Strategic Post-exam Preference Submission in the School Choice Game

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Previous studies

- Matching models generally:
  for survey, Roth (1982)

- Matching with incomplete information:
  Liu, Mailath, Postlewaite, Samuleson (2014), Chen and Hu (2023), Liu (2024)

- Job matching settings:
  Kojima, Sun, and Yu (2020)

Outline

1. The model
2. Two jobs
3. Multiple jobs
4. An experiment (in process)

The model

Notation and assumptions

- $N = \{1, \ldots, n\}$: a set of homogeneous candidates choosing a job.
- $J = \{J_1, \ldots J_m\}$: a set of jobs from different employers.

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- Candidates choose jobs simultaneously (each chooses only one job), but observe only their own scores.
- If several candidates choose J_j, only the candidate with higher score will get it.
- If a candidate gets J_j, his payoff is r_j, otherwise he receives zero payoff.

Two jobs

Strategies and Nash equilibrium

$J_1 \prec_i J_2, \ \ r_1 < r_2$

$s_i(x_i)$: candidate i's strategy—a probability distribution on J.

A threshold strategy $s_i(x_i)$ of a candidate i with:

$$s_i(x_i) = \begin{cases} (p, 1-p), & x_i \in [a, a_1], \\ (0, 1), & x_i \in (a_1, b]. \end{cases}$$

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Proposition. In the case of two jobs, Nash equilibrium behavior prescribes for a candidate $i \in N$ to adopt his threshold strategy

$$s_i^*(x_i) = \begin{cases} (p^*, 1-p^*) = \left(\dfrac{1}{1+F(a_1^*)}, \dfrac{F(a_1^*)}{1+F(a_1^*)}\right), & x_i \in [a, a_1^*], \\ (0,1), & x_i \in (a_1^*, b], \end{cases}$$

where a_1^* solves

$$F(a_1^*) = \left(\frac{r_1}{r_2}\right)^{\frac{1}{n-1}} = R.$$

Comparative statics analysis: Threshold and strategies

$$F(a_1^*) = \left(\frac{r_1}{r_2}\right)^{\frac{1}{n-1}} = R.$$

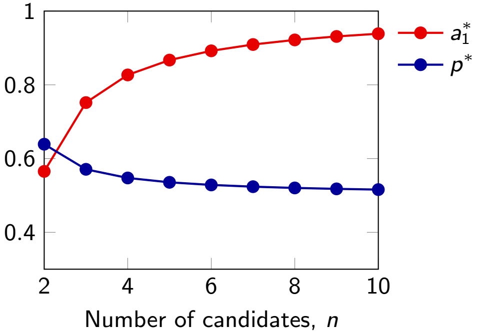
- a_1^* is increasing in n and $a_1^* \xrightarrow[n\to\infty]{} b$.
- a_1^* is convex in n when $2 + \left(\frac{R}{F'(F^{-1}(R))}\right)' \cdot \ln R < 0$.
- p^* is decreasing in n and $p^* \xrightarrow[n\to\infty]{} \frac{1}{2}$.
- p^* is convex in n when $2\left(1 - \frac{R \ln R}{1+R}\right) < -\ln R$.

An example

- $m = 2$ jobs.
- $n = 2$ candidates.
- $r_1 = 0.65$,
  $r_2 = 1.15$.
- $x_i \sim \mathbb{U}[0, 1]$.

Symmetric Nash equilibrium:

$$s_i(x_i) = \begin{cases} (0.639, 0.361), & x_i \in [0, 0.565], \\ (0, 1), & x_i \in (0.565, 1]. \end{cases}$$



Multiple jobs

Strategies

$J_1 \prec_i \ldots \prec_i J_m, \;\; r_1 < \ldots < r_m$

A threshold strategy $s_i(x_i) = (s_{i1}(x_i), \ldots, s_{im}(x_i))$ of a candidate i:

$$s_i(x_i) = \begin{cases} (s_i(1,1), \ldots, s_i(1,m)), & x_i \in [a, a_1], \\ (0, s_i(2,2), \ldots, s_i(2,m), & x_i \in (a_1, a_2], \\ \ldots & \\ (0, \ldots, 0, s_i(\ell,\ell), \ldots, s_i(\ell,m), & x_i \in (a_{\ell-1}, a_\ell], \\ \ldots & \\ (0, \ldots, 0, 1), & x_i \in (a_{m-1}, b], \end{cases}$$

Proposition. In the case of m jobs, Nash equilibrium behavior prescribes for a candidate $i \in N$ to adopt his threshold strategy

$$s^*(\ell, k) = \begin{cases} 0, & \ell \leqslant m,\ k < \ell - 1, \\ \dfrac{1}{1 + R_\ell + \ldots + R_\ell \cdots R_{m-1}}, & \ell < m,\ k = \ell, \\ \dfrac{R_\ell \cdots R_{k-1}}{1 + R_\ell + \ldots + R_\ell \cdots R_{m-1}}, & \ell < m,\ \ell < k < m, \\ 1, & \ell = k = m, \end{cases}$$

where $R_\ell = (\frac{r_\ell}{r_{\ell+1}})^{\frac{1}{n-1}}$ and a_ℓ^* solves

$$F(a_\ell^*) = -(m - \ell - 1) + R_\ell + \ldots + R_\ell \cdots R_{m-1}, \quad \ell < m.$$

Symmetric Nash equilibrium – 2

A remark

It may be true that for some value of ℓ

$$-(m-\ell-1)+R_\ell+\ldots+R_\ell\cdots R_{m-1}\geqslant 0,$$
$$-(m-\ell)+R_{\ell-1}+\ldots+R_{\ell-1}\cdots R_{m-1}<0.$$

In this case the candidates need to apply only to the jobs $J_\ell,\ldots,J_m$.

Comparative statics analysis: Thresholds

- a_ℓ^*, $\ell < m$, is increasing in n and $a_\ell^* \xrightarrow[n\to\infty]{} b$.

- a_ℓ^* is convex in n when

$$2 - (R_\ell \ln R_\ell + \ldots + R_\ell \ldots R_{m-1} \ln(R_\ell \cdots R_{m-1})) \cdot \frac{F''(a_\ell^*)}{(F'(a_\ell^*))^2} < -\frac{R_\ell (\ln R_\ell)^2 + \ldots + R_\ell \cdots R_{m-1} (\ln(R_\ell \cdots R_{m-1}))^2}{R_\ell \ln R_\ell + \ldots + R_\ell \cdots R_{m-1} \ln(R_\ell \cdots R_{m-1})}.$$

Comparative statics analysis: Strategies

- $s^*(\ell, \ell)$ is decreasing in n.

- $s^*(\ell, \ell)$ is convex in n when

$$2\left(1 - \frac{R_\ell \ln R_\ell + \ldots + R_\ell \cdots R_{m-1} \ln(R_\ell \cdots R_{m-1})}{1 + R_\ell + \ldots + R_\ell \cdots R_{m-1}}\right) < -\frac{R_\ell (\ln R_\ell)^2 + \ldots + R_\ell \cdots R_{m-1} (\ln(R_\ell \cdots R_{m-1}))^2}{R_\ell \ln R_\ell + \ldots + R_\ell \cdots R_{m-1} \ln(R_\ell \cdots R_{m-1})}.$$

- $s^*(\ell, k)$, $\ell < k < m$, is decreasing in n when

$$\frac{R_\ell \ln R_\ell + \ldots + R_\ell \cdots R_{m-1} \ln(R_\ell \cdots R_{m-1})}{1 + R_\ell + \ldots + R_\ell \cdots R_{m-1}} > \ln(R_\ell \cdots R_{k-1}).$$

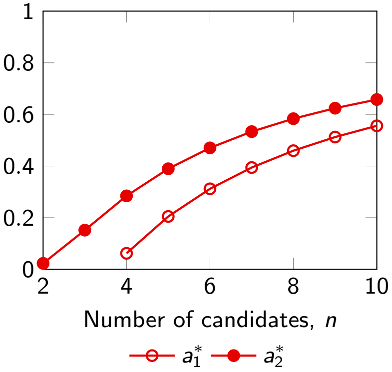
- $s^*(\ell, k)$, $\ell < k < m$, is convex in n when [*condition*].

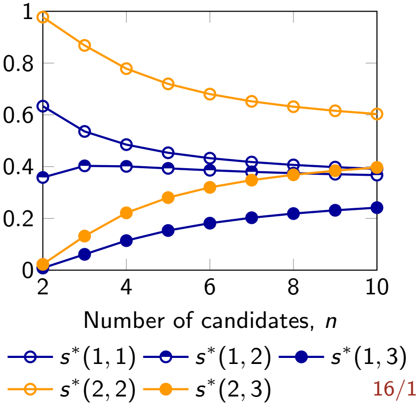
An example

- $m = 3$ jobs.
- $n = 8$ candidates.
- $r_1 = 0.65$,
  $r_2 = 1.15$,
  $r_3 = \mathbf{50}$.
- $x_i \sim \mathbb{U}[0,1]$.

Symmetric Nash equilibrium:

$$s_i^*(x_i) = \begin{cases} (0.406, 0.375, 0.219), & x_i \in [0, 0.459], \\ (0, 0.632, 0.368), & x_i \in (0.459, 0.583], \\ (0, 0, 1), & x_i \in (0.583, 1]. \end{cases}$$





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- $m = 2$ jobs.
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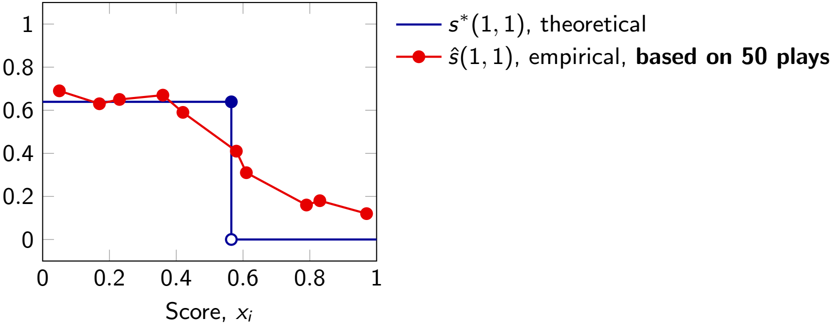

Thank you.