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ELEMENTARY PARTICLES AND FIELDS
Theory

Dynamical $O(4)$-Symmetry in the Light Meson Spectrum within the Framework of the Regge Approach

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Abstract—The light mesons tend to cluster near certain values of mass. As was noticed some time ago, the emergent degeneracy is of the same type as the dynamical $O(4)$-symmetry of the Coulomb potential in the hydrogen atom. The meson mass spectrum can be well approximated by the linear Regge trajectories of the kind $M^2 = al + bn_r + c$, where l and n_r are angular momentum and radial quantum numbers and a, b, c are parameters. Such a spectrum arises naturally within the hadron string models. Using 2024 data from the Particle Data Group, various fits for $M^2(l, n_r)$ were performed. Our analysis seems to confirm that $a \approx b$ in the light non-strange mesons, i.e., their masses depend on the sum $l + n_r$ as prescribed by the hydrogen-like $O(4)$-symmetry. Using the semiclassical approximation, we discuss which kind of hadron string models are more favored by the experimental data.

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1. INTRODUCTION

The light hadron spectrum has been extensively studied since the appearance of the Quark Model 60 years ago. A large interest in this topic is not surprising—the search for possible theoretical models describing the spectrum of the simplest bound states of quarks is tightly related with unveiling the nature of non-perturbative strong interactions and may lead to discoveries of new manifestations of strong interactions. The light meson spectrum is especially interesting in this quest since such particles consist of u- and d-quarks, just like nucleons.

The discrete spectrum of light meson resonances is somewhat simple: almost all known excited meson states included in the Particle Data Group (PDG) Tables [1] are clustered near certain values of mass. Apparently, there are four such clusters, see Table 1.

The degeneracies in the spectrum of light meson excitations was broadly discussed about 15 years ago (see, e.g., [2–9]). The following relation between the mass of a hadron and its quantum numbers can be written,

$$M^2 = al + bn_r + c, \quad (1)$$

where l and n_r are the angular momentum and the radial quantum number, and a, b and c are some constants of dimensionality $[M^2]$. It is important to notice that although the separation of total angular momentum J into orbital angular momentum L and intrinsic quark–antiquark spin S is not Lorentz-invariant, the spin–orbital correlations seem to be highly suppressed inside excited light mesons because their typical lifetime is much less than the typical time required for noticeable effects of spin–orbital interactions. As a result, the standard quantum-mechanical relationship, $J = L + S$, is fulfilled with good accuracy. The $q\bar{q}$ pairs can form either singlet or triplet states with $S = 0$ or $S = 1$, respectively. However, it appears that the observed degeneracy is independent of S, and both singlet and triplet states are clustered together, with the positions of the clusters depending only on the angular momentum number l.

Long ago, Chew and Frautschi observed that there exist linear Regge trajectories for angularly excited mesons. Later, it was observed that the daughter Regge trajectories are almost equidistant, i.e., the radial meson trajectories are approximately linear as well [10]. The analysis of the experimental data showed that the slopes of angular and radial trajectories are almost the same as predicted by the Veneziano-like dual amplitudes and related hadron string models [4, 5]. In view of the appearance of

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new data in the last 15 years, it is interesting to reanalyze the experimental data and check to what extent they are compatible with the aforementioned exciting pattern of string-like spectrum.

The paper is organized as follows. The modern light meson spectrum is briefly discussed in Section 2. In Section 3, we describe a simple semiclassical hadron string model which will be used later. The theoretical models describing the spectrum are presented in Section 4, and the last Section 5 contains a short summary and some conclusions.

2. THE MESON SPECTRUM

For our analysis, the PDG tables [1] were used. Since there is not enough basic data, we used not only the main meson table but also included resonances from the "Further States" section. Most of the mesons in that section were detected in the Crystal Barrel experiment and need further confirmation.

The internal quark structure of some meson resonances is not known yet. For example, we know for sure that the π-mesons contain either u or d-quarks and antiquarks. We also know that the isosinglet $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states are mixed together. For example, in the case of isosinglet vector mesons, $I^G(J^{PC}) = 0^-(1^{--})$, the mixing angle is close to 35.3°. Hence, the ω and ϕ mesons can be considered as almost pure $u\bar{u}$-$d\bar{d}$ and $s\bar{s}$-states. In most cases, the mixing angle is not known, so we have to make some assumptions about the amount of the $s\bar{s}$-component.

There are other difficulties related to the intrinsic structure of observed resonances. Some of the states that are seen in the experiment look more complicated than just a bound state of a quark and an antiquark. They might represent a tetraquark, a meson molecule, a glueball, or something else. A detailed discussion of each state will be given elsewhere. The resonances that are of interest should, first, be of a meson nature and, secondly, be a mixture of light quarks and antiquarks, without any hidden strangeness.

Some states, such as $f_1(1285)$ and $f_1(1420)$ or $h_1(1170)$ and $h_1(1415)$, have relatively close masses (a typical excitation energy in hadrons is about 500 MeV), and all the quantum numbers are the same. As soon as we do not know for sure the internal quark structure of the mesons, we may assume that these two states represent a mixture of two components, the $u\bar{u}$-$d\bar{d}$-one and the $s\bar{s}$-one, with an unknown mixing angle. The heavier state is believed to contain a bigger $s\bar{s}$-part. In some cases, the difference between the "heavy" and "light" states is close to the doubled mass of the s-quark.

Table 1. The approximate positions of clusters of light non-strange mesons

N	M, MeV	Amount
1	≈1300	11 states
2	≈1600	8 states
3	≈2000	22 states
4	≈2250	24 states

The first task is to assign two quantum numbers, n_r and l, to each observed state depending on its mass. The results of our prescriptions are shown in Fig. 4. Then, using the multiple linear regression method, several fits for Eq. (1) were performed, with results shown in Table 2. Masses of resonances in the PDG tables are known with different precision. Thus, weights in the fit should be different for them, depending on the reliability of the point included in the analysis.

The data is presented in two datasets. The first one includes only resonances with $I = 1$, because they should not contain $s\bar{s}$ component by definition of isospin. The second dataset contains all the resonances from the first dataset and additionally all the mesons with $I = 0$, which are believed to have relatively small $s\bar{s}$-component. The results obtained are slightly different for these two cases. In the second dataset, the slope for the angular trajectory is a bit smaller than for the radial one. The difference, however, lies within the experimental uncertainty.

The first regression was done for the "raw" masses without any weighting. In the second and the third fit, the weight of the resonance is defined as $w_i = \frac{1}{\sigma_i^2}$. The parameter σ_i can be the experimental error ΔM^2_{exp}, known from the PDG table, or the difference ΔM^2 between the average mass squared in every cluster and the observed mass squared for each state. In the last fit, the weighting function is introduced manually, satisfying the following conditions.

Firstly, if the resonance is well-established and its mass is known with high precision, the weight must be close to 1. Secondly, the weighting function should not fall off too rapidly. By changing the parameter A, we can obtain slightly different weight distributions.

The ground states with $n_r = l = 0$ were excluded in all fits.

The obtained average slopes for l and n_r turned out to be very close, numerically (in GeV^2)

$$a = 1.09 \pm 0.02, \quad b = 1.14 \pm 0.02$$

for the first dataset,

$$a = 1.13 \pm 0.05, \quad b = 1.15 \pm 0.07$$

for the second one.

L	S	J	$n_r = 0$	$n_r = 1$	$n_r = 2$	$n_r = 3$	$n_r = 4$
0	0	0	π	$\pi(1300)$	$\pi(1800)$	$\pi(2070)$	$\pi(2360)$
			η	$\eta(1295)$	$\eta(1760)$	$\eta(2010)$	$\eta(2320)$
	1	1	ρ	$\rho(1450)$	$\rho(1700)$	$\rho(2000)$	$\rho(2270)$
			ω	$\omega(1420)$	$\omega(1650)$	$\omega(1960)$	$\omega(2290)$
1	0	1	$b_1(1235)$	b_1?	$b_1(1960)$	$b_1(2240)$	
			$h_1(1170)$	$h_1(1595)$	$h_1(1965)$	$h_1(2215)$	
	1	0	$a_0(1450)$	$a_0(1710)$	$a_0(2020)$	a_0?	
			$f_0(1370)$	$f_0(1770)$	$f_0(2020)$	$f_0(2200)$	
		1	$a_1(1260)$	$a_1(1640)$	$a_1(1930)$	$a_1(2270)$	
			$f_1(1285)$	f_1?	$f_1(1970)$	$f_1(2310)$	
		2	$a_2(1320)$	$a_2(1700)$	$a_2(2030)$	$a_2(2255)$	
			$f_2(1270)$	$f_2(1750)$	$f_2(1950)$	$f_2(2300)$	
2	0	2	$\pi_2(1670)$	$\pi_2(2005)$	$\pi_2(2285)$		
			$\eta_2(1645)$	$\eta_2(2030)$	$\eta_2(2250)$		
	1	1	ρ?	ρ?	ρ?		
			ω?	ω?	$\omega(2220)$		
		2	ρ_2?	$\rho_2(1940)$	$\rho_2(2225)$		
			ω_2?	$\omega_2(1975)$	$\omega_2(2195)$		
		3	$\rho_3(1690)$	$\rho_3(1990)$	$\rho_3(2250)$		
			$\omega_3(1670)$	$\omega_3(1945)$	$\omega_3(2255)$		
3	0	3	$b_3(2030)$	$b_3(2245)$			
			$h_3(2025)$	$h_3(2275)$			
	1	2	$a_2(1990)$	a_2?			
			$f_2(2010)$	f_2?			
		3	$a_3(2030)$	$a_3(2275)$			
			$f_3(2050)$	$f_3(2300)$			
		4	$a_4(1970)$	$a_4(2255)$			
			$f_4(2050)$	f_4?			
4	0	4	$\pi_4(2250)$				
			$\eta_4(2330)$				
	1	3	ρ_3?				
			$\omega_3(2285)$				
		4	$\rho_4(2230)$				
			$\omega_4(2250)$				
		5	$\rho_5(2350)$				
			$\omega_5(2250)$				

$N = 0$
$N = 1$
$N = 2$
$N = 3$
$N = 4$

$I = 1$
$I = 0$

$N = n_r + L$

Fig. 1. Classification of the light unflavored mesons used in the present analysis. The missing states are marked with a question. The poor-known states are shown on the pale background.

Thus we can conclude that, at least in the first approximation, the whole spectrum seems to depend on a linear combination of $n_r + l$ as prescribed by the $O(4)$-symmetry. As in the hydrogen atom, we can introduce the principal quantum number N,

$$N \equiv l + n_r, \quad M^2 \approx 1.14 \cdot N + 0.5,$$

Table 2. Examples of fits for the relation $M^2 = al + bn_r + c$

I	Weight	a	b	c	χ^2/DoF
1	none	1.08 ± 0.03	1.13 ± 0.03	0.69 ± 0.09	0.0052
	$1/(\Delta M^2_{\text{exp}})^2$	1.10 ± 0.03	1.16 ± 0.03	0.63 ± 0.03	0.0050
	$1/(\Delta M^2)^2$	1.113 ± 0.005	1.126 ± 0.005	0.62 ± 0.01	0.0051
	$\exp\left(-A\frac{\Delta M^2_{\text{exp}}}{M^2}\right)$, $A = 7$	1.08 ± 0.03	1.13 ± 0.03	0.69 ± 0.09	0.0051
0 and 1	none	1.13 ± 0.20	1.15 ± 0.04	0.58 ± 0.03	0.0045
	$1/(\Delta M^2_{\text{exp}})^2$	1.14 ± 0.03	1.13 ± 0.04	0.51 ± 0.03	0.0057
	$1/(\Delta M^2)^2$	1.12 ± 0.11	1.16 ± 0.09	0.52 ± 0.03	0.0051
	$\exp\left(-A\frac{\Delta M^2_{\text{exp}}}{M^2}\right)$, $A = 7$	1.13 ± 0.08	1.15 ± 0.03	0.56 ± 0.08	0.0045

where M^2 is given in GeV^2. Strictly speaking, $N = n_r + l + 1$ in the hydrogen atom, but here it is convenient to start N with zero value.

3. HADRON STRING MODELS

Various hadron string models are often exploited for the description of radially and angularly excited mesons. The idea is to consider a quark and antiquark as massless string ends bonded together with a gluonic flux tube. Then, the radial excitation of this string can be compared to the radially excited meson state, and the same may be done for the angular excitations.

Using hadron string models, the spectrum as in Eq. (1) can be obtained. In the reference frame in which the meson is at rest, the meson mass operator $\widehat{M}$ equals the hamiltonian $\widehat{H}$. For two ultrarelativistic quarks with small masses, the Bethe–Salpeter equation may be written, which is a relativistic analog of the Schrödinger equation:

$$\left(\sqrt{\hat{p}^2 + m_q^2} + \sqrt{\hat{p}^2 + m_{\bar{q}}^2} + V(r)\right)\psi = E\psi.$$

The quark masses are much smaller than their momenta, $\frac{m_q}{|p|} \ll 1$, and are considered equal, $m_q = m_{\bar{q}}$. The relativistic kinetic term is then

$$\sqrt{p^2 + m_q^2} + \sqrt{p^2 + m_{\bar{q}}^2} \approx 2p.$$

In the above relation, the momentum operator contains both radial and angular parts, $\hat{p} = \sqrt{-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2}}$. The potential part of the hamiltonian is the Cornell potential,

$$V(r) = \sigma r - C_F\frac{\alpha_s}{r} + c.$$

The first term here is the linear confining potential which dominates at large distances between the quark and the antiquark. In further analysis, it will play the main role, since it coincides with the typical potential of the string with tension σ. The second term describes the Coulomb-like interaction between quarks, which comes into play at short distances when quarks undergo a gluonic exchange. α_s is the strong running coupling, and the fact that it is not a constant should entail some complications. However, in the further analysis, the Coulomb-like term will always be negligible compared to the linear one. The constant C_F depends on the number of colors, $C_F \equiv (N_c^2 - 1)/2N_c$, that yields $C_F = 4/3$ for $N_c = 3$. The last term in the Cornell potential is a negative constant c. The necessity to add c is dictated by the nonrelativistic limit of the Bethe–Salpeter equation, and numerically it depends on the type of interacting quarks. Like with the Coulomb-like term, this constant will not play a role in the further analysis.

3.1. The Radial Quantum Number

For simplicity, we set $l = 0$ and consider only radial excitations of a hadron string. In this case, the momentum operator contains only the radial part $\hat{p} \equiv \hat{p}_r$. The potential part of the hamiltonian, as discussed previously, consists of three terms. At large distances, the linear term σr dominates, as if it were a classic string. Then, the meson mass is given by the relation

$$M = 2p_r + \sigma r.$$

The constant σ is the effective string tension. If the maximal flux tube length is ℓ, then the total mass of the string can be calculated as $M = \sigma \cdot \ell$. The linear trajectories corresponding to the radial excitations can be derived using the semiclassical approximation.

In the semiclassical approximation, the quark wave function may be written in the form

$$\psi(r,t) = \text{Const} \cdot e^{iEt} e^{-i \oint p(r)dr}.$$

This wave function is antisymmetric due to the fermionic nature of quarks. Taking this into account, the Bohr–Sommerfeld quantization condition reads as

$$\oint p_r(r)dr = 2\int_0^{\ell} p_r(r)dr = \pi(n_r + \gamma).$$

Substituting the momentum in the form $p_r = \frac{M-\sigma r}{2}$ one obtains the expected linear relation $M^2 = 2\pi\sigma(n_r + \gamma)$. The constant γ is of order of unity, and it depends on the turning points of the system, which are the ends of the string. For example, in the case of potentials that possess the central symmetry, $\gamma = \frac{1}{2}$.

The antisymmetry of the wave function discussed above is important [11]: in most of the previous works (for example, [5, 12, 13]), this was not taken into account, as a result the obtained slope between M^2 and n_r was equal to $4\pi\sigma$ instead of $2\pi\sigma$.

There is another way to derive the needed linear relation between M^2 and n_r. In this approach, the radial excitation is believed to be due to the collective gluon excitation the quarks are exchanging with (a pomeron, for example) at a constant separation. The kinetic term is now twice smaller. This collective excitation is believed to be a boson. Then, the quantization condition takes the form

$$\oint p(r)dr = 2\int_0^{\ell} p(r)dr = 2\pi(n + \gamma).$$

Although this approach is conceptually different from the previous one, the result is the same,

$$M^2 = 2\pi\sigma(n_r + \gamma).$$

Either way, the slope for the radial Regge trajectories should be equal to $\frac{1}{2\pi\sigma}$.

3.2. The Angular Momentum

The spectrum of the rotating relativistic string is known to be well described by the Chew–Frautschi formula,

$$M^2 = 2\pi\sigma l.$$

Consider a gluonic flux tube as a solid body of known length ℓ, rotating at the speed $v(r) = 2r/\ell$. The total mass M and the angular momentum l of such a tube may be calculated as

$$M = 2\int_0^{\ell/2} \frac{\sigma dr}{\sqrt{1 - v^2(r)}} = \frac{\pi\sigma\ell}{2},$$

$$l = 2\int_0^{\ell/2} \frac{\sigma r v(r) dr}{\sqrt{1 - v^2(r)}} = \frac{\pi\sigma\ell^2}{8}.$$

Then, combining these two equations, one gets the Chew–Frautschi formula. This nice result was first derived by Nambu [14]. The coefficient between M^2 and the angular momentum is the same as for the radial excitation, $2\pi\sigma$, and this equality agrees well with the experimental data as discussed in the previous section.

There is another way to obtain the linear relation between M^2 and the angular momentum using semiclassical methods, although the Regge slope happens to be different. If two quarks remain in circular orbits, then the quark wave function should not change within the phase changing to $\Delta\Phi = 2\pi rp = lh$. The Bohr–Sommerfeld quantization condition reads as

$$\oint p(r)dr = l\hbar. \quad (2)$$

The quarks moving on circular orbits undergo the centripetal acceleration $a = \frac{v^2}{r}$. The second Newton law is then

$$\frac{\partial V(r)}{\partial r} = F = ma$$
$$= \frac{mv^2}{r} = \frac{mv^2}{2}\frac{2}{r} = E_{\text{kin}} \cdot \frac{2}{r}.$$

As before, consider the linear part of the Cornell potential that gives $\frac{\partial V(r)}{\partial r} = \sigma$, and

$$\sigma = p\frac{2}{r} \quad \Rightarrow \quad p = \frac{\sigma r}{2}. \quad (3)$$

Combining (2) and (3), we obtain a similar formula for the angular momentum,

$$l = rp = 2 \cdot \frac{r}{2}p = 2 \cdot \frac{\sigma r}{2} \cdot \frac{r}{2} = \frac{\sigma r^2}{2}.$$

After a simple calculation that includes squaring of the relation $M = 2p + \sigma r$ and some substitutions, we finally get

$$M^2 = 8\sigma l.$$

As predicted by the experimental data, there is a linear dependence of M^2 on the angular momentum. The slope, however, is different from the one for the radial quantum number. Nevertheless, there are reasons to doubt the adequacy of this second semiclassical model. In reality, when a meson resonance appears, its typical lifetime is too short for a quark—antiquark pair to have time to complete a full "revolution" around a common center of mass. Thus, the second model looks unrealistic. On the other side, when quarks move past each other, the forming gluon string between them must actually "turn" through a certain angle as it happens within the Nambu string picture. Thus, the first model should better capture the underlying physics.

4. CONCLUSIONS

Our new Regge analysis of experimental data confirms the existence of a broad mass degeneracy in the light unflavored mesons.

In the first approximation, this spectrum depends linearly on the radial quantum number n_r and the orbital number l, with

$$M^2 = al + bn_r + c.$$

The analysis of the experimental data shows that $a \approx b \approx 1.14$ GeV^2. This means that the considered meson mass spectrum must depend on a single quantum number $N = n_r + l$. This leads to the $O(4)$-like degeneracy of the same kind as in the hydrogen atom.

The observed degeneracy has a simple qualitative explanation within the framework of semiclassical hadron string approach resulting in the relation $a = b = 2\pi\sigma$ for the frequencies of orbital and oscillatory motions.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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