Exploitation and Recovery Periods in Dynamic Resource Management Problem

Vladimir Mazalov[1,2] and Anna Rettieva[1,2](✉)

[1] Institute of Applied Mathematical Research, Karelian Research Center of RAS, 11 Pushkinskaya str., Petrozavodsk 185910, Russia
{vmazalov,annaret}@krc.karelia.ru

[2] Saint Petersburg State University, 7/9 Universitetskaya nab., Saint Petersburg 199034, Russia

Abstract. Dynamic game related to resource management problem is considered. The planning horizon is assumed to be divided into the periods of exploitation where many players use a common resource and the periods of recovery where the resource stock is evolving according to the natural growth rule. Both noncooperative and coordinated players' behaviors are investigated. The conditions linking the values of exploitation and recovery periods in order to maintain the sustained resource usage are determined. To illustrate the presented approaches, a dynamic bioresource management problem (harvesting problem) with many players and compound planning horizon is investigated. AQ1

Keywords: Dynamic games · Resource management problem · Exploitation period · Recovery period

1 Introduction

Resource management problems (exploitation of common renewable resources) is one of the real-life challenges that are very important for ecology and economics. It encompasses a number of different issues including the phenomenon known as the tragedy of the commons. The commons denotes a natural resource extracted by many individuals and the tragedy means that the participants tend to overexploitation in the absence of regulation [15]. There is an extensive literature on renewable resources management in economics, operations research and optimal control theory. AQ2

Real-life problems such as the exploitation processes involve dynamics of the renewable resource and a number of decision makers. Hence, they can be investigated applying the technics of optimal control theory and dynamic games. The game-theoretic approach to resource exploitation was pioneered by Munro

This research was supported by the Russian Science Foundation grant No. 22-11-00051, https://rscf.ru/en/project/22-11-00051/.

M. Khachay et al. (Eds.): MOTOR 2023, LNCS 13930, pp. 1–12, 2023.
https://doi.org/10.1007/978-3-031-35305-5_20

[14] and Clark [1], who applied the Nash equilibrium for the models of fisheries. The optimal behavior of players in harvesting problems is investigated by many scientists including Hamalainen et al. [5,8], Kaitala and Lindroos [11], Levhari, Fisher and Mirman [6,7,10], Petrosyan and Zackharov [17], Plou [18], Tolwinski [19], Zaccour et al. [2,3]. Several types of optimal behavior regulation have been suggested such as incentive equilibrium [5,12], time-consistent imputation distribution procedure [13,16], moratorium regimes [1,2,14], prohibited for exploitation areas [13] and others.

Some resource management problems involve periodic processes when a natural resource is exploited over a certain period of time, and then a time period for resource recovery or moratorium regime [2] is established. The matter of environmental concernment is the balance between time intervals of exploitation and recovery for sustained resource usage.

Such periodic exploitation processes are wide spread in harvesting problems. For example, in lake Onego in 2004, the moratorium for exploitation of the Shuya salmon population was withdrawn. Salmon fishing has become popular among fishing firms in northwestern Russia. However, in 2010, the size of salmon population that was about a hundred tons of fish according to official statistics, began to decline very sharply. Already by 2014, it reached, according to the data of Karelian harvesting agency, forty tons. The catch, primarily illegal, significantly exceeded reproduction. Therefore, in 2015, a new edition of the fishing rules introduced a new moratorium for salmon exploitation. Similar problems arise the forest management issues and other renewable resource exploitation processes.

The main goal of this paper is to determine the ratio between exploitation and recovery periods depending on the natural environment parameters and the exploitation load in order to maintain the sustainable nature resource evolution. For that purpose a dynamic bioresource management problem (harvesting problem) with many players and compound planning horizon is investigated. Both egoistic (noncooperative) and coordinated players' behaviors are investigated. The obtained conditions linking the values of exploitation and recovery periods allow to establish optimal for renewable resource evolution moratorium regimes.

Further exposition has the following structure. Section 2 describes the main model where the planning horizon is divided into the periods of exploitation and recovery. Two-periods dynamic resource management problem is presented in Sect. 3 while Sect. 4 describes the infinite horizon problem with interchanging periods of exploitation and recovery. The ratio between exploitation and recovery periods is determined for both models in noncooperative and coordinated settings. Finally, Sect. 5 provides the basic results and their discussion.

2 Main Model

Consider a renewable natural resource evolving according to

$$x'(t) = f(x(t)), \quad x(0) = x_0, \quad t \in (0, \infty), \tag{1}$$

where $x(t) \geq 0$ denotes the resource stock at time $t \geq 0$, $f(x)$ is the resource natural growth function.

Let $N = \{1, \ldots, n\}$ players exploit a common resource during time period $t \in [0, T]$. The state dynamics takes the form

$$x'(t) = f(x(t)) - u_1(t) - \ldots - u_n(t)\,, \;\; x(0) = x_0\,, \;\; t \in [0, T]\,, \tag{2}$$

where $u_i(t) \geq 0$ gives the exploitation rate of player i at time t, $i \in N$.

Then, the moratorium for exploitation is established during time interval $(T, T+\tau]$ when the resource recovers and evolves according to the dynamics (1). Section 3 considers only two periods of exploitation and recovery while Sect. 4 describes a periodic exploitation process where after the moratorium the players continue resource usage till the next recovery period and so on.

The payoff functions of the players on a finite planning horizon $[0, T]$ have the form

$$J_i(x, u_1, \ldots, u_n) = \int_0^T e^{-\rho t} g_i(x(t), u_1(t), \ldots, u_n(t))\, dt\,, \tag{3}$$

where $g_i(\cdot) \geq 0$, $i \in N$, are the instantaneous payoff functions, $\rho \in (0, 1)$ denotes the discount factor.

For sustainable resource evolution we suppose that after the recovery period the size of the population shouldn't be less than the initial one x_0[1]. The main goal of this paper is to determine the ratio between exploitation and recovery periods (T and τ) depending on the environmental and economical parameters.

Linear natural resource growth function as well as quadratic instantaneous payoff functions are considered in the next sections. The noncooperative (egoistic) case where each player wishes to maximize individual payoff (3) and the coordinated one where the players combine their exploitation rates and optimize the joint payoff are investigated.

3 Two-Periods Model

We begin with two-periods model where players exploit the resource during time interval $[0, T]$ and the resource recovers during time interval $(T, T+\tau]$. The main goal is to determine the conditions linking the values of exploitation and recovery periods (T and τ) in order to maintain the sustainable resource usage. The conditions for noncooperative and coordinated cases are constructed and compared.

Let $N = \{1, \ldots, n\}$ players exploit a natural renewable resource during time period $t \in [0, T]$. The evolution of the resource takes the form

$$x'(t) = \varepsilon x(t) - u_1(t) - \ldots - u_n(t)\,, \;\; x(0) = x_0\,, \;\; t \in [0, T]\,, \tag{4}$$

---

[1] The results obtained in the paper are naturally extended for the case when the desired size of the population has the form kx_0 for any $k \in (-\infty, \infty)$. The dependence on x_0 is induced by the fact that the only information that the regulator possesses at the beginning of the planning period is the initial size of the population.

where $x(t) \geq 0$ denotes the resource stock at time $t \geq 0$, $\varepsilon \geq 1$ denotes the natural birth rate and $u_i(t) \geq 0$ gives the exploitation rate of player i at time t, $i \in N$.

During the recovery period a renewable resource evolves according linear dynamics

$$x'(t) = \varepsilon x(t)\,, \quad x(0) = x_0\,, \quad t \in [T, T+\tau]\,. \tag{5}$$

Each player wishes to maximize the revenue from resource sales and to minimize the exploitation costs. Assume that the players have the same market prices and costs that depend quadratically on the exploitation rate. The payoff functions of the players take the forms

$$J_i(x, u_1, \ldots, u_n) = \int_0^T e^{-\rho t}[p u_i(t) - h u_i(t)^2]\,dt\,, \tag{6}$$

where $p \geq 0$ is the market price of the resource, $h \geq 0$ indicates the exploitation cost, and $\rho \in (0,1)$ denotes the discount factor. Further assume that $\varepsilon \geq n\rho$ (see Proposition 1).

Due to the symmetry of the instantaneous payoff functions dividing (6) by p and denoting $c = h/p$ gives the players payoff functions in the form

$$J_i(x, u_1, \ldots, u_n) = \int_0^T e^{-\rho t}[u_i(t) - c u_i(t)^2]\,dt\,. \tag{7}$$

3.1 Noncooperative Behavior

First, we consider noncooperative behavior and construct the Nash equilibrium strategies u_i^N that satisfy Nash inequalities

$$J_i(x, u_1^N, \ldots, u_{i-1}^N, u_i, u_{i+1}^N, \ldots, u_n^N) \leq J_i(x, u_1^N, \ldots, u_n^N)\ \forall u_i \in U_i = [0, \infty),\ i \in N.$$

Note, that the Nash equilibrium will be constructed in the feedback form $u_i(t) = u_i(x(t))$, $i \in N$.

Proposition 1. *The Nash equilibrium strategies in problem (4), (5), (7) have the form*

$$u_i^N(x) = \frac{2\varepsilon - \rho}{2n-1}x - \frac{\varepsilon - n\rho}{2\varepsilon c(2n-1)} \tag{8}$$

and the resource size is given by

$$x^N(t) = \frac{n}{2\varepsilon c} + \big(x_0 - \frac{n}{2\varepsilon c}\big)e^{-\frac{\varepsilon - n\rho}{2n-1}t}\,, \quad t \in [0, T]\,. \tag{9}$$

Proof. To construct an equilibrium we apply dynamic programming principle and construct Hamilton-Jacobi-Bellman (HJB) equations. Since players are symmetric assume, that all the players except the player i use feedback strategies

$u_j(x) = \phi(x)$, $j \in N$, $j \neq i$ and find the player i's optimal behavior. To maximize the individual payoff (7) the player i's value function $V_i(x,t)$ satisfy the next HJB equation:

$$\rho V_i(x,t) - \frac{\partial V_i}{\partial t} = \max_{u_i}\Big\{u_i - cu_i^2 + \frac{\partial V_i}{\partial x}(\varepsilon x - u_i - (n-1)\phi(x))\Big\}. \tag{10}$$

The solution of (10) will be constructed in quadratic form

$$V_i(x,t) = V_i(x) = A_i x^2 + B_i x + D_i\,.$$

Substitution to (10) gives

$$\rho A_i x^2 + \rho B_i x + \rho D_i = \max_{u_i}\Big\{u_i - cu_i^2 + (2A_i x + B_i)(\varepsilon x - u_i - (n-1)\phi(x))\Big\} \tag{11}$$

which yields

$$u_i(x) = -\frac{A_i}{c}x + \frac{1-B_i}{2c}\,. \tag{12}$$

The symmetry of the problems lead to the symmetry of the strategies $u_i(x) = \phi(x)$, $i \in N$. Substituting to (11), the system to define parameters becomes

$$\begin{cases} \rho A_i = (\frac{2n-1}{c}A_i^2 + 2\varepsilon A_i)\,, \\ \rho B_i = (\frac{2n-1}{c}A_i B_i - \frac{n}{c}A_i + \varepsilon B_i)\,, \\ \rho D_i = \frac{(B_i-1)((2n-1)B_i-1)}{4c}\,, \end{cases}$$

which yields

$$A_i = -\frac{c(2\varepsilon-\rho)}{2n-1}\,,\ B_i = \frac{n(2\varepsilon-\rho)}{\varepsilon(2n-1)}\,,\ D_i = \frac{(\varepsilon-n\rho)(n(2\varepsilon-\rho)-\varepsilon)}{4\varepsilon^2 c\rho(2n-1)}\,.$$

As $A_i \leq 0$, to have the nonnegative payoff for player i it is necessary the top of the parabola $A_i x^2 + B_i + D_i$ to lie above the X-line. The abscissa of the top is equal $\bar{x} = \frac{n}{c\varepsilon}$, hence the ordinate takes the form

$$\bar{y} = A_i\frac{n^2}{c^2\varepsilon^2} + B_i\frac{n}{c\varepsilon} + D_i = D_i\,.$$

It yields that D_i should be nonnegative that is the fact when $\varepsilon \geq n\rho$.

The noncooperative equilibrium strategies become

$$u_i^N(x) = \frac{2\varepsilon-\rho}{2n-1}x - \frac{\varepsilon-n\rho}{2\varepsilon c(2n-1)}\,,\ i \in N\,.$$

Substituting (8) into the state dynamics corresponding to the exploitation regime (4), we get

$$x'(t) = -\frac{\varepsilon-n\rho}{2n-1}x(t) + \frac{n(\varepsilon-n\rho)}{2\varepsilon c(2n-1)}\,,\ x(0) = x_0\,. \tag{13}$$

The solution of (13) takes the form

$$x^N(t) = \frac{n}{2\varepsilon c} + \big(x_0 - \frac{n}{2\varepsilon c}\big)e^{-\frac{\varepsilon-n\rho}{2n-1}t}\,,\ t \in [0,T]\,.$$

3.2 Coordinated Behavior

To coordinate the resource usage the players combine their exploitation rates and optimize the joint payoff function that takes the form

$$J(x, u_1, \ldots, u_n) = \int_0^T e^{-\rho t}[\sum_{j=1}^n u_j(t) - c\Big(\sum_{j=1}^n u_j(t)\Big)^2]\, dt\,. \quad (14)$$

Proposition 2. *The coordinated strategies in problem (4), (5), (14) have the form*

$$u_i^c(x) = \frac{2\varepsilon - \rho}{n} x - \frac{\varepsilon - \rho}{2n\varepsilon c} \quad (15)$$

and the resource size is given by

$$x^c(t) = \frac{1}{2\varepsilon c} + \big(x_0 - \frac{1}{2\varepsilon c}\big) e^{-(\varepsilon - \rho)t}\,, \;\; t \in [0, T]\,. \quad (16)$$

Proof. Applying HJB equation similar to Proposition 1 we construct coordinated behavior. Note that the joint payoff function takes the form

$$V(x, t) = V(x) = Ax^2 + Bx + D\,,$$

where

$$A = -c(2\varepsilon - \rho)\,, \;\; B = \frac{(2\varepsilon - \rho)}{\varepsilon}\,, \;\; D = \frac{(\varepsilon - \rho)^2}{4\varepsilon^2 c\rho}\,.$$

3.3 Sustainable Resource Exploitation

For sustainable resource evolution after the recovery period the size of the population shouldn't be less than the initial one. Denote the resource size achieved at the end of the moratorium regime lasting for τ^N time steps in noncooperative case or τ^c steps in the coordinated one as $x^N(T + \tau^N)$ or $x^c(T + \tau^c)$. According to the condition for sustained usage mentioned above the next equality should be fulfilled:

$$x^N(T + \tau^N) = x^c(T + \tau^c) = x_0\,. \quad (17)$$

Since during the recovery period the population evolves according to dynamics (5) the condition (17) becomes

$$x^N(T) e^{\varepsilon \tau^N} = x^c(T) e^{\varepsilon \tau^c} = x_0\,. \quad (18)$$

Consider the sustainable condition for noncooperative case. According to (9) it takes the form

$$\big(\frac{n}{2\varepsilon c} + \big(x_0 - \frac{n}{2\varepsilon c}\big) e^{-\frac{\varepsilon - n\rho}{2n-1}T}\big) e^{\varepsilon \tau^N} = x_0 \quad (19)$$

which yields that the time period for resource recovery shouldn't be less that

$$\tau^N = \frac{1}{\varepsilon} \ln\Big(\frac{2c\varepsilon x_0}{(2c\varepsilon x_0 - n) e^{-\frac{\varepsilon - n\rho}{2n-1}T} + n}\Big)\,. \quad (20)$$

As the dynamics (4) and (5) can describe real fish population possess the huge sizes we consider the condition (20) for large x_0. Under this assumption it gives the ratio between exploitation and recovery periods in the form

$$\frac{\tau^N}{T} \approx \frac{\varepsilon - n\rho}{\varepsilon(2n-1)}\,. \tag{21}$$

Now consider the sustainable condition for the coordinated case. According to (16) it takes the form

$$\Big(\frac{1}{2\varepsilon c} + \big(x_0 - \frac{1}{2\varepsilon c}\big)e^{-(\varepsilon-\rho)T}\Big)e^{\varepsilon\tau^c} = x_0 \tag{22}$$

which yields that the time period for resource recovery shouldn't be less that

$$\tau^c = \frac{1}{\varepsilon}\ln\Big(\frac{2c\varepsilon x_0}{(2c\varepsilon x_0 - 1)e^{-(\varepsilon-\rho)T} + 1}\Big)\,. \tag{23}$$

Again, for large x_0 the condition (23) gives the ratio between exploitation and recovery periods in the form

$$\frac{\tau^c}{T} \approx \frac{\varepsilon - \rho}{\varepsilon}\,. \tag{24}$$

Theorem 1. *The time period for resource recovery in noncooperative case is less than in the coordinated one.*

Proof. Comparing (21) and (24) observe that

$$\tau^c - \tau^N = \frac{(2\varepsilon - \rho)(n-1)}{\varepsilon(2n-1)}T \geq 0\,.$$

Hence, the noncooperative behavior is better for population state. This observation differs from the cooperation preference in the "fish wars" model as the population density $0 \leq x \leq 1$ instead of the size of the population was investigated there.

4 Model with Many Periods

Now, consider the case when the periods of exploitation and recovery are repeated many times. As above, the state dynamics (4) correspond to the period of exploitation, while (5) – to the period of recovery.

In this model, the sequence of events is as follows: players exploit the resource for time interval $t \in [0, T]$. Then, the recovery period is implemented for $t \in (T, T+\tau]$, during which the players get zero payoffs. At $t = T + \tau$, the stock level is back to the desired level x_0, and the players can again exploit the resource. Hence, the players' planning horizon is infinite and the payoff functions take the form

$$J_i(x_0, u) = \int_0^\infty e^{-\rho t}(\hat{u}_i(t) - c\hat{u}_i(t)^2)\,dt\,, \tag{25}$$

where

$$\hat{u}_i(t) = \begin{cases} u_i(t), & t \in [k(T+\tau), (k+1)T + k\tau], \\ 0, & t \in ((k+1)T + k\tau), (k+1)(T+\tau)], \ k = 0, 1, \dots . \end{cases}$$

Since after the recovery period the resource size is equal to the initial one x_0 player i's payoff in the game where the moratorium regime is firstly applied at $t = T$ is

$$J_i(x_0, u) = \int_0^T e^{-\rho t}(u_i(t) - cu_i(t)^2)\, dt + e^{-\rho(T+\tau)} J_i(x_0, u)\,,$$

which yields

$$J_i(x_0, u) = \frac{1}{1 - e^{-\rho(T+\tau)}} \int_0^T e^{-\rho t}(u_i(t) - cu_i(t)^2)\, dt \tag{26}$$

with state dynamics (4) for $t \in [0, T]$ and (5) for $t \in (T, T+\tau]$. In noncooperative case player $i \in N$ maximizes (26) with respect to u_i and under coordination the players wish to maximize the joint payoff with combined exploitation rates.

As before, we consider both types of players' behavior and construct the equilibrium strategies in feedback form $u_i(t) = u_i(x(t))$, $i \in N$.

4.1 Noncooperative Behavior

First, define the Nash equilibrium in the feedback strategies when each player i maximizes individual payoff (26).

Proposition 3. *The Nash equilibrium strategies in problem (4), (5), (26) have the form*

$$u_i^N(x) = \frac{2\varepsilon - \rho(1 - e^{-\rho(T+\tau^N)})}{2n-1} x - \frac{\varepsilon - n\rho(1 - e^{-\rho(T+\tau^N)})}{2\varepsilon c(2n-1)} \tag{27}$$

and the resource size is given by

$$x^N(t) = \frac{n}{2\varepsilon c} + \big(x_0 - \frac{n}{2\varepsilon c}\big) e^{-\frac{\varepsilon - n\rho(1 - e^{-\rho(T+\tau^N)})}{2n-1} t}, \ \ t \in (0, \infty)\,. \tag{28}$$

Proof. To construct an equilibrium we apply HJB equation again. Since players are symmetric assume, that all the players except the player i use feedback strategies $u_j(x) = \phi(x)$, $j \in N$, $j \neq i$ and find the player i's optimal behavior. To maximize the individual payoff (26) the player i's value function $V_i(x, t)$ satisfy the next HJB equation:

$$\rho V_i(x,t) - \frac{\partial V_i(x,t)}{\partial t} = \frac{1}{1 - e^{-\rho(T+\tau^N)}} \max_{u_i} \Big\{ u_i - cu_i^2 + \frac{\partial V_i(x,t)}{\partial x}(\varepsilon x - u_i - (n-1)\phi(x)) \Big\}\,. \tag{29}$$

As before we seek the value function in quadratic form

$$V_i(x,t) = V_i(x) = A_i x^2 + B_i x + D_i\,.$$

Similarly to Proposition 1 we get

$$u_i(x) = -\frac{A_i}{c}x + \frac{1-B_i}{2c}$$

and

$$A_i = -\frac{c(2\varepsilon - \rho(1-e^{-\rho(T+\tau^N)}))}{2n-1}\,,\ B_i = \frac{n(2\varepsilon - \rho(1-e^{-\rho(T+\tau^N)}))}{\varepsilon(2n-1)}\,,$$
$$D_i = \frac{-(n-1)^2\varepsilon^2 + n^2(\varepsilon - \rho(1-e^{-\rho(T+\tau^N)}))}{4c\rho\varepsilon^2(1-e^{-\rho(T+\tau^N)})}\,.$$

Hence, the noncooperative equilibrium strategies become

$$u_i^N(x) = \frac{2\varepsilon - \rho(1-e^{-\rho(T+\tau^N)})}{2n-1}x - \frac{\varepsilon - n\rho(1-e^{-\rho(T+\tau^N)})}{2\varepsilon c(2n-1)}\,.$$

Substituting (27) into the state dynamics corresponding to the exploitation regime (4) we get

$$x'(t) = -\frac{\varepsilon - n\rho(1-e^{-\rho(T+\tau^N)})}{2n-1}x(t) + \frac{n(\varepsilon - n\rho(1-e^{-\rho(T+\tau^N)}))}{2\varepsilon c(2n-1)}\,,\ x(0)=x_0\,. \tag{30}$$

The solution of (30) takes the form

$$x^N(t) = \frac{n}{2\varepsilon c} + \big(x_0 - \frac{n}{2\varepsilon c}\big)e^{-\frac{\varepsilon - n\rho(1-e^{-\rho(T+\tau^N)})}{2n-1}t}\,.$$

4.2 Coordinated Behavior

Now, define the coordinated equilibrium in the feedback strategies when players maximizes the joint payoff with combined exploitation rates:

$$J(x_0,u) = \frac{1}{1-e^{-\rho(T+\tau)}}\int_0^T e^{-\rho t}\Big[\sum_{i=1}^n u_i(t) - c\Big(\sum_{i=1}^n u_i(t)\Big)^2\Big]\,dt\,. \tag{31}$$

Proposition 4. *The coordinated strategies in problem (4), (5), (31) have the form*

$$u_i^c(x) = \frac{2\varepsilon - \rho(1-e^{-\rho(T+\tau^c)})}{n}x - \frac{\varepsilon - \rho(1-e^{-\rho(T+\tau^c)})}{2n\varepsilon c} \tag{32}$$

and the resource size is given by

$$x^c(t) = \frac{1}{2\varepsilon c} + \big(x_0 - \frac{1}{2\varepsilon c}\big)e^{-(\varepsilon - \rho(1-e^{-\rho(T+\tau^c)}))t}\,,\ \ t\in(0,\infty)\,. \tag{33}$$

Proof. Applying HJB equation similar to Proposition 3 we construct cooperative behavior.

Note that the joint payoff function takes the form

$$V(x,t) = V(x) = Ax^2 + Bx + D\,,$$

where

$$A = -c(2\varepsilon - \rho(1 - e^{-\rho(T+\tau^c)}))\,,\ B = \frac{2\varepsilon - \rho(1 - e^{-\rho(T+\tau^c)})}{\varepsilon}\,,$$
$$D = \frac{(\varepsilon - \rho(1 - e^{-\rho(T+\tau^c)}))^2}{4\varepsilon^2 c\rho(1 - e^{-\rho(T+\tau^c)})}\,.$$

4.3 Sustainable Resource Exploitation

As before assume that for sustainable resource evolution after the recovery period the size of the population should be equal to the initial one x_0. For both types of players' behavior the corresponding condition take the form

$$x^N(T)e^{\varepsilon\tau^N} = x^c(T)e^{\varepsilon\tau^c} = x_0\,. \tag{34}$$

Consider the sustainable condition for noncooperative case. According to (28) it takes the form

$$\Big(\frac{n}{2\varepsilon c} + \big(x_0 - \frac{n}{2\varepsilon c}\big)e^{-\frac{\varepsilon - n\rho(1-e^{-\rho(T+\tau^N)})}{2n-1}T}\Big)e^{\varepsilon\tau^N} = x_0 \tag{35}$$

which yields for large x_0 the ratio between exploitation and recovery periods in the form

$$\frac{\tau^N}{T} \approx \frac{\varepsilon - n\rho}{\varepsilon(2n-1)} + \frac{1}{\rho T}W\Big(\frac{\rho^2 nT}{\varepsilon(2n-1)}e^{-\frac{\rho n(2\varepsilon-\rho)T}{\varepsilon(2n-1)}}\Big)\,, \tag{36}$$

where $W(\cdot)$ is the Lambert function.

Now consider the sustainable condition for the coordinated case. According to (33) it takes the form

$$\Big(\frac{1}{2\varepsilon c} + \big(x_0 - \frac{1}{2\varepsilon c}\big)e^{-(\varepsilon - \rho(1-e^{-\rho(T+\tau^c)}))T}\Big)e^{\varepsilon\tau^c} = x_0 \tag{37}$$

which yields (for large x_0) the ratio between exploitation and recovery periods in the form

$$\frac{\tau^c}{T} \approx \frac{\varepsilon - \rho}{\varepsilon} + \frac{1}{\rho T}W\Big(\frac{\rho^2 T}{\varepsilon}e^{-\frac{\rho(2\varepsilon-\rho)T}{\varepsilon}}\Big)\,. \tag{38}$$

Theorem 2. *The time period for resource recovery in noncooperative case is less than in the coordinated one for the model with many periods.*

Proof. Let us compare (36) and (38). From (35) and (37) (for large x_0) we get

$$\tau^c = T(\frac{\varepsilon-\rho}{\varepsilon} + \frac{\rho}{\varepsilon}e^{-\rho(T+\tau^c)}),$$
$$\tau^N = T(\frac{\varepsilon-n\rho}{\varepsilon(2n-1)} + \frac{\rho n}{\varepsilon(2n-1)}e^{-\rho(T+\tau^N)}),$$

which yields

$$\tau^c - \tau^N = \frac{(2\varepsilon-\rho)(n-1)}{\varepsilon(2n-1)}T + \frac{T\rho e^{-\rho(T+\tau^N)}}{\varepsilon(2n-1)}\big((2n-1)e^{-\rho(\tau^c-\tau^N)} - n\big). \quad (39)$$

The solution of (39) can be also obtained via Lambert function in the next form

$$\tau^c - \tau^N = \frac{T}{\varepsilon(2n-1)}\big((2\varepsilon-\rho)(n-1) - \rho n e^{-\rho(T+\tau^N)}\big)$$
$$+ \frac{1}{\rho}W\Big(\frac{\rho^2 T}{\alpha}e^{-\rho(T+\tau^N)}e^{-\frac{\rho((2\varepsilon-\rho)(n-1)-\rho n e^{-\rho(T+\tau^N)})}{\varepsilon(2n-1)})}\Big). \quad (40)$$

As the Lambert function is positive for nonnegative argument and $(2\varepsilon-\rho)(n-1) - \rho n e^{-\rho(1+\tau^N)}$ is larger that $\rho n(1-e^{-\rho(1+\tau^N)}) > 0$ (40) yields that

$$\tau^c - \tau^N \geq 0.$$

5 Conclusions

Dynamic game related to resource management problem (renewable resource exploitation process) is considered. The evolution of the resource and exploitation processes are assumed to be periodic. Namely, the periods of extraction of the renewable resource are interchanged with recovery periods in order to maintain the sustained resource usage.

The desired resource size after the recovery period is assumed to be equal to the initial one for long-term exploitation. First, the model with one extraction and one recovery periods are considered. Then, the extension with many rotated exploitation periods and moratorium regimes is presented. Both egoistic (noncooperative) and coordinated players' behaviors are investigated. The conditions linking the values of exploitation and recovery periods are derived analytically. It is shown that the time period needed for resource recovery in noncooperative case is less than in the coordinated one. The obtained ration between exploitation and recovery periods allow to establish optimal for renewable resource evolution moratorium regimes.

Note that the solutions are obtained under assumption that the value of exploitation period T is externally given. The problem of optimal extraction period size determination in order to maintain sustained exploitation process is planned for near future work.

References

1. Clark, C.W.: Bioeconomic Modelling and Fisheries Management. Wiley, New York (1985)
2. Dahmouni, I., Parilina, E.M., Zaccour, G.: Great fish war with moratorium. Technical report, Les Cahiers du GERAD G-2022-07. GERAD, HEC Montreal, Canada (2022)
3. Dahmouni, I., Vardar, B., Zaccour, G.: A fair and time-consistent sharing of the joint exploitation payoff of a fishery. Nat. Resour. Model. **32**, 12216 (2019)
4. Dockner, E.J., Jorgensen, S., Long, N.V., Sorger, G.: Differential Games in Economics and Management Science. Cambridge University Press, Cambridge, UK (2000)
5. Ehtamo, H., Hamalainen, R.P.: A cooperative incentive equilibrium for a resource management problem. J. Econ. Dyn. Control **17**, 659–678 (1993)
6. Fisher, R.D., Mirman, L.J.: Strategic dynamic interaction: fish wars. J. Econ. Dyn. Control **16**, 267–287 (1992)
7. Fisher, R.D., Mirman, L.J.: The complete fish wars: biological and dynamic interactions. J. Environ. Econ. Manag. **30**, 34–42 (1996)
8. Hamalainen, R.P., Kaitala, V., Haurie, A.: Bargaining on whales: a differential game model with Pareto optimal equilibria. Oper. Res. Lett. **3**(1), 5–11 (1984)
9. Haurie, A., Tolwinski, B.: Acceptable equilibria in dynamic games. Large Scale Syst. **6**, 73–89 (1984)
10. Levhari, D., Mirman, L.J.: The great fish war: an example using a dynamic Cournot-Nash solution. Bell J. Econ. **11**(1), 322–334 (1980)
11. Lindroos, M., Kaitala, V.T., Kronbak, L.G.: Coalition games in fishery economics. In: Advances in Fishery Economics. Blackwell (2007)
12. Mazalov, V.V., Rettieva, A.N.: Incentive equilibrium in discrete-time bioresource sharing model. Dokl. Math. **78**(3), 953–955 (2008)
13. Mazalov, V.V., Rettieva, A.N.: Fish wars and cooperation maintenance. Ecol. Model. **221**, 1545–1553 (2012)
14. Munro, G.R.: The optimal management of transboundary renewable resources. Can. J. Econ. **12**(8), 355–376 (1979)
15. Ostrom, E.: Governing the Commons: The Evolution of Institutions for Collective Action. Cambridge University Press, Cambridge, UK (1990)
16. Petrosjan, L.A., Danilov, N.N.: Stable solutions of nonantogonostic differential games with transferable utilities. Viestnik Leningrad Univ. **1**, 52–59 (1979)
17. Petrosyan, L.A., Zakharov, V.V.: Mathematical Models in Ecology. Saint-Petersburg University Press, Saint-Petersburg, Russia (1997)
18. Plourde, C.G., Yeung, D.: Harvesting of a transboundary replenishable fish stock: a noncooperative game solution. Mar. Resour. Econ. **6**, 57–70 (1989)
19. Tolwinski, B., Haurie, A., Leitmann, G.: Cooperative equilibria in differential games. J. Math. Anal. Appl. **119**, 182–202 (1986)

Author Queries

Chapter 20

Query Refs.	Details Required	Author's response
AQ1	This is to inform you that corresponding author has been identified as per the information available in the Copyright form.	
AQ2	References [4 and 9] are given in the list but not cited in the text. Please cite these in text or delete these from the list.	