D. MINGAZOV, M. KOZIN[1]
(Saint Petersburg State University, Saint Petersburg)

RESEARCH ON CONSENSUS AND FIXED-TIME CONTROL METHODS FOR MULTI-AGENT SYSTEMS

This work examines consensus and fixed-time control algorithms for multi-component systems. The study explores mathematical models using iterative consensus and control methods, and proposes a fixed-time approach based on nonlinear functions. Numerical experiment results are provided to demonstrate the effectiveness of the proposed methods.

Introduction. Modern multi-agent systems (MAS) are a critical area of research, particularly in their applications within robotics, autonomous vehicle control, and communication networks. Coordinating agents, each following local behavioral rules and interacting with neighbors, presents a significant challenge. A key issue is achieving a consistent state (consensus) across all agents in the system, despite limited computational resources and time constraints.

Traditional consensus methods, such as linear information exchange models between agents, were introduced in the work of Olfati-Saber et al. [3]. These methods enable asymptotic convergence to a common state, making them useful for various practical applications. However, a major drawback of these algorithms is the requirement of unlimited time to reach consensus. This limitation reduces their effectiveness in systems that require convergence within a finite time.

To address this issue, fixed-time control (Fixed-Time Convergence, FTC) methods have been proposed, ensuring system convergence within a fixed time interval regardless of the agents' initial conditions. This approach is based on nonlinear control functions and was thoroughly discussed in the works of Polyakov and Parsegov [4]. These methods guarantee rapid state alignment among agents, allowing consensus to be achieved in fixed time, which makes them appealing for tasks where response speed is critical.

This study investigates combined consensus and FTC methods for multi-agent systems. Specifically, an algorithm is considered that employs nonlinear control functions to accelerate convergence. The control expression is written as follows:

$$U_t = \sum_{j \neq i} K_1(\sqrt[3]{|x_i - x_j|}) + K_2(x_i - x_j)^3 \qquad (1)$$

where: x_i and x_j are the states of agents, and K_1 and K_2 are constants that regulate the rate of convergence. Using such control functions enables consensus to be reached faster and within a finite time compared to classical approaches.

This report is focused on examining the effectiveness of combined consensus and fixed-time control methods for multi-agent systems. Numerical experiments are presented to demonstrate the advantages of the proposed approach in managing systems with limited time resources.

Problem Statement. We consider a system with N agents, each with a state $x_i(t)$ at time t, where $i = 1, 2, ..., N$. The goal is to develop an algorithm that ensures consensus among all agents within a fixed time. This implies that there exists a time Tafter which the states of all agents coincide:

$$x_i(t) = x_j(t) \quad \forall i, j \in \{1, 2, ..., N\} \qquad (2)$$

The task is to develop a control algorithm that ensures the alignment of all agents' states within a finite time T, regardless of their initial conditions.

Problem Conditions. To solve the problem, it is assumed that agents can interact with a limited number of neighbors according to a communication graph. The interactions between agents are described by an adjacency matrix $A = [a_{ij}]$, where $a_{ij} \neq 0$ indicates a connection between agents i

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[1] **Academic Advisor:** Dr. Oleg Granichin, Doctor of Physical and Mathematical Sciences, Professor in the Department of System Programming, Faculty of Mathematics and Mechanics, Saint Petersburg State University

and j, and $a_{ij} = 0$ indicates its absence. The dynamics of the agents' state changes are described as follows:

$$x_i^{(t+1)} = x_i^{(t)} + \sum_{j\neq i} a_{ij}[K_1(\sqrt[3]{\left|x_j^{(t)} - x_i^{(t)}\right|}) + K_2(x_j^{(t)} - x_i^{(t)})^3] \quad (3)$$

where: K_1 and K_2 are positive constants that regulate the rate of convergence. This nonlinear control function includes two components: the first component, involving the cubic root, is responsible for the "soft" alignment of agent states, while the second accelerates the consensus process through the cubic difference of states.

Stability Analysis. To analyze the stability of the system, let us consider the Lyapunov function $V(t)$, defined as:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{J=1}^{N} a_{ij}(x_j(t) - x_i(t))^2 \quad (4)$$

This function characterizes the divergence of agent states and must decrease over time to ensure convergence. The time derivative of the Lyapunov function is calculated as:

$$\widehat{V}(t) =- K_1\sum_{i=1}^{N}\sum_{J=1}^{N} a_{ij}(\sqrt[3]{\left|x_i - x_j\right|})^2 - K_2\sum_{i=1}^{N}\sum_{J=1}^{N} a_{ij}(x_j(t) - x_i(t))^6 \quad (5)$$

Both terms are decreasing, which confirms the stability of the proposed algorithm and guarantees that all agents converge to a single state within a finite time T.

Conclusion. This study examined various dynamic models for controlling multi-agent systems, specifically the consensus protocol, the Round-Robin algorithm, and a modified protocol incorporating Fixed-Time Convergence (FTC) elements. The findings indicate that FTC significantly enhances the convergence rate of the system due to its adjustable parameters, K_1 and K_2, which allow for precise control over the consensus achievement time.

Optimizing parameters K_1 and K_2 further demonstrated that FTC surpasses traditional methods in convergence speed. Future research could focus on fine-tuning these parameters through adaptive techniques or machine learning approaches. It is also crucial to investigate the application of FTC in the context of partially connected agent networks and under resource constraints.

In summary, this work illustrates the efficacy of FTC algorithms for achieving rapid consensus, and continued research will likely extend their applicability to more complex systems.

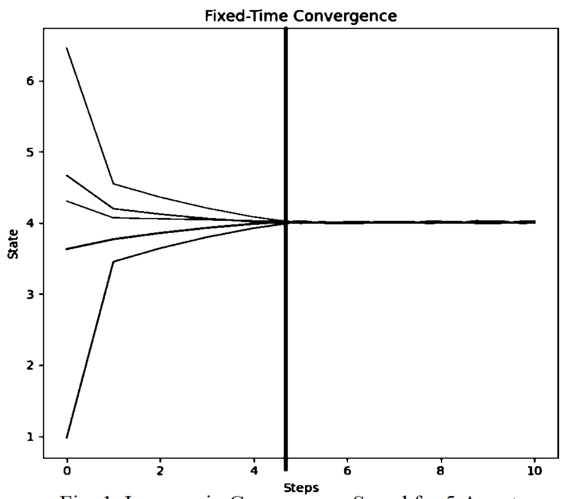

Fig. 1. Increase in Convergence Speed for 5 Agents

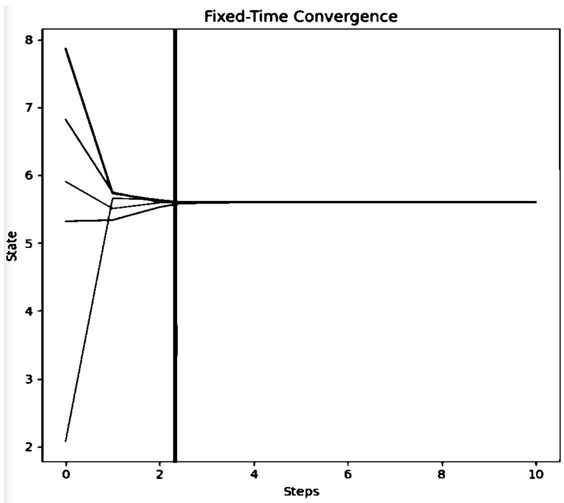

Fig. 2. Increase in Convergence Speed for 5 Agents

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D. Mingazov, M. Kozin (Saint Petersburg State University. St. Petersburg)

Research on consensus methods and fixed-time control for multi-agent systems

Abstract: The paper examines consensus algorithms and fixed-time control methods for multi-component systems. Mathematical models are explored using iterative consensus and control techniques, and a fixed-time approach based on nonlinear functions is proposed. Numerical experiments are presented, demonstrating the effectiveness of the proposed methods.