Sirius
Mathematics Center

015w: Probabilistic Techniques in Analysis: Spaces of Holomorphic Functions

December 6-10 | 2021

Sirius University of Science and Technology

Sirius Mathematics Center

International Conference

Probabilistic Techniques in Analysis: Spaces of Holomorphic Functions

December 6–10, 2021

Program and Abstracts

Sirius, 2021

Speakers:

- Natalia Abuzyarova, *Bashkir State University, Ufa*
- Nikita Alexeev, *ITMO University, Saint Petersburg*
- Iskander Azangulov, *Saint Petersburg State University*
- Viacheslav Borovitskiy, *Saint Petersburg State University*
- Vladimir Božin, *University of Belgrade, Serbia*
- Alexander Bufetov, *Steklov Institute, Moscow, and CNRS, France*
- Yalchin Efendiev, *Texas A&M University, USA*
- Konstantin Fedorovskiy, *Moscow State University*
- Haakan Hedenmalm, *Royal Institute of Technology, Stockholm, and Saint Petersburg State University*
- Konstantin Isaev, *Institute of Mathematics, Ufa*
- Nikita Karagodin, *Saint Petersburg State University*
- Ilgiz Kayumov, *Kazan Federal University*
- Bulat Khabibullin, *Bashkir State University, Ufa*
- Alexander Kuznetsov, *Saint Petersburg State University*
- Mark Lawrence, *Nazarbayev University, Nur-Sultan city, Republic of Kazakhstan*
- Guido Mazzuca, *Royal Institute of Technology, Stockholm, Sweden*
- Petar Melentijević, *University of Belgrade*
- Alfonso Montes-Rodríguez, *University of Sevilla, Spain*
- Peter Mostowsky, *Saint Petersburg State University*
- Pavel Mozolyako, *Saint Petersburg State University*
- Sergei Nikitin, *Saint Petersburg State University*
- Ekaterina Noskova, *ITMO University, Saint Petersburg*
- Joaquim Ortega-Cerdà, *University of Barcelona, Spain*
- Yulia Petrova, *IMPA, Rio de Janeiro, Brazil, and Saint Petersburg State University*
- Alexei Poltoratski, *University of Wisconsin, USA*
- Roman Romanov, *Saint Petersburg State University, Russia*

- Eero Saksman, *University of Helsinki, Finland*
- Vladimir Shalgin, *Saint Petersburg State University*
- Mikhail Skopenkov, *Higher School of Economics, Moscow*
- Arman Tadevosyan, *Saint Petersburg State University*
- Vesna Todorčević, *Mathematical Institute SANU, Belgrade, Serbia*
- Jani Virtanen, *University of Reading, United Kingdom*
- Steven Zelditch, *Northwestern University, Evanston, USA*

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- Vladimir Peller, *Saint Petersburg State University, Russia*

CONFERENCE PROGRAM

MONDAY 6 DECEMBER

10^{00} – 10^{45} Haakan Hedenmalm, *Soft Riemann-Hilbert problems and planar orthogonal polynomials*

10^{50} – 11^{35} Viacheslav Borovitskiy, *Gaussian processes in machine learning*

BREAK

11^{55} – 12^{40} Alexander Bufetov, *Determinantal point processes, quasi-symmetries and interpolation*

12^{45} – 13^{30} Roman Romanov, *Division subspaces, integrable kernels and determinantal processes*

LUNCH

15^{00} – 15^{45} Mikhail Skopenkov and Alexei Ustinov, *Feynman checkers: quantum field theory on a checkered paper and quantum computations*

15^{50} – 16^{35} Alfonso Montes-Rodríguez, *The Gauss–Kuzmin–Wirsing operator on the Bergman space*

BREAK

17^{10} – 17^{55} Jani Virtanen, *Hankel operators on weighted Fock spaces*

18^{00} – 19^{00} Steven Zelditch, *How are zeros of random polynomials distributed?*

TUESDAY 7 DECEMBER

10^{50} – 11^{35} Mark Lawrence, *Szegö–Bergman formulas for planar domains*

BREAK

11^{55} – 12^{40} Ilgiz Kayumov, *On the integral of the modulus of the derivative of a finite Blaschke product in the unit disk*

LUNCH

Section A (3E)

14^{10} – 14^{55} Alexander Kuznetsov, *Upper and lower densities of Gabor Gaussian systems*

15^{00} – 15^{45} Vladimir Božin, *On Brennan conjecture*

15^{50} – 16^{35} Evgeny Abakumov, *Chui's conjecture in Bergman spaces*

BREAK

17^{10} – 17^{55} Guido Mazzuca, *Generalized Gibbs ensemble of the Ablowitz–Ladik lattice, circular β-ensemble and double confluent Heun equation*

Section B (1E)

14^{10} – 14^{55} Iskander Azangulov, *Matern Gaussian process on graphs*

15^{00} – 15^{45} Nikita Alexeev, *Random graphs: models and applications*

15^{50} – 16^{35} Ekaterina Noskova, *Bayesian optimization for demographic history inference*

BREAK

17^{10} – 17^{55} Peter Mostowsky, *Probabilistic numerics and solving PDEs*

Plenary lecture (3E)

18^{00} – 18^{45} Alexei Poltoratski, *Pointwise convergence for the scattering data and non-linear Fourier transform*

THURSDAY 9 DECEMBER

Section A (3E)

10^{00} – 10^{45} Natalia Abuzyarova *Zero (sub)sets of slowly decreasing functions and Division Theorem in Schwartz algebra*

10^{50} – 11^{35} Petar Melentijević, *Best constants inequalities for Riesz and co-analytic projections with applications*

BREAK

11^{55} – 12^{40} Konstantin Isaev, *On Riesz bases of normalized reproducing kernels in radial Hilbert spaces of entire functions*

12^{45} – 13^{30} Pavel Mozolyako, *Hardy operator on the bi-tree and some unexpected combinatorial properties of planar measures*

Section B (1E)

10^{00} – 10^{45} Vladimir Shalgin, *Solving linear equations on adiabatic quantum computer*

10^{50} – 11^{35} Arman Tadevosyan, *Gaussian assignment process*

BREAK

11^{55} – 12^{40} Sergei Nikitin, *Large deviations of Telecom processes*

12^{45} – 13^{30} Nikita Karagodin, *On the distribution of the last exit time over a slowly growing boundary for a Gaussian process*

LUNCH

Plenary lectures (3E)

15^{00} – 15^{45} Vesna Todorčević, *Harmonic quasiconformal mappings and Ahlfors regular domains*

15^{50} – 16^{35} Yulia Petrova, *Small ball probabilities for Gaussian processes*

BREAK

17^{10} – 17^{55} Bulat Khabibullin, *Envelopes of subsets in vector lattices and balayage with applications to plurisubharmonic and holomorphic functions*

19^{00} CONFERENCE PARTY

FRIDAY 10 DECEMBER

10^{00} – 10^{45} Yalchin Efendiev, *Multiscale methods for stochastic problems*

10^{50} – 11^{35} Joaquim Ortega-Cerdà, *Hilbert points in Hardy spaces*

BREAK

11^{55} – 12^{40} Konstantin Fedorovskiy, *Approximation problems for polyanalytic functions*

12^{30} – 13^{30} Eero Saksman, *On a class of random inner functions*

LUNCH

Abstracts of the talks

Zero (sub)sets of slowly decreasing functions and Division Theorem in Schwartz algebra

Natalia Abuzyarova

Bashkir State University, Ufa

The talk is about slowly decreasing functions in the Schwartz algebra P defined as the Fourier-Laplace transform image of the Schwartz space of all compactly supported distributions on the real line. Slowly decreasing functions in P are exactly those ones for which Division Theorem in P holds. We discuss the properties of their zero (sub)sets. In particular, we draw parallels to already known properties of zero (sub)sets of sine-type functions.

Random graphs: models and applications

Nikita Alexeev

ITMO University, Saint Petersburg

We will discuss different random graphs models, in particular, I will explain how the graph structure depends on its density. I will formulate the classic theorem about the number of trees in Erdős–Rényi graphs and their (new) counterparts for other random graph models. If we have time we will discuss the applications of these results in computational biology and other fields.

Matern Gaussian process on graphs

Iskander Azangulov

Saint Petersburg State University

Gaussian processes are widely used in machine learning. Most of the models based on Gaussian processes assume that the data is embedded in a Euclidean space. In this talk, I will show how, using the SPDE characterization of Matern Gaussian processes, to generalize these to the case of weighted undirected graphs. We will discuss the properties of such processes and the adaptation of variational inference methods that allow the use of such models for classification tasks (and not only for them).

Gaussian Processes in machine learning

Viacheslav Borovitskiy

Saint Petersburg State University

Gaussian random fields (Gaussian processes) are beautiful mathematical objects that are useful in a number of areas outside of pure mathematics. In machine learning, they are widely accepted as a model of choice in scenarios where decision making under uncertainty is required e.g. in geological modeling, black-box optimization and robotics. In this talk I will briefly overview Gaussian processes as a machine learning tool and discuss some of their real-world applications.

On Brennan conjecture

Vladimir Božin
University of Belgrade, Serbia

Brennan conjecture concerns finiteness of p-norms of the derivatives of Riemann map from a plane domain onto the unit disk for $4/3 < p < 4$. It is related to the dome maps and Thurston and Sullivan conjecture, disproven by Epstein and Markovic in 2005. We will discuss some new results and proofs in case of conformal maps, harmonic quasi-conformal maps and other generalizations.

Determinantal point processes, quasi-symmetries and interpolation

Alexander Bufetov
Steklov Institute, Moscow, and CNRS, France

The study of point processes, that is, random subsets of a Polish space, goes back at least to the 1662 work of John Graunt on mortality in London. Matrices whose entries are given by chance were studied by Ronald Fisher in 1915 and John Wishart in 1928 and used by Freeman Dyson who in 1962 observed that "the statistical theory (...) will describe the degree of irregularity (...) expected to occur in any nucleus". The Weyl character formula implies that the correlation functions for the eigenvalues of a Haar-random unitary matrix have determinantal form – the Ginibre-Mehta theorem, – and in 1973 Odile Macchì started the systematic study of point processes whose correlation functions are given by determinants.

This level of abstraction has proved very fruitful: on the one hand, examples of determinantal point processes arise in diverse areas such as asymptotic combinatorics (Burton-Pemantle, Benjamini-Lyons-Peres-Schramm, Baik-Deift-Johansson, Borodin-Okounkov-Olshanski), representation theory of infinite-dimensional groups (Olshanski, Borodin-Olshanski), random series (Hough- Krishnapur-Peres-Virág) and, of course, random matrices; on the other hand, the general theory of determinantal point processes includes limit theorems (Soshnikov), a characterization of Palm measures (Shirai-Takahashi), the Kolmogorov as well as the Bernoulli property (Lyons, Lyons-Steif), and rigidity (Ghosh, Ghosh-Peres).

In this talk, the correlation kernels of our determinantal point processes will be assumed to induce orthogonal projections: for example, the sine-kernel of Dyson induces the projection onto the Paley-Wiener space of functions whose Fourier transform is supported on the unit interval, while the Bessel kernel of Tracy and Widom induces the orthogonal projection onto the subspace of square-integrable functions whose Hankel transform is supported on the unit interval.

What is the relation between the point process and the Hilbert space that governs it?

Extending the earlier work of Lyons and Ghosh, in joint work with Qiu and Shamov, it is proved that almost every realization of a determinantal point process is a uniqueness set for the underlying Hilbert space. For the sine-process, almost every realization with one particle removed is a complete and minimal set for the Paley-Wiener space, whereas if two particles are removed, then one obtains a zero set for the Paley-Wiener space. Quasi-invariance of the sine-process under compactly supported diffeomorphisms of the line plays a key role.

The 1933 Kotelnikov theorem samples a Paley-Wiener function from its restriction onto the integers. How to reconstruct a Paley-Wiener function from a realization of the sine-process?

In joint work with Borichev and Klimenko, it is proved that if a Paley-Wiener function decays at infinity as a sufficiently high negative power of the distance to the origin, then the Lagrange interpolation formula yields the desired reconstruction. Similar results are also obtained for the Airy kernel, the Bessel kernel, and the Ginibre kernel of orthogonal projection onto the Fock space.

In joint work with Qiu, the Patterson-Sullivan construction is used to interpolate Bergman functions from a realization of the determinantal point process with the Bergman kernel, in other words, by the Peres-Virág theorem, the zero set of a random series with independent complex Gaussian entries. The invariance of the zero set under the isometries of the Lobachevsky plane plays a key role.

Multiscale methods for stochastic problems

Yalchin Efendiev

Texas A&M University, USA

In this talk, we will focus on applications of multiscale methods to stochastic problems. Various techniques related to basis construction, upscaling, and machine learning will be discussed. Numerical results for porous media flows related to oil recovery applications will be presented.

Approximation problems for polyanalytic functions

Konstantin Fedorovskiy

Moscow State University

Polyanalytic functions occupy a special prominent place in the theory of approximations by analytic functions. Such famous problems, as Verdera's conjecture on the triviality of the approximation conditions for uniform polyanalytic rational approximation for classes of functions on compact sets in the complex plane (that was proved in 2008), or the problem on uniform polyanalytic polynomial approximation (which was solved in many special cases, but which is still open in the general case) gave impetus to the development of several fields in modern mathematical analysis. In the talk we will draw a picture of the state-of-the-art of the theory of approximation by polyanalytic functions, explain some recent results and discuss open questions.

Soft Riemann-Hilbert problems and planar orthogonal polynomials

Haakan Hedenmalm

Royal Institute of Technology, Stockholm, and Saint Petersburg State University

The asymptotics of orthogonal polynomials has a long history, with early contributions in the 1850–1900 period. Renewed interest began in the 1920s, with Szegö's and Carleman's new insight. Later, in the 1990s, a general framework was found for orthogonal polynomials on the line, with work by Fokas–Its–Kitaev and Deift–Zhou. For the corresponding case in the plane (with exponentially varying weights) not much was known. Work of Ameur–Hedenmalm–Makarov was able to derive convergence of the corresponding probability wave to harmonic measure. Later, the corresponding asymptotics of orthogonal polynomials was derived by Hedenmalm–Wennman using a complicated algorithm which required the construction of a flow of loops. We now show how to avoid those loops and obtain the asymptotics using an approximate solution of a soft Riemann-Hilbert problem introduced by Its–Takhtajan.

On Riesz bases of normalized reproducing kernels in radial Hilbert spaces of entire functions

Konstantin Isaev

Institute of Mathematics, Ufa

We consider a reproducing kernel radial Hilbert space of entire functions and prove necessary and sufficient conditions for the existence of unconditional bases of reproducing kernels in terms of norms of monomials. The results obtained are applied to weighted Fock spaces. We prove a criterion for the existence of unconditional bases of reproducing kernels in Fock spaces with radial regular weight.

On the distribution of the last exit time over a slowly growing boundary for a Gaussian process

Nikita Karagodin

Saint Petersburg State University

Consider a stationary Gaussian process with continuous trajectories and its "last exit time over a moving boundary", i.e. the last instant when the process hits a line $af(t)$ for a fixed function f and changing parameter a. We are interested in the asymptotic distribution of the last exit time when the parameter a goes to zero.

In this work a limit theorem on the convergence of the distribution of the properly centered and scaled last exit time to a double exponential (Gumbel) law is proven. A special case of this problem, for a particular process, emerged in recent works [1,2] providing a mathematical study of a physical model (Brownian chain break).

The last exit time is a sufficiently popular object in the problems of economical applications such as studies of ruin probabilities. In risk theory the process

$$R(t) = u + af(t) - Y(t),$$

where $Y(t)$ is a centered Gaussian process with continuous trajectories, represents a company balance. For instance, this is a basic model of an insurance company with a starting balance u, fixed income per time a (corresponding to a function $f(t) = t$) and stochastic expenses Y. In this setting the ruin time $\inf\{t : R(t) < 0\}$ the ultimate recovery time $\max\{t : R(t) \leq 0\}$ are of economic interest. In those settings, however, as a rule, one considers processes with stationary increments and trend is fixed, see [3,4,5], while in this work the ultimate recovery time for the case of the stationary process without starting balance and small trend is considered. As far as we know, the problem setting handling the small trend is new.

In order to succeed one has to choose between the variety of processes and the variety of boundaries. We consider weak assumptions on the correlation function of $Y(t)$ and strong assumptions on the boundary function $f(t)$, although covering the cases $f(t) = t^d, d > 0$ and $f(t) = \exp\{x^q\}$, $0 < q < 1$.

The case of the linear boundary is handled in [6].

The talk is based on the joint work with Mikhail Lifshits (St. Petersburg State University, Department of Mathematics and Computer Sciences).

References

[1] F. Aurzada, V. Betz, M. Lifshits, Breaking a chain of interacting Brownian particles. Ann. Appl. Probab. Published online: https://doi.org/10.1214/20-AAP1658.
[2] F. Aurzada, V. Betz, M. Lifshits, Universal break law for chains of Brownian particles with nearest neighbour interaction.
[3] K. Debicki, P. Liu, The time of ultimate recovery in Gaussian risk model. Extremes, 22(3), 499–521, 2019.
[4] J. Hüsler, Y. Zhang, On first and last ruin times of Gaussian processes. Statist. Probab. Lett., 78(10):1230–1235, 2008.
[5] Ch. Paroissin, L. Rabehasaina, First and last passage times of spectrally positive Lévy processes with application to reliability. Methodology and Computing in Applied Probability, 17, 351–372, 2015.
[6] N. Karagodin, M. Lifshits, On the distribution of the last exit time over a slowly growing linear boundary for a Gaussian process. https://arxiv.org/abs/2012.03222.

On the integral of the modulus of the derivative of a finite Blaschke product in the unit disk

Ilgiz Kayumov

Kazan Federal University

I am going to describe recent results obtained jointly with Anton Baranov on a sharp order of the growth for the integral of the modulus of the derivative of a finite Blaschke product in the unit disk. A relation with the Makarov law of the iterated logarithm will be demonstrated.

Envelopes of subsets in vector lattices and balayage with applications to plurisubharmonic and holomorphic functions

Bulat Khabibullin

Bashkir State University, Ufa

We consider the problem on the existence of the upper (lower) envelope of a subset on the projective limit of vector lattices with values in the completion of the Kantorovich space or on the extended real line. Our main scheme is a dual approach based on the concept of abstract balayage used in both potential theory and probability theory. We propose vectorial, ordinal, and topological dual interpretations of the existence conditions for such envelopes. For these purposes, we introduce the concepts of linear and affine balayage. Applications to the problem on the existence of a nontrivial (pluri)subharmonic and/or (pluri)harmonic minorant for functions in domains of a finite-dimensional real or complex space are considered. We also propose general approaches to problems on the nontriviality of weight classes of holomorphic functions, describing zero (sub)sets for such classes of holomorphic functions, and to the problem of representing a meromorphic function as a ratio of holomorphic function from a given weight class.

Upper and lower densities of Gabor Gaussian systems

Alexander Kuznetsov
Saint Petersburg State University

Given a set Λ of points in $\mathbb{R}^2$ we define the corresponding Gabor Gaussian system

$$\mathcal{G}_\Lambda = \{e^{2i\pi\omega x}\varphi(x-t) : (t,w) \in \Lambda\}, \quad \varphi(x) = 2^{1/4}e^{-\pi x^2}.$$

We give a construction of a complete and minimal Gabor Gaussian system with the upper density $\frac{1}{\pi}$ (in a classical lattice case the density is equal to 1). We reformulate this result in a language of the Fock space of square-integrable entire functions with the weight $e^{-\pi|z|^2/2}$ and construct Λ as the zero set of some entire function. Based on the joint paper: Y. Belov, A. Borichev and A. Kuznetsov, Upper and lower densities of Gabor Gaussian systems, Appl. Comput. Harmon. Anal., 49 (2020), pp. 438–450.

Szegö-Bergman formulas for planar domains

Mark G. Lawrence
Nazarbayev University, Nur-Sultan city, Republic of Kazakhstan

The setting for the theorems presented her is the use of the 1-dimensional extension property in complex analysis. One has a family of curves C_λ on a domain in $\mathbb{C}^n$ or on the boundary of a domain in $\mathbb{C}^n$. It is supposed that each curve bounds an analytic disc contained in the domain. If a function f has analytic extension from each curve to the analytic disc it bounds, one wants to know that the function is holomorphic, or in the case of working from a boundary, one wants to know that the function has an analytic extension to the domain. Two examples of this are the famous strip theorem of V. Tumanov, and the speaker's "CR Hartogs theorem" which deals with extension from convex boundaries in $\mathbb{C}^n$. By proving theorems in L^p and weighted L^p spaces, one is able to do some new analysis. In this talk I discuss how to recover weighted weighted Bergman projections on planar domains. This uses improvements on the strip problem, and also, via projection, the CH Hartogs theorem in $\mathbb{C}^2$. One obtains explicit asymptotic expressions for the Bergman projection which involve Szegö projections on the curves from the one dimensional extension set-up.

Generalized Gibbs ensemble of the Ablowitz–Ladik lattice, circular β-ensemble and double confluent Heun equation

Guido Mazzuca
Royal Institute of Technology, Stockholm, Sweden

In this talk, I will focus on the Ablowitz–Ladik lattice, which is an integrable system. I will introduce the Generalized Gibbs ensemble for this lattice and relate its density of states with the one of the Circular beta ensemble in the high temperature regime. This connection allows me to obtain an explicit formula for the density of states of the Ablozit–Ladik lattice as a solution of the double confluent Heun equation.

This talk is mainly based on my recent paper with Tamara Grava, https://arxiv.org/abs /2107.02303.

Best constants inequalities for Riesz and co-analytic projections with applications

Petar Melentijević
University of Belgrade

We address the problem of finding the best constants in inequalities of the form:

$$\||P_+f|^s + |P_-f|^s\|_{L^p(\mathbb{T})} \leq C_{s,p}\|f\|_{L^p(\mathbb{T})},$$

where P_+f and P_-f denote analytic and co-analytic projection of a function f, for $p \geq 2$ and $s \leq \frac{1}{\sin^2 \frac{\pi}{2p}}$, thus proving Hollenbeck–Verbitsky conjecture. Moreover, for $2 \leq p \leq 4$, we are able to prove optimal version of this inequality for all $s \in \mathbb{R}^+$.

The proof uses method of plurisubharmonic minorants and an approach of proving the appropriate "elementary" inequalities that seem to be new. Some applications of the main results are given.

The Gauss–Kuzmin–Wirsing operator on the Bergman space

Alfonso Montes-Rodríguez
University of Sevilla

The operator

$$Pf = \sum_{n\in\mathbb{Z}\setminus\{0\}} \frac{1}{(x+2k)^2} f\left(\frac{-1}{x+2k}\right)$$

is called the Gauss–Kuzmin–Wirsing operator. When acting on $L^1[-1,1]$, it is a Perrón-Frobenius operator, that is, Pf is positive for positive f and it is an isometry on the cone of the positive functions. Between many other things. the study of the dynamics of this operator allowed Hedenmalm and Montes to solve the Goursat problem in the affirmative. Consider now the Berman space $\mathcal{A}^2(\mathbb{D})$ of those functions analytic on the unit disk $\mathbb{D}$ of the complex function that are of square modulus integrable with respect to the area measure. Formula (1) defines a linear bounded operator on $A(\mathbb{D})$. Indeed it is also normal. We will show that P acting in the Bergman space is also compact. One of the consequences of this is that there are plenty of functions $f \in L^2[-1,1]$ for which $\|T^n f\|_1$ tends to 0 geometrically. The compactness of P acting on $\mathcal{A}^2$ is intimately related with the Stieljes integral.

Probabilistic numerics and solving PDEs

Peter Mostowsky
Saint Petersburg State University

I will talk about probabilistic numerics which started to actively develop recently. Many "classical" numerical methods may be obtained from a probability theory viewpoint. This allows control of numerical error and propagating uncertainty through levels of computations, for example. I will talk about applying probabilistic numerics approach to solving partial differential equations.

Hardy operator on the bi-tree and some unexpected combinatorial properties of planar measures

Pavel Mozolyako

Saint Petersburg State University

We consider embeddings of various spaces of analytic and harmonic functions on the polydisc into Lebesgue spaces with respect to a measure on the polydisc. These problems can often be moved to a discrete setting by considering weighted Dirichlet spaces on (poly-) trees and weighted dyadic multi-parameter Hardy operators. We find necessary and sufficient conditions for this operator to be bounded in the n-parameter case, when n is 1, 2, or 3. The answer is rather strange – it is a certain combinatorial property of all measures in dimension 2 and 3 – and seemingly goes against the well-known difference between box and Chang–Fefferman condition. Time permitting we also discuss some possible applications of our model.

Large Deviations of Telecom Processes

Sergei Nikitin

Saint Petersburg State University

We study large deviation probabilities of Telecom processes appearing as limits in a critical regime of the infinite source Poisson model elaborated by I. Kaj and M. Taqqu. We examine three different regimes of large deviations (LD) depending on the deviation level. A Telecom process $(Y_t)_{t\geq 0}$ itself scales as $t^{1/\gamma}$, where $t^{1/\gamma}$ denotes time and $\gamma \in (1, 2)$ is the key parameter of Y.

When studying the large deviation probability $\mathbb{P}(Y_t \geq q)$ one must distinguish moderate LD with $t^{1/\gamma} \ll q \ll t$, intermediate LD with $q \approx t$, and ultralarge LD with $q \gg t$. The results we obtain essentially depend on another parameter of Y, namely resource distribution. We solve completely the cases of moderate and intermediate LD (the latter being the most technical one), whereas the ultralarge deviation asymptotics is found for the case of regularly varying resource distribution tails. In all considered cases, the large deviation level is essentially reached by the minimal necessary number of "service processes".

This is a joint work with M.A. Lifshits. The results are published in the preprint arXiv:2107.11846.

Bayesian optimization for demographic history inference

Ekaterina Noskova

ITMO University, Saint Petersburg

Demographic history of populations is a record of their evolutionary history that includes parameters like population size dynamics, divergence times, rates of migration and selection over time. With the rise of the next generation sequencing technologies and emergence of the abundant genome data, it has become possible to explore complex and parameter-rich demographic models.Existing methods for inference of demographic history solve the “forward” problem. They model population statistics for a given demographic history. A researcher then has to try a number of plausible histories and choose the one for which the modeled statistics align well with the observed data. In order to take the human out of the loop we are interested in automatically solving the inverse problem that is reduced to an optimization one: find the parameters of the demographic history that maximize the likelihood of the observed data.Bayesian optimization is one of the most popular algorithms for optimization with a limited time budget. In my talk, I will tell about the application of the Bayesian optimization method for fast inference of the demographic history parameters.

Hilbert points in Hardy spaces

Joaquim Ortega-Cerdà

University of Barcelona, Spain

A Hilbert point in $H^p(\mathbb{T}^d)$, is a nontrivial function g in $H^p(\mathbb{T}^d)$ such that $\|g\|_p \leq \|g + f\|_p$ whenever f is in $H^p(\mathbb{T}^d)$ and orthogonal to g in the usual L^2 sense. We find a complete description in one variable and investigate the case of 1-homogenous polynomials in several variables. As a byproduct we obtain a new proof of the sharp Khintchin inequality for Steinhaus variables in the range $2 < p < \infty$.

Small ball probabilities for Gaussian processes

Yulia Petrova

IMPA, Rio de Janeiro, Brazil, and Saint Petersburg State University

In the talk we will consider a problem of small ball probabilities for Gaussian processes, which consists in finding the asymptotics of probability that a norm of a process is less than ε as ε tends to zero. This question arises in different areas: quantization of Gaussian vectors, metric entropy, etc. We will consider what is already known in general situation and talk about more advanced results in L_2-norm, for which the distribution is totally defined by eigenvalues of the covariance operator. For a wide class of Green Gaussian processes (with covariance function being a Green function for some ODE) we can use the powerful methods of spectral theory for ODEs to get the exact small ball probabilities. Another question we will address is how to relate the small ball probabilities of some process and it’s finite dimensional perturbations.

Pointwise convergence for the scattering data and non-linear Fourier transform

Alexei Poltoratski

University of Wisconsin, USA

This talk is about applications of complex and harmonic analysis in spectral and scattering theory for differential operators. The scattering transform for the Dirac system of differential equations can be viewed as the non-linear version of the classical Fourier transform. This connection raises many natural problems on extensions of classical results of Fourier analysis to non-linear settings. In this talk I will discuss one of such problems, an extension of Carleson's theorem on pointwise convergence of Fourier series to the non-linear case.

Division subspaces, integrable kernels and determinantal processes

Roman Romanov

Saint Petersburg State University

We will discuss the relationships between division invariant function spaces, integrability of the corresponding reproducing kernels and the quasi-invariance property of determinantal point processes.

On a class of random inner functions

Eero Saksman

University of Helsinki, Finland

We consider some properties of random inner functions, related to Gaussian multiplicative chaos. The talk is based on joint work with Yichao Huang (U. Helsinki).

Solving linear equations on adiabatic quantum computer

Vladimir Shalgin

Saint Petersburg State University

We consider the problem of solving the equation $ax = b$ using the adiabatic quantum computer D-Wave 2000Q. Due to the specifics of the computer, the task is reduced to finding the minimum of the function $H(x) = (ax-b)^2$, where $x = c\sum_{i=0}^{R-1} 2^{-i}q_i - d$ is a R-bit representation of the desired minimum point up to scaling and shifting. A matrix of the quadratic form $H(q_0, \dots, q_{R-1})$ it is sent to the computer as an input. The result of the computer is a random variable having a Boltzmann distribution. It is proved that in the case when R, c, $d \to +\infty$, the limiting distribution of solutions of the equation $ax = b$ is a normal distribution, or a truncated normal distribution when only $R \to +\infty$. The parameters of the distribution of solutions to one equation were found experimentally by minimizing the distance between the empirical distribution and the normal distribution in one case and the Boltzmann distribution in the other. An algorithm for refining the solution of the equation is constructed. Sufficient conditions for its convergence are found in the case of the assumption of a normal and truncated normal distribution of solutions. A generalization of the above results is developing for the case of a system of linear equations, including systems with no solutions or having infinitely many solutions.

Feynman checkers: quantum field theory on a checkered paper and quantum computations

Mikhail Skopenkov, Alexei Ustinov

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We study the most elementary model of electron motion introduced by R.Feynman. It is a game, in which a checker moves on a checkerboard by simple rules, and we count the turns. It is also known as a one-dimensional quantum walk or an Ising model at imaginary temperature. Quantum walks are one of the universal models of a quantum computer. We solve mathematically a problem by R. Feynman from 1965, which was to prove that the discrete model is consistent with the continuum one, namely, reproduces the usual quantum-mechanical free-particle kernel for large time, small average velocity, and small lattice step. Most of the talk is accessible to undergraduate students; no knowledge of physics is assumed.

Gaussian assignment process

Arman Tadevosyan

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Consider the following *random assignment problem*. Let (X_{ij}) be a $n \times n$ random matrix with i.i.d. random entries having a common distribution $\mathcal{P}$. Let $\mathcal{S}_n$ denote the group of permutations $\pi : \{1, 2, \dots, n\} \to \{1, 2, \dots, n\}$. For every $\pi \in \mathcal{S}_n$ let

$$S(\pi) = \sum_{i=1}^{n} X_{i\pi(i)}.$$

We are interested in the study of $\max_{\pi \in \mathcal{S}_n} S(\pi)$ in case, where $\mathcal{P} = \mathcal{N}(0, 1)$, and in finding of the optimal permutation π^*, such that $\mathbb{E}\, S(\pi^*) = \mathbb{E} \max_{\pi \in \mathcal{S}_n} S(\pi)$. We call $\{S(\pi), \pi \in \mathcal{S}_n\}$ *a Gaussian assignment process.*

The setting with (X_{ij}) uniformly distributed on $[0, 1]$ was studied by Steele [1] and Mézard and Parisi [2], where the authors proved that

$$\mathbb{E} \min_{\pi \in \mathcal{S}_n} S(\pi) = \zeta(2) - \frac{\zeta(2) + 2\zeta(3)}{n} + O\left(\frac{1}{n^2}\right), \quad n \to \infty,$$

$\zeta(\cdot)$ being Riemann's zeta function. Mézard et al. [3] also conjectured that in the exponential case ($\mathcal{P} = \mathrm{Exp}(1)$) it is true that

$$\mathbb{E} \min_{\pi \in \mathcal{S}_n} S(\pi) \to \zeta(2). \tag{1}$$

Aldous [4] gave a rigorous proof of conjecture (1). His approach is based on the assignment analysis of a graph with edges provided with exponentially distributed weights, see [5].

We show that in the Gaussian case

$$\mathbb{E} \max_{\pi \in \mathcal{S}_n} S(\pi) \sim n\sqrt{2 \log n}, \quad n \to \infty.$$

The talk based on a joint work with Mikhail Lifshits (St. Petersburg University, Department of Mathematics and Computer Sciences).

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Harmonic quasiconformal mappings and Ahlfors regular domains

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Geometry of domains has been an important topic in Geometric Function Theory. Lavrentiev or chord-arc domains are domains which are both quasidiscs and Ahlfors regular domains. A result of Pommerenke has been generalized by Zinsmeister from chord-arc to Ahlfors domains and by Kalaj from conformal to harmonic quasiconformal mappings. We will consider these questions and further possible generalizations.

Hankel operators on weighted Fock spaces

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In this talk I discuss the space IDA of locally integral functions whose "Integral Distance to Analytic functions" is finite, and use it to completely characterize boundedness and compactness of Hankel operators on weighted Fock spaces. As an application, for bounded symbols, we show that the Hankel operator H_f is compact if and only if $H_{\bar{f}}$ is compact, which complements the classical compactness result of Berger and Coburn. This phenomenon is a unique feature in the theory of Fock spaces and is not true for Hankel operators on Bergman or Fock spaces. This leads to a question of whether an analogous phenomenon holds true for Hankel operators in the Schatten classes. This was answered in the affirmative for the Hilbert–Schmidt class by Bauer in 2004. To deal with the remaining cases, we give a complete characterization of the Schatten class membership of single Hankel operators using the space IDA and Hörmander's $\bar{\partial}$-theory. As a further application of the above results, the Berezin–Toeplitz quantization may be discussed briefly. This is joint work with Zhangjian Hu.

How are zeros of random polynomials distributed?

Steven Zelditch

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Random polynomials of degree N are polynomials $p_N(z) = \sum_{j=0}^{N} a_j z^j$ whose coefficients a_j are random. That is, there is a probability measure on the coefficients a_j, which could be real or complex numbers. The Kac random polynomials has i.i.d. real or complex Gaussian coefficients. It was proved that the complex zeros of such random polynomials concentrate on the unit circle. What made that happen? Any N complex numbers are the zeros of some p_N. My talk will explain the answer and show how it led to a highly developed theory of 'random complex geometry' in all dimensions, involving Hermitian metrics on line bundles and equilibrium measures. No priori knowledge of probability theory is assumed.

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