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Cumulative production at central rapidities due to interactions involving fluctons

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Keywords: Dense cold nuclear matter; quark-gluon clusters; NICA collider.

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1. Introduction

Cumulative production is the emission of particles in interactions involving nuclei in regions that are kinematically forbidden for reactions with free nucleons. This phenomenon was first observed experimentally in the works.[1,2] Then, as a consequence, the hypothesis[3] was put forward that a theoretical explanation for this effect could be the assumption of the presence in nuclei of heavy objects — fluctons consisting of several nucleons, which periodically arise in nuclei when nucleons approach each other at distances smaller than the radius of the nucleon.

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From a modern point of view, such multi-nucleon clusters inherent in nuclei can be considered as multi-quark bags — baryon-enriched quark-gluon clusters, similar to drops of cold quark-gluon plasma. A consequence of the existence of such cold dense nuclear matter clusters in nuclei is the emission of particles in regions kinematically forbidden for reactions with free nucleons (cumulative production). Until now, these effects have been studied experimentally[4–11] and theoretically[12–17] mainly only in the region of fragmentation of one of the colliding nuclei (target nucleus or projectile nucleus).

The construction of the NICA collider,[18,19] whose characteristic feature in comparison with the RHIC and LHC colliders is relatively low energies of colliding nuclei and high luminosities, opens up the possibility of studying this phenomenon in a new cumulative region of central rapidities and large transverse momenta.[20–24] It is important to note that the study of cumulative phenomena in this new region opens up the possibility of experimentally studying a new interesting process of flucton–flucton interaction in MPD and SPD experiments at NICA, which cannot be studied in the region of fragmentation of one of the nuclei.

In this regard, it is also necessary to note that studying this process in dd collisions at NICA SPD[25] has some additional advantages. In this case, there will be no contribution from additional nucleon–nucleon collisions, which will make it possible to study this process in its purest form, when both interacting deuterons are in the state of 6-quark bags at the moment of interaction.

In this paper, we calculate the asymptotic behavior (at high initial energies) of the inclusive cross-section for the emission of a quark with a large transverse momentum near the cumulative kinematic boundary of an AA collision in the region of central rapidities (it is assumed that later, at the hadronization stage, this quark fragments into a cumulative pion). We show that the main contribution in this region comes from the interaction of two fluctons. In our analysis, we use a generalization of the approach[17,26–30] developed earlier to describe particle production from a flucton and based on an estimate of the asymptotic behavior of the QCD diagrams near thresholds.

As a result, we find the quark counting rules for the inclusive cross-section, characterizing the asymptotic dependence of this cross-section on two parameters — the initial energy and the deviation of the momentum of the produced particle from the kinematic threshold. We also show that the proposed approach, in the case of its application to the analysis of the asymptotic behavior of elastic and quasi-elastic reactions, reproduces the already known[31–34] quark counting rules for the differential scattering cross-section in the region $|t| \sim s \gg m^2$.

2. General Formulae

The amplitude of the process leading to the production of a pion in the cumulative region with a large transverse momentum $k_\perp \sim \sqrt{s_{A_1A_2}}/2$ in a collision of two nuclei A_1 and A_2 is schematically shown in Fig. 1. Here, P_1 and P_2 are the momenta of the

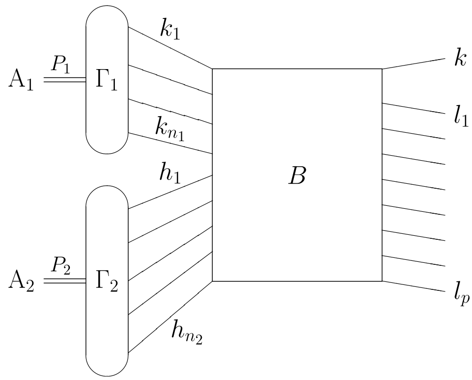

Fig. 1. Schematic illustration of the amplitude of the process leading to the production of a quark in the cumulative region with a large transverse momentum $k_\perp \sim \sqrt{s_{A_1A_2}}/2$ in a collision of two nuclei A_1 and A_2.

colliding nuclei and n_1 and n_2 are the numbers of their constituent "elementary" objects (constituent quarks and, possibly, diquarks[35]). It is also convenient to introduce the momenta per nucleon of the nucleus: $p_1 = P_1/A_1$, $p_2 = P_2/A_2$, then

$$s_{A_1A_2} \equiv (P_1 + P_2)^2 = A_1A_2s + (A_1 - A_2)^2m_N^2, \quad s \equiv s_{NN} = (p_1 + p_2)^2. \qquad (1)$$

For the case of a collision of identical nuclei $(A_1 = A_2 = A)$ $\sqrt{s_{AA}} = A\sqrt{s}$. The approach presented below is supposed to be used to describe pion production in nuclear collisions in the new cumulative region of large transverse momenta and central rapidities, accessible for observation in the SPD and MPD experiments at the NICA collider.[20–24] For example, to analyze dd and pp collisions, it is sufficient to set $A = 2$ and $A = 1$, respectively, in the general formulas obtained.

In this paper, we will also limit ourselves to calculating the inclusive cross-section of quark production with momentum k in the specified cumulative region, leaving aside the description of the process of its fragmentation into the observed cumulative pion, as well as the process of fragmentation of recoil quarks with momenta $l_1, \ldots, l_p$ into hadrons. In this case, the inclusive cross-section $I(\mathbf{k})$, corresponding to the amplitude T, presented in Fig. 1, will be equal to

$$I(\mathbf{k}) \equiv (2\pi)^3 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{1}{J}\int |T|^2 d\tau_p, \qquad (2)$$

where

$$d\tau_p \equiv (2\pi)^4\delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^3\mathbf{l}_i}{2l_{i0}(2\pi)^3}, \qquad (3)$$

and

$$J \equiv 4\sqrt{(P_1P_2)^2 - P_1^2P_2^2} = 4A_1A_2\sqrt{(p_1p_2)^2 - m_N^4} = 2A_1A_2\sqrt{s(s - 4m_N^2)}. \qquad (4)$$

Here, m_N is the mass of the nucleon.

We will estimate the amplitude T corresponding to the diagram in Fig. 1 for the case of high energies, when $s \gg m_N^2$. We will carry out the analysis in the center-of-mass system of the NN collision. In this system $\mathbf{p}_1 + \mathbf{p}_2 = 0$ and $|\mathbf{p}_1| = |\mathbf{p}_2| \equiv |\mathbf{p}|$.

$$\sqrt{s} = 2\sqrt{\mathbf{p}^2 + m_N^2} \approx 2|\mathbf{p}| \gg m_N. \tag{5}$$

In this case, moving on to the light front variables

$$x_i \equiv k_{i+}/p_{1+}, \quad k_{i+} \equiv (k_{i0} + k_{iz})/\sqrt{2}, \tag{6}$$

we can connect the vertex Γ_1 in Fig. 1 with the partonic wave function of the nucleus A_1 on the light front, similar to how it was done in the works[17,36]:

$$\psi_1(x_i, \mathbf{k}_{i\perp}) \equiv \frac{\Gamma_1(x_i, \mathbf{k}_{i\perp})}{\sum\limits_{i=1}^{n_1} \frac{m^2 + \mathbf{k}_{i\perp}^2}{x_i} - A_1 m_N^2}, \tag{7}$$

where m is the mass of the constituent quark. It is important that in this ultra-relativistic case the dependence of the vertex Γ_1 on the small components k_{i-} can be neglected, just as in the nonrelativistic case the dependence of the vertex on the components k_{i0} can be neglected.[37]

From the condition that the form factor should tend to 1 for small transferred momenta, the following normalization condition[17] can be obtained for the wave function defined according to (7):

$$\int |\psi_1(x_i, \mathbf{k}_{i\perp})|^2 2\delta\left(\sum_{i=1}^{n_1} x_i - A_1\right)(2\pi)^3 \delta^{(2)}\left(\sum_{i=1}^{n_1} \mathbf{k}_{i\perp}\right) \prod_{i=1}^{n_1} \frac{dx_i}{2x_i} \frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^3} = A_1. \tag{8}$$

Similarly, for a nucleus that flies in the opposite direction, we can neglect the dependence of the vertex Γ_2 on the small components h_{i+} and introduce the partonic wave function $\psi_2(y_i, \mathbf{h}_{i\perp})$, where $y_i \equiv h_{i-}/p_{2-}$.

3. Asymptotic Behavior of the Inclusive Cross-Section

Block B in the diagram for the amplitude T in Fig. 1 contains hard interactions between partons of colliding nuclei with the transfer of large momenta $\sim \sqrt{s}$ which provide the emission in the region of central rapidities of a quark with a large transverse momentum $k_\perp \simeq \sqrt{s_{AA}}/2 = A\sqrt{s}/2$, as well as $p = n_1 + n_2 - 1$ recoil quarks with momenta $\mathbf{l}_i$. In the center-of-mass system used

$$\mathbf{k} + \sum_{i=1}^{p} \mathbf{l}_i = 0, \tag{9}$$

and

$$\sqrt{\mathbf{k}^2 + m^2} + \sum_{i=1}^{p} \sqrt{\mathbf{l}_i^2 + m^2} = \sqrt{s_{AA}} = A\sqrt{s}, \tag{10}$$

where m is the constituent quark mass. The maximum possible value of $k \equiv |\mathbf{k}|$ is determined by the energy conservation condition. It is easy to show that it is achieved when all momenta $\mathbf{l}_i = -\mathbf{k}_{\max}/p$ are equal. The value of $k_{\max} \equiv |\mathbf{k}_{\max}|$ is determined from the condition

$$\sqrt{k_{\max}^2 + m^2} + \sqrt{k_{\max}^2 + (pm)^2} = A\sqrt{s}. \tag{11}$$

In the leading approximation in $m^2/s \ll 1$ we have $k_{\max} \approx A\sqrt{s}/2$.

We will seek the asymptotic behavior of the inclusive cross-section (2) for the process in Fig. 1 over two small parameters: $m^2/s \ll 1$ and $(1 - k/k_{\max}) \ll 1$, i.e., near the kinematic boundary for AA collisions, similar to how it was done in Ref. 17. In this case, when calculating block B, we can set the values of the parton momenta in the final state equal to their limiting values. In addition, when estimating the contribution of block B, we also neglect small differences of the initial momenta k_i and h_i, which arise due to the relative motion of the constituents in the nuclei A_1 and A_2, from their average values P_1/n_1 and P_2/n_2:

$$B(k_i, h_i; \mathbf{k}, \mathbf{l}_i) \approx B(P_1/n_1, P_2/n_2; \mathbf{k}_{\max}, -\mathbf{k}_{\max}/p). \tag{12}$$

In this approximation, the contribution of the diagram in Fig. 1 to the amplitude T is factorized:

$$T = J_{n_1} J_{n_2} B. \tag{13}$$

Here, J_{n_1} and J_{n_2} are the contributions from the vertices Γ_1 and Γ_2 together with the attached propagators. For them, we find

$$J_{n_1} = \int \psi_1(x_i, \mathbf{k}_{i\perp}) 2\delta\left(\sum_{i=1}^{n_1} x_i - A_1\right)(2\pi)^3 \delta^{(2)}\left(\sum_{i=1}^{n_1} \mathbf{k}_{i\perp}\right) \prod_{i=1}^{n_1} \frac{dx_i}{2x_i} \frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^3}, \tag{14}$$

and a similar formula for J_{n_2}. It is clear that if we move from momentum to coordinate space, then J_{n_1} will be proportional to the value of the wave function with the relative spatial coordinates of the constituents equal to zero.

$$J_{n_1} \sim \psi_1(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0). \tag{15}$$

This corresponds to the results obtained in the works[32,33] in estimating the asymptotic behavior of elastic and quasi-elastic processes in the region of large momentum transfers, when $t \sim s$. Physically, this means that the hard process described by block B requires that nuclei (e.g., deuterons) be in the flucton state at the moment of interaction.

A more accurate estimate of J_{n_1} using formula (14) and the normalization condition (8) yields

$$J_{n_1} = \frac{C_1}{m^{(n_1-1)/2} R_1^{3(n_1-1)/2}}, \tag{16}$$

where R_1 is the radius of the nucleus A_1 in the system in which it is at rest (for $A_1 = 1$, R_1 is the nucleon radius). C_1 is a dimensionless coefficient. A similar estimate is obtained for J_{n_2}.

Physically, this estimate is understandable, since it follows from (15) that J_n^2 is proportional to the probability for n constituents of a nucleus of volume V_A to gather in some small volume V_0:

$$J_n^2 \sim |\psi(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)|^2 \sim \left(\frac{V_0}{V_A}\right)^{n-1} = \frac{r_0^{3(n-1)}}{R^{3(n-1)}}. \tag{17}$$

To estimate the asymptotic behavior of block B for $s \gg m^2$, we will use the considerations expressed earlier in Refs. 17, 32, 33 and 36. This block contains hard interactions between partons of colliding nuclei with the transfer of large momenta $\sim \sqrt{s}$, which ensure the emission of a quark with a large transverse momentum $k_\perp \simeq k_{\max} \approx A\sqrt{s}/2$ (11) in the region of central rapidities, as well as p recoil quarks with momenta $\mathbf{l}_i = -\mathbf{k}_{\max}/p$ (12). We will assume, as was done in the Refs. 17, 32, 33 and 36, that hard exchanges in block B occur between all $n = n_1 + n_2$ quarks. This means that the interaction between all n quarks occurs at one point — at distances much smaller than the characteristic nuclear (or nucleon) distances R_1 and R_2.

On the other hand, since each interaction in block B involves transfer of large momenta $\sim \sqrt{s}$, then due to the property of asymptotic freedom in QCD it introduces an additional smallness. Therefore, the main contribution to B will be given by diagrams with a minimum number of exchanges that connect all n quarks, i.e., which contain $n - 1$ exchange. Examples of such diagrams are shown in Fig. 2.

In Ref. 33, considerations were put forward, verified for the case of quantum electrodynamics, that in any renormalizable theory the effective quark interaction potential $V(q^2)$ tends to a constant (with an accuracy of up to taking into account

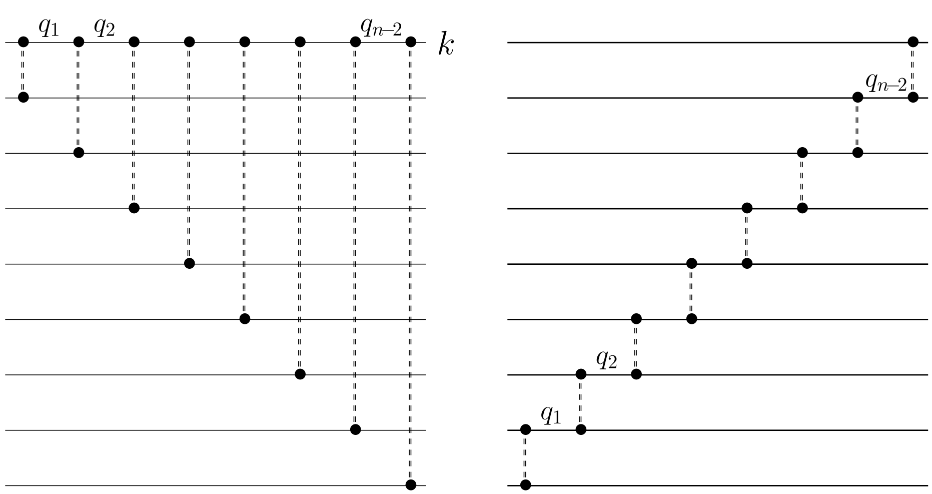

Fig. 2. Examples of the diagrams contributing to the hard block B in Fig. 1.

powers of $\log|q^2|$) when $q^2 \to -\infty$. In Ref. 17 this fact was confirmed for the case of scalar quarks. In particular, it was shown that the product of the vertices of gluon attachment to scalar quarks by its propagator tends to a constant when $q^2 \to -\infty$.

Using this observation to estimate the asymptotic behavior of the diagrams contributing to block B, we conclude that the main dependence of block B at high initial energy ($s \gg m^2$) and large momentum transfers ($|q^2| \sim s$) will come from $n-2$ "internal" quark propagators with momenta $|q_i^2| \sim s$, which enter the diagrams in Fig. 2 between gluon exchanges. This leads to the following estimate of the dependence of B on the initial energy:

$$B = \frac{C_B}{s^{n-2}} = \frac{C_B}{s^{n_1+n_2-2}}\,, \tag{18}$$

where C_B is a dimensionless constant.

Substituting the obtained expressions into Eq. (13), we find the following expression for the asymptotic behavior of the amplitude T in the region under consideration:

$$T = \frac{C_1C_2C_B}{m^{(n-2)/2}R_1^{3(n_1-1)/2}R_2^{3(n_2-1)/2}s^{n-2}} = \frac{C_1C_2C_B}{m^{(n-2)/2}R^{3(n-2)/2}s^{n-2}}\,. \tag{19}$$

The last transition is performed for the case of identical nuclei $R_1 = R_2 \equiv R$. Substituting this asymptotic expression for the amplitude T into the formula (2) for the inclusive cross-section, we obtain

$$I(\mathbf{k}) = \frac{1}{J}|T|^2\tau_p, \tag{20}$$

where

$$\tau_p = (2\pi)^4 \int \delta^4\left(P_1 + P_2 - k - \sum_{i=1}^{p} l_i\right) \prod_{i=1}^{p} \frac{d^3\mathbf{l}_i}{2l_{i0}(2\pi)^3}\,, \tag{21}$$

4. Calculation of Phase Volume

It remains to calculate the phase volume τ_p Eq. (21). As mentioned in the previous section, we will calculate it for momenta k near the kinematic boundary for an AA collision (we will restrict ourselves to the case $A_1 = A_2 \equiv A$), using an additional small parameter $(1 - k/k_{\max}) \ll 1$. To describe the magnitude of the deviation of momentum k from the kinematic boundary, the so-called cumulative number x is usually used. In our case this variable is defined as follows:

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)m]^2}. \tag{22}$$

Comparing this formula with Eq. (11), we see that $x = A$ corresponds to the kinematic boundary of the AA reaction, provided that $p(A) = n_1 + n_2 - 1$.

It is important that in order for this variable *simultaneously* to coincide with the kinematic limits of the reaction for smaller integer values $(1, \ldots, A-1)$, the dependence of the number of recoil quarks on the cumulative number, $p = p(x)$, must be entered into the definition (22). If we do not yet introduce diquarks into consideration and assume that each nucleon contains three constituent quarks, then $p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1$. In this case, for example, for $x = 1$, Eq. (22) gives the correct kinematic limit for pp collisions. For noninteger values of x, Eq. (22) uniquely determines the value of the cumulative number x for a particle (in our case, a parton) as a function of its momentum k.

By calculating τ_p (21) for $A - x \ll 1$, we obtain the following asymptotic expression for the integral of the phase volume:

$$\tau_p = \frac{1}{2^{4p-5} p^{3p/2-1} m^{p-1}} \frac{[\frac{A}{\pi} s(A-x)]^{\frac{3}{2}p-\frac{5}{2}}}{(\frac{3}{2}p - \frac{5}{2})!}, \tag{23}$$

where for half-integer values instead of the factorial $(3p/2 - 5/2)!$ you need to use the Gamma function $\Gamma(3(p-1)/2)$.

Substituting this result into Eq. (20) and taking into account Eq. (19), as well as the fact (4), that $J \approx 2sA^2$, we find the asymptotic dependence of the inclusive cross-section of production of a quark with a large transverse momentum in the cumulative region on two small parameters m^2/s and $A - x$:

$$I(x) \equiv (2\pi)^3 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p-\frac{5}{2}}}{(m^2R^3)^{p-1} s^{(p+3)/2}}, \tag{24}$$

where x is a cumulative number that is determined from k using the formula (22), $p = n - 1 = n_1 + n_2 - 1$ and C is a dimensionless constant that is independent of the dimensional parameters of the model.

5. Quark Counting Rules for Quasi-Elastic Processes

Earlier, in the works,[31–34] quark counting rules for elastic and quasi-elastic processes were obtained. Note that within the framework of the presented approach, they are also correctly reproduced. Indeed, acting similarly to how it was done above when finding the asymptotic behavior of the inclusive cross-section, for the differential cross-section of quasi-elastic scattering $(A_1 + A_2 \to A_1' + A_2')$ at $t \sim s \gg m^2$, we obtain the following expression:

$$\frac{d\sigma}{dt} = \frac{\bar{C}}{s^{2n-2} m^{2n-4} R_1^{3(n_1-1)} R_2^{3(n_2-1)} R_1'^{3(n_1'-1)} R_2'^{3(n_2'-1)}}, \tag{25}$$

here $\bar{C}$ is a dimensionless constant that is independent of the dimensional parameters of the model.

Taking into account that in our case $n = n_1 + n_2 = n_1' + n_2'$, this result agrees with the predictions[31–34] of quark counting rules for elastic and quasi-elastic processes:

$$\frac{d\sigma}{dt} \sim \frac{1}{s^{n_1+n_2+n_1'+n_2'-2}}. \tag{26}$$

Note that, unlike the inclusive cross-section, in this case $t \sim s$ and there is only one small parameter $m^2/s \ll 1$.

6. Conclusion

In this paper, an analysis of the process of pion production in AA collisions at central rapidities with a large transverse momentum in the region kinematically forbidden for reactions with free nucleons (cumulative production) is carried out. It is shown that in this region the dominant contribution is from the flucton–flucton interaction, leading to the emission of a quark, which then fragments into a pion. The asymptotic behavior of the inclusive cross-section at high initial energies near the kinematic boundary of the process is calculated and quark counting rules for the inclusive cross-section in this region are formulated.

The found patterns can be tested in the MPD and SPD experiments at the NICA collider. Note that for reliable registration of the very rare production of particles in the cumulative region and separation of their tracks from various kinds of false background tracks, a signal from the Internal Tracking System is highly desirable, allowing confirmation of the exit of the cumulative particle track from the primary interaction vertex.

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