}{dt^2} = \mathbf{F}_G + \mathbf{F}_G^{dst} + \mathbf{F}_L^B + \mathbf{F}_L^E + \mathbf{F}_{\text{pr}} + \mathbf{F}_{\text{drag}}. \tag{2}$$

Here, $\mathbf{F}_G$ is the force acting upon the MP from the central gravitational field of the Earth, $\mathbf{F}_G^{dst}$ is the perturbation of the force $\mathbf{F}_G$ caused by polar flattening of the Earth, $\mathbf{F}_L^B$ and $\mathbf{F}_L^E$ are, respectively, the magnetic and electric components of the Lorentz force caused by the interaction of the charge of a MP $q = q(t)$ moving with a velocity $\mathbf{v}$ with the magnetic $\mathbf{B}$ and electric $\mathbf{E}$ fields of NES, $\mathbf{F}_{\text{pr}}$ is the force of solar light pressure on the particle, and $\mathbf{F}_{\text{drag}}$ is the drag force of the neutral component of the background gas. In the first approximation, the geomagnetic field can be approximated by a field of a magnetic dipole placed at the center of the Earth with a magnetic moment $\mathbf{M}_d$ oriented oppositely to the vector of the angular velocity of Earth's rotation $\mathbf{\Omega}$. Researchers also used approximation by the sum of N spherical harmonics of the Gauss series [43]. The electric field in NES can be represented by a superposition of fields of corotation $\mathbf{E}_{\text{cor}}$ and convection $\mathbf{E}_{\text{cross-tail}}$, where $\mathbf{E}_{\text{cor}} = \frac{1}{c}(\mathbf{r} \times \mathbf{\Omega}) \times \mathbf{B}$, whereas the electric field of convection can be approximated by a constant electric field directed from the morning to the evening side of the magnetosphere and depending on the geomagnetic activity. The value of the solar pressure force was computed through the solar pressure intensity Q_{pr} averaged over the solar spectrum. For the considered particle dimensions comparable with the wavelength in the maximum of the solar-radiation spectrum, the intensity Q_{pr} was prescribed based on numerical summation of the series, which are the formal exact solution to the problem concerning the absorption and scattering of light by a homogeneous spherical particle obtained in the so called Mie theory.

The electric charge $q(t)$ induced on the surface of a microparticle evolves according to the equation

$$\frac{dq}{dt} = \sum_i J_i = J_{\text{tot}}, \tag{3}$$

where J_{tot} is the total charging current and J_i are the charging currents corresponding to the possible mechanisms of MP charging in space. In the general case, the indicated mechanisms include the following:

(i) charging of the MP by plasma electrons and ions of thermal energies incident on its surface;

(ii) charging by high-energy charged particles of corpuscular streams;

(iii) secondary electron emission in the case of collisions of electrons and ions with the MP surface;

(iv) electron backscattering;

(v) photoelectron emission;

(vi) field-electron emission (for metallic MPs at low negative potentials) [22].

The charging currents J_i on the right-hand side of Eq. (2) are determined by the expressions given in works [44–51]. Because the charging currents are determined by local values of the density and temperature of electrons and ions of the background plasma, which vary in the course of the orbital motion of MPs in NES, in our numerical modeling of the motion of a particle in NES based on joint solution of the equation of motion (2) and charging equation (3), we used the model of the Earth's plasma sheath described in [22]. It was assumed that it consists of three structural regions: ionosphere, plasmasphere, and plasma layer.

Numerical experiments for determining the conditions of long-term orbital existence of MPs injected into NES were performed for circular and nearcircular orbits passing at different altitudes over the Earth's surface, as well as for elongated elliptical orbits with a low perigee, in particular, with the parameters corresponding to the orbital parameters of the "Molniya" spacecraft. Submicron particles made of aluminum, aluminum oxide, and carbon (material with a low yield of photoemission) having radii from a few nanometers to tens of micrometers were considered. The orbital parameters, the size, and the material of the particles were chosen corresponding to the available experimental data [52–56].

As shown in [33], the long-term orbital existence of ultrafine carbon MPs in NES can be divided into three modes. In the first mode the spatial trajectory of MPs traces out a torus with an axis coinciding with Earth's magnetic axis symmetric with respect to the plane of the magnetic equator. The physical nature of the long-term orbital existence of ultrafine MPs in NES in the considered mode is caused by adiabatic invariancy of the equivalent magnetic moment of the MP. As shown in [57], the latter condition for the dipole magnetic field turns out to be true for motion at distances from the Earth's center that are much less than the Størmer length.

The second mode describes motion in the vicinity of the equatorial plane. The specified motions are a special case of the above considered mode, but are characterized by longer times of orbital existence of MPs in NES. They can occur upon the injection of ultrafine MPs on parent orbits with zero or small angle of inclination to the equatorial plane. The principal possibility of the long-term orbital existence of ultrafine MPs in planar orbits and near-planar spatial orbits was first demonstrated in [23].

The third mode of long-term orbital existence (magnetic—gravitational trapping) in NES of ultrafine carbon MPs occurs when the condition of weak local heterogeneity of the magnetic field is fulfilled. In this case Earth's gravitational field makes a noticeable contribution to the effective potential $U_{eff}(s)$ describing the motion of the leading center along the arclength coordinate of the local force line in the leading plane. The contribution of the gravitational field to the effective potential leads to the appearance of two local minima of the effective field potential in the Northern and Southern Hemispheres. This leads to the occurrence of a possible mode of motion in which the leading center of an ultrafine MP performs oscillations along a segment of the force line in the Northern or Southern Hemisphere in the corresponding potential well, simultaneously drifting along the azimuth due to inhomogeneity of the magnetic field and a variation in the electric charge of the MP along the trajectory of its motion in NES. Here, the spatial trajectory of the MP traces out a torus located in the Northern or Southern Hemisphere and having an axis coinciding with Earth's magnetic axis. As the results of computational experiments showed, the phenomena of magnetic—gravitational trapping take place for carbon MPs with radii up to 7 nm inclusively that are injected into NES on parent orbits with different altitudes and angles of inclination.

The results of numerical modeling of the motion in NES of aluminum oxide MPs (material with a high yield of photoemission) with radii from 5 to 30 nm injected into NES to circular orbits with altitudes from 1.5 to 50 thousand km and different angles of inclination to the equatorial plane show that the orbital existence is the longest (up to several years) in the considered case for ultrafine MPs moving in NES in the mode of magnetic—gravitational trapping (MGT). As follows from the obtained calculation data, the MGT phenomena can occur for ultrafine aluminum oxide MPs with radii up to 21 nm inclusively injected into the plasmasphere on parent orbits with different altitudes and angles of inclination to the equatorial plane. Long-term motions in the vicinity of the equatorial plane appear only for MPs with radii up to 8 nm injected in NES on orbits with zero and small angles of inclination to the equatorial plane and altitudes from 6 to 30 thousand km. Long-term motions characterized by oscillations of the leading center of MPs between the mirror points in the Northern and Southern Hemispheres were detected only in a few computational experiments for the finest MPs with a radius of 5 nm.

In addition to that, a series of computational experiments was intended to determine the possible modes of long-term orbital existence of ultrafine aluminum oxide MPs injected into NES in the plasma layer of the Earth. The obtained calculation data show that upon the injection of ultrafine aluminum oxide MPs into NES in the plasma layer of the Earth, the only type of orbital motion of aluminum oxide MPs at which we can have the long-term (longer than 1 month) retention of a MP in NES is MP motion in the magnetic—gravitational trapping mode.

In [58, 59] Kolesnikov and Chernov presented the results of modeling the motion in NES of aluminum and aluminum oxide microparticles with radii from 1 to 100 μm injected into NES in the perigee of an elliptical orbit with the parameters corresponding to the orbital parameters of the *Molniya* spacecraft, which showed the principal possibility of the long-term orbital existence of such man-made microparticles (with lifetimes longer than 1 month). Moreover, the results of numerical experiments implies that in the corresponding conditions the microparticles injected into NES on elongated elliptical orbits with a low perigee can have extremely long times of orbital existence, on the order of a year and longer, and must be regarded as one of the dangerous sources of man-made pollution of near space. Kolesnikov and Chernov carried out numerical experiments in works [58, 59], in which they simulated the motion in NES of large MPs with radii greater than 1 μm, ignoring the effect of electromagnetic forces on the dynamics of MPs. After that, in [32] they performed a series of computational experiments intended to determine the times of orbital existence of submicron aluminum and aluminum oxide particles injected into NES on an elongated elliptical orbit with a low perigee. The calculations were performed taking into account the effect of electromagnetic forces on the motion of MPs under conditions of a low level of solar and geomagnetic activity.

The possibility of the long-term orbital existence of MPs in geostationary orbits was considered in [43, 60]. In [43], based on the results of numerical modeling, Kolesnikov and Chernov for the first time showed such a possibility for aluminum oxide particles separating from the surface of an operating geostationary satellite or an object decommissioned in the vicinity of the geostationary orbit. They showed that under conditions of low solar and geomagnetic activity, those particles have long times of orbital existence (longer than 1 month) whose radii are larger than a threshold value close to 1.1 μm. The times of orbital existence of MPs with radii larger than this threshold value are almost independent of the initial position of their injection point in geostationary orbit and rapidly increase as the MP radius grows. It was also demonstrated that under conditions of low solar and geomagnetic activity, submicron particles with radii of less than 0.1 μm can also have long times of orbital existence. It was established that ultrafine MPs with radii less than 0.01 injected into geostationary orbit can exist on that orbit for more than 2 years. Analysis shows that in this case a MP moves in the so called magnetic—gravitational trapping mode.

In [61], Kolesnikov and Chernov used the drift equations of motion to analytically determine the conditions of two possible modes of long orbital existence of man-made nanoparticles injected into NES at a high circular orbit in the region of the ring current and not leaving this region in the course of their orbital motion. They showed that in each of the indicated modes the leading center of a nanoparticle, not reaching the dense atmosphere, periodically oscillates in the leading plane along the segment of a force line of the geomagnetic field between the mirror points that are located in the Northern and Southern Hemispheres in one mode and in the same hemisphere as the injection point in the other mode.

To study the possibility of pathogenic MBOs returning to Earth, in [62] Kolesnikov and Chernov constructed a mathematical model of the aerodynamic heating of a spherical carbon microparticle with a radius from 0.5 to 3 μm (endospores of terrestrial or virtual extraterrestrial bacteria) upon its entry to Earth's atmosphere. The model is based on joint numerical solution of the equations of motion of a particle in NES and the equation describing the variation in the internal energy of the MBO. They showed that the mentioned endospores of bacteria can retain their viability even upon entry to the atmosphere with initial velocities higher than the escape velocity.

2.2. Analytical Approach to Modeling the Orbital Motion of Submicron Man-Made Particles Injected into Near-Earth Space

Although the main method for the theoretical study of peculiarities of the orbital motion of MPs in the vicinity of Earth is the above considered numerical simulation method, the physical estimates and results of computational experiments show that in certain practically interesting cases the number of factors affecting the orbital dynamics decreases substantially, which simplifies ballistic problem formulation in

NES and allows using the corresponding analytical methods to determine the peculiarities of orbital motion near the Earth.

In works [35, 63], Vavilov and Kolesnikov considered the ballistics problem for a MP in NES under the model assumption that the main force perturbing the motion of the MP in Earth's central gravitational field is the magnetic component of the Lorentz force acting upon the MP charge from the dipole geomagnetic field. The action on the microparticle of other perturbing forces was ignored and it was assumed that the charge of the MP in the course of its orbital motion in NES remains unchanged.

Analysis performed in [63], which used the experimental data on the plasma parameters in the day- and nighttime ionosphere given in [64], showed that the above mentioned assumptions turn out to be valid for the practically interesting motions of submicron particles with radii of less than 0.2 μm made of a material with a high yield of photoemission (aluminum or aluminum oxide) along low near-Earth orbits at altitudes of approximately 1000 km.

In the considered model formulation of the ballistics problem, the Hamiltonian function describing the motion of a MP in NES in the spherical geomagnetic coordinate system (r, ϑ, φ) with the origin at Earth's center is given by

$$H = \frac{1}{2m}\left[p_r^2 + \frac{p_\vartheta^2}{r^2} + \frac{1}{r^2\sin^2\vartheta}\left(p_\varphi + \frac{qM_E\sin^2\vartheta}{cr}\right)^2\right] - \frac{m\mu_E}{r}, \tag{4}$$

where μ_E is the gravitational parameter of the Earth and M_E is the dipole magnetic moment of the Earth oriented to the south magnetic pole.

Due to the cyclicity of the azimuthal coordinate φ, the generalized momentum p_φ is the integral of motion.

The Hamiltonian function (4) can be represented as

$$H = H_0 + H_1 + H_2, \tag{5}$$

where

$$H_0 = \frac{p_r^2}{2m} + \frac{p_\vartheta^2}{2mr^2} + \frac{p_\varphi^2}{2mr^2\sin^2\vartheta} - \frac{m\mu_E}{r}, \tag{6}$$

$$H_1 = \frac{qM_E p_\varphi}{mcr^3}, \tag{7}$$

$$H_2 = \frac{q^2M_E^2\sin^2\theta}{2mc^2r^4}. \tag{8}$$

In (5) the function H_0 corresponds to the Hamiltonian function describing unperturbed motion in the central gravitational field and the terms H_1 and H_2 characterize the perturbing effect of the Lorentz force. Under conditions when this effect is weak, the main contribution to (42) is made by the function H_1, whereas the term H_2 (quadratic in q) can be ignored. In this case, when solving the considered ballistics problem, instead of the complete Hamiltonian function (5), in [65] we used a reduced Hamiltonian:

$$H^* = H_0 + H_1. \tag{9}$$

The Hamiltonian function H^* allows writing the Hamilton–Jacobi equation as

$$H^*\left(q_i, \frac{\partial S}{\partial q_i}, t\right) + \frac{\partial S}{\partial t} = 0, \tag{10}$$

whose complete integral has the form

$$S = -\alpha_1 t + \alpha_3\varphi + \int\sqrt{P\left(\frac{1}{r}\right) - \frac{2qM_E\alpha_3}{cr^3}}dr + \int\sqrt{\alpha_2 - \frac{\alpha_3^2}{\sin^2\vartheta}}d\vartheta, \tag{11}$$

where $P\left(\frac{1}{r}\right) = 2m\left(\alpha_1 + \frac{m\mu_E}{r}\right) - \frac{\alpha_2^2}{r^2}$ and the constants α_1 and α_3, respectively, are the total energy E and the azimuthal component of the generalized momentum p_φ, which are the integrals of motion.

Construction of the complete integral of the Hamilton–Jacobi equation allows finding the solution to the corresponding system of equations:

$$\beta_i = \frac{\partial S}{\partial \alpha_i} \quad (i = 1, 2, 3). \tag{12}$$

For $q \neq 0$ the minimum and maximum geocentric distances r_1 and r_2 can be determined from the equation

$$P\left(\frac{1}{r}\right) - \frac{2qM_E\alpha_3}{cr^3} = 0. \tag{13}$$

Equations (12) allow drawing the trajectory of motion of a particle for an oscillation period in r from $r = r_1$ to $r = r_2$ and back to $r = r_1$. We can write the solutions to these equations as

$$\sin\left(\frac{\pi}{2} - \vartheta\right) = \sin i \sin \omega, \tag{14}$$

$$\tan\left(\frac{\pi}{2} - \vartheta\right) = \tan i \sin \Delta\Omega, \tag{15}$$

where $i = \arccos(\alpha_3/\alpha_2)$ is the inclination of the equatorial plane, the angle ω (the so-called argument) is the angular distance of the particle from the ascending node in the orbital plane, and the angle $\Delta\Omega$ (the increment of the right ascension of the ascending node) is the angular distance between the projection of the particle onto the equatorial plane and the ascending node. The angles ω and $\Delta\Omega$ in (14) and (15) can be expressed through the roots of Eq. (13) and the normal Legendre elliptic integral of the first kind.

Thus, the perturbing action of the Lorentz force on the motion of a particle leads to two secular variations in the spatial position of the particle's Kepler orbit: rotation of the orbit in its own plane and precession of the orbit about the geomagnetic axis. At the same time, the geometric characteristics of the orbit remain close to the parameters of the elliptic of an uncharged particle.

However, due to the approximate nature of Eqs. (12) obtained using the shortened Hamiltonian (9), there arose a question concerning how reliable the formulated qualitative pattern of perturbed motion for large time spans is, at which the action of the ignored term H_2 in the exact Hamiltonian (5), even under the condition $\left|\frac{H_2}{H_1}\right|$, can become significant.

To answer this questions, Vavilov and Kolesnikov proceeded [35] from the canonical variables (r, ϑ, φ, p_r, p_ϑ, p_φ) to the canonical Delaunay variables (L_D, G_D, H_D, l_D, g_D, h_D) [66] associated with the osculating elements of the perturbed motion a, e, M, Ω, i, and ω. In [35] they showed that in this case the Hamiltonian function is independent of the generalized momentum h_D, and, therefore, the coordinate H_d is an integral of motion $H_D = H_{D0}$. Using a special canonical transformation, they further proceeded from the Delaunay variables to a new system of canonical variables $\bar{L}$, $\bar{G}$, $\bar{l}$, and $\bar{g}$ with the Hamiltonian $\bar{H}(\bar{L}, \bar{G}, \bar{L}, \bar{g}) = \bar{H}_0(\bar{L}) + \varepsilon\bar{H}_1(\bar{L}, \bar{G}) + \varepsilon^2\bar{H}_2(\bar{L}, \bar{G}, \bar{l}, \bar{g})$ 2π-periodic in the variables $\bar{l}$ and $\bar{g}$, where the dimensionless parameter $\varepsilon = \dfrac{M_E q \mu_E}{mcH_{D0}^3}$. The smallness of the perturbing action of the Lorentz force implies that the parameter $\varepsilon \ll 1$.

Using $\bar{l}$ instead of time allowed derivation of the system of canonical equations

$$\begin{cases} \dfrac{d\bar{G}}{d\bar{l}} = \Phi_{\bar{g}} = \dfrac{\bar{H}_{\bar{g}}}{\bar{H}_{\bar{L}}} \\ \dfrac{d\bar{g}}{d\bar{l}} = -\Phi_{\bar{G}} = \dfrac{\bar{H}_{\bar{G}}}{\bar{H}_{\bar{L}}}, \end{cases} \tag{16}$$

specifying the family of trajectories lying on the isoenergetic surface $\bar{H}(\bar{L}, \bar{G}, \bar{L}, \bar{g}) = C$. The explicit form of the mapping $\tilde{\varphi}_\varepsilon$ was obtained in the phase plane $\bar{G}$, $\bar{g}$ for system (16), which is determined by the displacement along the trajectories of this system of equations from $\bar{l} = 0$ to $\bar{l} = 2\pi$. It was shown that the mapping $\tilde{\varphi}_\varepsilon$ satisfies the conditions of Moser's theorem [67] on the properties of near twist mapping of a plane annulus guaranteeing the existence of an infinite set of invariant curves of the mapping $\tilde{\varphi}_\varepsilon$ for sufficiently small ε. The trajectories of the studied system of canonical equations in the four-dimensional phase space $\overline{L}$, $\overline{G}$, $\overline{l}$, $\overline{g}$ that emerge from the points of the invariant curve form a two-dimensional invariant torus. The motions over the surface of invariant tori are quasi-periodic with two frequencies. Any trajectory beginning between the two invariant tori on the same isoenergetic surface must endlessly remain between them. For any solution to the considered system of canonical equations for sufficiently small ε, this guarantees endless proximity of the current value of the variable $\overline{G}(t)$ to the initial value $\overline{G}(0)$, that is, $|\overline{G}(t) - \overline{G}(0)| \leq \delta(\varepsilon)$, where $\delta(\varepsilon) \to 0$ as $\varepsilon \to 0$. Thus, for sufficiently small ε the variable $\overline{G}$ and the corresponding Delaunay variable G_D can be regarded as quasi-integrals of motion. The variables $\overline{L}$ and L_D have an analogous property. Then, taking into account the expressions associating the Delaunay variables with the osculating elements of the perturbed orbit, the proved implies that for sufficiently small ε we must remain perpetually close to the initial values of the current values of the osculating elements of the perturbed orbit a and e characterizing its geometric parameters, as well as the endless proximity to the initial value of the angle of inclination i of the orbit to the plane of the geomagnetic equator.

It is well-known that the charge of a microparticle moving in NES varies, and the character of this variation depends on which region of NES is the performed motion in. For a particle with a low yield of photoemission, we can have situations when its electric charge can be considered quasi-equilibrium with a high accuracy, that is, depending only on the position of the particle along the trajectory:

$$Q = Q(\mathbf{r}), \tag{17}$$

where $\mathbf{r}$ is the position vector of the current position of the MP.

For this case, in [40, 41] Yakovlev and Kolesnikov generalized the results obtained in [35]. First of all, they determined the applicability conditions of the canonical problem formulation about the motion of a particle with a locally equilibrium electric charge in NES.

The general condition of correctness of the canonical problem formulation about the motion of a microparticle in NES is the possibility of representing all forces $\mathbf{F}^j$ acting upon the MP as

$$\mathbf{F}^j = -\nabla V^j \quad \text{or} \quad F_i^j = -\frac{\partial U^j}{\partial x_i} + \frac{d}{dt}\frac{\partial U^j}{\partial \dot{x}_i} \quad (i = 1,2,3),$$

where $V^j = V^j(x_1, x_2, x_3, t)$ is the common potential and $U^j(x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3, t)$ is the generalized potential. In the course of the motion of a MP at sufficiently high altitudes, we can ignore the drag force of background gas of the upper atmosphere, and, therefore, the main forces acting upon the MP are the gravitational force $\mathbf{F}^{\mathrm{gr}}$, the solar pressure force $\mathbf{F}^{\mathrm{pr}}$, and the Lorentz force $\mathbf{F}^L$. The velocity-independent gravitational force and solar pressure force (for a spherical MP) are represented by the common potentials V^{gr} and V^{pr}. Here, the Lorentz force acting in general with a variable charge on a MP from the side of the magnetic and electric fields of NES, as shown in [38], admits a representation by means of a generalized potential U^L when the corresponding restrictions on the character of variation in the MP charge in the course of its orbital motion, as well as on the peculiarities of the spatial distribution of the magnetic and electric fields of NES, are fulfilled. The first of the indicated restrictions consists in fulfillment of the quasi-equilibrium condition (17) for the electric charge of the MP, which is determined by the local values of the background plasma parameters and depends only on the current coordinates of the MP.

The second restriction (on the geometric characteristics of the fields) consists in fulfillment of the following conditions:

$$\mathbf{B} \perp \nabla q \tag{18}$$

$$\mathbf{E} \parallel \nabla q. \tag{19}$$

As shown in [38], when conditions (18) and (19) are met, the Lorentz force can be represented by the generalized potential

$$U^L = Y_1 - \frac{\mathbf{v} \cdot \mathbf{Y}_2}{c} \tag{20}$$

where Y_1 and $\mathbf{Y}_2$ are the scalar and vector functions of coordinates satisfying the equations

$$\begin{cases} \nabla Y_1 = Q \nabla \varphi \\ \mathrm{curl}\mathbf{Y}_2 = Q \mathrm{curl}\mathbf{A}. \end{cases} \tag{21}$$

In (21) φ and **A**, respectively, are the scalar and vector potentials of the electric and magnetic fields of NES acting upon a MP. Such a method of introducing the generalized charge-field potentials is natural, because for a constant charge the functions Y_1 and $\mathbf{Y}_2$ are determined by

$$Q_1 = Q\varphi, \quad \mathbf{Y}_2 = Q\mathbf{A}$$

which at their substitution into (20) yield the well-known expression for the generalized potential of the Lorentz force in the case of constant charge.

As shown in [38], taking into account (20), the Hamiltonian function describing the dynamics of a microparticle with a variable charge in NES is determined by the formula

$$H = \frac{1}{2m}\sum_{i=1}^{3}\left(P_i - \frac{Y_{2i}}{c}\right)^2 + V^{\mathrm{gr}} + V^{\mathrm{pr}} + Y_1, \tag{22}$$

where the components of the generalized momentum vector $P_i = m\dot{x}_i + \dfrac{Y_{2i}}{c}$.

In [38], using the results of numerical modeling, Kolesnikov et al. demonstrated that in the course of motion of a MP in Earth's plasmasphere, condition (17) turns out to be fulfilled for microparticles with radii larger than 0.01 μm whose orbits pass at distances larger than one-and-a-half radius of the Earth from its surface. The geometric conditions (18) and (19) can be satisfied in Earth's plasmasphere, which follows from the peculiarities of the spatial distribution of plasma in it. It is known that this distribution can be described by the model of two-component plasma [68].

The equilibrium charge of a body, being a function of local density and temperature, is thus independent of the longitudinal coordinate along the force line of the magnetic field, which corresponds to fulfillment of condition (18). In this case $Q = Q(L)$, where L is the parameter of the local magnetic sheath. The main contribution to the geoelectric field at plasmasphere altitudes is made, as is known, by the electric field of corotation E_{cor}. We can easily prove that in the case of approximation of the geomagnetic field by a dipole magnetic field with a magnetic moment antiparallel to Earth's axis of rotation, the field of corotation fulfills the necessary condition (19).

For further consideration, in [40–42] several additional simplifying assumptions were made:

(i) MPs made of material with a low photoemission yield and having radii of approximately 100th fractions of a micron were considered.

(ii) The motion of a MP occurs along slightly elongated orbits (with an eccentricity less than 0.3–0.4) with a perigee altitude larger than Earth's radius.

Under these assumptions, the considered problem is reduced to the problem of the motion of a MP with variable electric charge in the superposition of the central gravitational field and the Lorentz force acting on the electric charge of the MP from the dipole geomagnetic field and electric field of corotation.

In the geomagnetic spherical coordinate system (r, ϑ, φ) with the origin at the center of the Earth and the polar axis passing through the south magnetic pole, the Hamiltonian function (22) can be written as follows:

$$H = \frac{1}{2m}\left[P_r^2 + P_\vartheta^2 + \frac{1}{r^2\sin^2\vartheta}\left(P_\varphi - \frac{Y_{2\varphi} r\sin\vartheta}{c}\right)^2\right] - \frac{m\mu_E}{r} + Y_1. \tag{23}$$

The value of the electric charge $Q(L)$ is determined by solving the balance equation of charging currents (3) numerically.

In [40], we proposed to approximate the charge of a MP by the polynomial

$$Q(L) = Q_0\left[1 + \sum_{k=1}^{n}\xi_k\left(\frac{L - L_0}{L}\right)^k\right], \tag{24}$$

where Q_0 is the charge at $L = L_0$. The values of the coefficients ξ_k can be determined by the substitution of expression (24) into Eq. (3). As shown in [40, 41], an increase in the degree of the polynomial substantially improves the accuracy of approximation even for a large range of values L. To derive the expressions for $Y_{2\varphi}$ and Y_1, the dipole coordinates L, Φ, and M [69], formula (17) for the charge, and the Lamé coefficients h_i for the dipole coordinates were used. Then

$$Y_{2\varphi}h_\Phi = -Q_0 B_E R_E^2 \int \left[1 + \sum_{k=1}^{n} \xi_k \left(\frac{L - L_0}{L}\right)^k\right] \frac{1}{L^2} dL, \quad (25)$$

$$Y_1 = Q_0 h_L \int \left[1 + \sum_{k=1}^{n} \xi_k \left(\frac{L - L_0}{L}\right)^k\right] E_{\text{cor}} dL, \quad (26)$$

where $B_E = \frac{M_E}{R_E^3}$ is the induction of the magnetic field in the plane of the magnetic equator $\left(\vartheta = \frac{\pi}{2}\right)$ on Earth's surface [63] and $E_{\text{cor}} = -\frac{CR_E}{r^2}$ is the field of corotation with $C = 92$ kV.

In the case of a small eccentricity of the unperturbed Keplerian elliptic orbit and three first terms in polynomial (24), from (25) and (26) the expressions for the generalized potentials in spherical coordinates were obtained in the form

$$Y_{2\varphi} = -\frac{Q_0 M_E \sin\vartheta}{r}\left[\frac{1 + \xi_1 + \xi_2}{r} - \frac{R_E L_0 \sin^2\vartheta(\xi_1 + 2\xi_2)}{2r^2} + \frac{\xi_2 R_E^2 L_0^2 \sin^4\vartheta}{3r^3}\right],$$

$$Y_1 = \frac{Q_0 C R_E \sin\vartheta}{\sqrt{1 + 3\cos^2\vartheta}}\left[\frac{1 + \xi_1 + \xi_2}{r} - \frac{R_E L_0 \sin^2\vartheta(\xi_1 + 2\xi_2)}{2r^2} + \frac{\xi_2 R_E^2 L_0^2 \sin^4\vartheta}{3r^3}\right].$$

If we ignore the field of corotation, then, as shown in [41, 42], the Hamiltonian of the problem can be represented as the sum:

$$H = H_0 + H_1 + H_2, \quad (27)$$

where

$$H_0 = \frac{1}{2m}\left(P_r^2 + P_\vartheta^2 + \frac{P_\varphi^2}{r^2\sin^2\vartheta}\right) - \frac{m\mu_E}{r}, \quad H_1 = \frac{P_\varphi Q_0 M_E \eta_1}{mcr^3},$$

$$H_2 = \frac{Q_0^2 M_E^2 \sin^2\vartheta\,\eta_2}{2mc^2r^4} + O\left(\frac{1}{r^4}\right), \quad \eta_1 = 1 + \xi_1 + \xi_2,$$

$$\eta_2 = 1 + 2\xi_1 + 2\xi_2 + \xi_1^2 + \xi_2^2 + 2\xi_1\xi_2 - \eta_3, \quad \eta_3 = \frac{P_\varphi L_0 R_E c}{Q_0 M_E}(\xi_1 + 2\xi_2).$$

For the values of the radius of hundredth fractions of a micrometer and the above specified values of the parameter L, all coefficients η_k are on the order of unity. Therefore, for the considered case the expressions for the components of the Hamiltonian coincide with their expressions for the case of a constant charge up to a multiplier on the order of unity. Hence, all conclusions made for the constant charge remain valid in the current case. In this regard, this raises the question of how taking into account the effect of the field of corotation can affect the behavior of the orbit. When Y_1 was substituted into (23) in [70, 71], the following expression was obtained:

$$H = H_{\text{cor}} = H_0 + H_1 + H_2 + \frac{Q_0 C R_E \sin\vartheta}{\sqrt{1 + 3\cos^2\vartheta}}\left[\frac{1 + \xi_1 + \xi_2}{r} - \frac{R_E L_0 \sin^2\vartheta(\xi_1 + 2\xi_2)}{2r^2} + \frac{\xi_2 R_E^2 L_0^2 \sin^4\vartheta}{3r^3}\right]. \quad (28)$$

Unlike the previous case, transition to Delaunay variables does not lead to a simple representation: $H_{\text{cor}} = H_{0,\text{cor}} + H_{1,\text{cor}} + H_{2,\text{cor}}$ with $H_{0,\text{cor}}$ depending only on L_D. Therefore, in the general case $\left(\text{for } \vartheta \neq \frac{\pi}{2}\right)$, even in the zeroth order of expansion of the Hamiltonian in the small parameter, preservation of the main parameters of the orbits is not provided. However, if $\vartheta = \frac{\pi}{2}$, for the Hamiltonian H_{cor} we can introduce the effective parameter $\mu_{E,\,\mathrm{eff}} = \mu_E - \dfrac{Q_0 C R_E}{m}(1 + \xi_1 + \xi_2)$, which allows writing $H_{0,\,\mathrm{cor}}$ in the same form as H_0 in formula (27), and, moreover,

$$H_{1,\mathrm{cor}} = -\frac{Q_0 C L_0 R_E^2(\xi_1 + 2\xi_2)}{2r^2},$$

$$H_{2,\mathrm{cor}} = \frac{\xi_2 Q_0 C L_0^2 R_E^3}{3r^3} + \frac{P_\varphi Q_0 M_E \eta_1}{mcr^3} + \frac{Q_0^2 M_E^2 \eta_2}{2mc^2 r^4} + O\left(\frac{1}{r^4}\right).$$

That is, during motion in the equatorial plane, taking into account the field of corotation reduces to conservation in the zeroth approximation of the semi-major axis, eccentricity, and angle of inclination. Taking into account the term $H_{2,\,\mathrm{cor}}$, proportional to r^{-3}, leads to small oscillations of the semi-major axis and eccentricity in the same manner as was previously shown in [41, 42] for the perturbing action of the magnetic field. At the same time, the term $H_{1,\,\mathrm{cor}}$, proportional to r^{-2}, causes the existence of secular variations in the orbital parameters.

These conclusions are confirmed by the results of numerical modeling published in [72]. In addition, in [73] Kolesnikov et al. used numerical modeling to study the effect of the initial angle of inclination and shadowing of a part of the orbit on the character of the variation in the orbital parameters. They showed that a nonzero initial angle of inclination causes oscillations of the semi-major axis, eccentricity, and angle of inclination, steady growth in the average value i, and the appearance of an additional frequency of oscillations of the MP potential coinciding with the frequency of oscillations a, i, and e. The effect of shadowing of a part of the orbit causes no significant variation in the behavior of the MP potential, eccentricity, and angle of inclination of the orbit and smoothens the pronounced character of the behavior of the semi-major axis.

Analysis of the results and method discussed above allowed us [36] to make a conclusion about the appearance of a new object of dynamic investigation, i.e, a body of constant mass and variable electric charge. Papers [74–76] were devoted to further development of the methods for studying this new object. Special attention in these works was drawn to generalization of the theory of the drift motion of particles with constant charge presented in [77] to the case of variable charge. In the model of locally equilibrium electric charge, the existence of a new type of drift was proved, called the charge drift in [36].

The pollution of space by components of space debris with dimensions from fractions of a micrometer to several meters has prompted research and development to create clean-up systems. In [78, 79], a method for removing fine-disperse components from NES using beams of high-energy neutral particles was considered. In [80, 81], an original approach was proposed to the study of using a tethered system for cleaning NES from the large-scale component of space debris. In particular, it was demonstrated that for stable functioning of the proposed system, we need to use a facility for electric discharge with a very broad range of variation in the current value.

A detailed review of the publications related to Section 2 can also be found in [82].

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CONFLICT OF INTEREST

The author of this work declares that he has no conflicts of interest.

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