Modification of PWM Algorithms in Continuous and Discontinuous Modes for a Three-Phase Inverter in an Oblique Coordinate System

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***Abstract*—Close relationship between Aaron method of two wattmeter measurement for three-phase grid and an oblique coordinate systems of electric machines is investigated. An oblique coordinate system allows completely eliminate direct and reverse Clarke and Gorev-Park transformations for three phase inverters. This also leads to a significant simplification of SVPWM techniques for continuous and discontinuous modes.**

Keywords—oblique coordinate system, inverter, SVPWM, pulse width modulation, covariant coordinates

I. An oblique coordinates in three-phase electric machine and their relationships with measured values

The inverter in Fig. 1 connected to a three-phase electric machine without neutral wire. Currents obeys Kirchhoff laws in the form:

$$i_A + i_B + i_C = 0. \qquad (1)$$

The inverter joint to industrial grid through inductors to smooth current and voltage ripples at industrial grid side of inductors. The value of inductors are picked up by equation:

$$L\frac{\Delta i}{\Delta t} = \Delta u$$

Here Δu – line voltage, Δt - hemiperiod of PWM, Δi – current ripples, usually 5-10% of I RMS. For example, for Δu=540V, PWM frequency 5kHz, current 1000A we get L=0.0005H.

Voltage and current sensors used in three-phase voltage inverter driven by an industrial three-phase grid are depicted in Fig. 1.

Current sensors can be installed at any end of the inductor. Line voltage sensors installed on the inductor side opposite inverter. Line voltages on the inductor at the inverter side are known from the control system.

By using set of sensors as depicted at Fig. 1, and by using Kirchhoff laws for currents and introducing ”zero” potential u_0 (value of u_0 could change in time) for non-measured directly by voltage sensors, we could develop formula for the power:

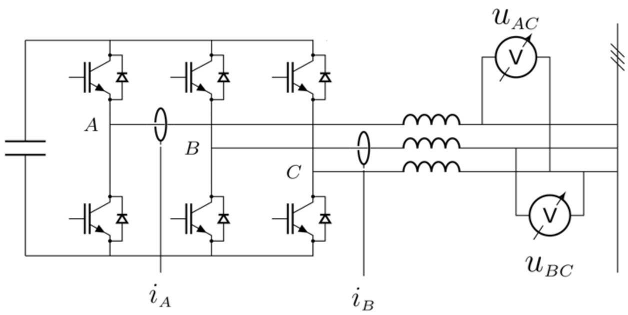

Fig. 1 Current ad voltage sensors in three-phase inverter

$$P = i_A u_{AC} + i_B u_{BC} =$$
$$= i_A(u_A - u_0 - (u_C - u_0)) + i_B(u_B - u_0 - (u_C - u_0)) =$$
$$= i_A u_A + i_B u_B - u_C(i_A + i_B) = i_A u_A + i_B u_B + i_C u_C,$$

and the power, as developed, equals to the sum of the powers of each three phases.

Expression for power

$$P = i_A u_{AC} + i_B u_{BC} \qquad (2)$$

literally agrees with the amount of power measured according to Aaron’s two wattmeter [1] method: sum of measured instantaneous values of i_A current winding and u_{AC} voltage winding of one wattmeter, and i_B and u_{BC} – of another wattmeter. ”Zero” voltages potential u_0 may be equal to the potential of the junction point of the three phases in the star circuit, or could be chosen arbitrarily and, usually, is selected from the condition:

$$u_A + u_B + u_C = 0 \qquad (3)$$

In an oblique coordinate system with units basis vectors along the axes $|\overrightarrow{e_1}| = 1$ for amplitude arbitrary vector with perpendicular projections of x_i and components of the decomposition of x^i of this vector over the basis vectors $\overline{e_i}$, where i runs through the axis’s numbers {1, 2}, we have:

$$|\overline{X}| = \sqrt{x_1 x^1 + x_2 x^2}.$$

Here and below i in the notation x^i is not a power but the upper index, similarly, x_i is the lower index [3].

Scalar the product of two vectors is written as:

$$(\overline{X}, \overline{Y}) = x^1 y_1 + x^2 y_2 = y^1 x_1 + y^2 x_2 \qquad (4)$$

Vectors $\overrightarrow{e^i}$, selected by the rule:

$$(\overrightarrow{e^i}, \overrightarrow{e_k}) = \delta_k^i,$$

specified conjugate or dual basis. Here i, k run through the axes {1, 2}, δ_i^j is the Kronecker symbol such that:

$$\delta_i^j = \begin{cases} 1, \text{при } i = j \\ 0, \text{при } i \neq j \end{cases}$$

The vector $\overrightarrow{e^1}$ is perpendicular to $\overrightarrow{e_2}$ and $\overrightarrow{e^2}$ is perpendicular to $\overrightarrow{e_1}$ hence the direction of the vectors $\overrightarrow{e^i}$ could be chosen in two ways. Actual direction is chosen to make the angle between $\overrightarrow{e^i}$ and $\overrightarrow{e_i}$ acute. This definition of dual vectors make amplitude of unit vector $\overrightarrow{e^i}$ of dual axes not equal the ones $\overrightarrow{e_i}$ of base axes:

$$\left|\overrightarrow{e^i}\right| \neq |\overrightarrow{e_i}| = 1$$

An arbitrary vector $\overline{X}$ is decomposed along the base axes:

$$\vec{X} = x^1 \overrightarrow{e_1} + x^2 \overrightarrow{e_2}.$$

and also along the dual axes

$$\vec{X} = x_1 \overrightarrow{e^1} + x_2 \overrightarrow{e^2}.$$

The coordinates x_i are called covariant, and x^i are contravariant.

Scalar product is

$$(\overline{X}, \overline{X}) = (x^1 \overrightarrow{e_1} + x^2 \overrightarrow{e_2})\left(x_1 \overrightarrow{e^1} + x_2 \overrightarrow{e^2}\right) = x^1 x_1 + x^2 x_2$$

here the coordinates x_i and x^i are of different unit sizes.

Covariant coordinates could be obtained from contravariant:

$$x_k = \sum_{j=1}^{2} g_{kj}\, x^j \qquad (5)$$

where g_{kj} – metric tensor:

$$g_{kj} = \begin{pmatrix} (\overrightarrow{e_1}, \overrightarrow{e_1}) & (\overrightarrow{e_1}, \overrightarrow{e_2}) \\ (\overrightarrow{e_2}, \overrightarrow{e_1}) & (\overrightarrow{e_2}, \overrightarrow{e_2}) \end{pmatrix}$$

Formula (5) is called "juggling" of indices.

As the base coordinate system, we choose the axes consist of measured currents i_A, i_B and phase voltages u_A, u_B. The instant measured value of current i_A is a perpendicular projection of the representing vector $\vec{\imath}$, drawn from the origin. For any vector $\vec{x}$ drawn from the origin of coordinates, the projections onto the phase axes A, B, C satisfy the equation

$x_A + x_B + x_C = 0$, which is true for the representing current vector $\vec{\imath}$ which has the sum of the projections on the axes A, B, C is equal to zero according to the Kirchhoff law (1). Due to the choice of potential u_0 (3), the representing phase voltage vector $\vec{u}$ is also drawn from the origin. Perpendicular projections of the phase voltage vector u_k are not directly measurable. Perpendicular projections i_k and u_k are covariant coordinates of the representing vectors $\vec{\imath}$ and $\vec{u}$.

The line voltage axes $u_A - u_C$ and $u_B - u_C$ appeared to be codirectional to the dual axes U^A and U^B, as are shown in Fig. 2. From an engineer's point of view, the dimension of

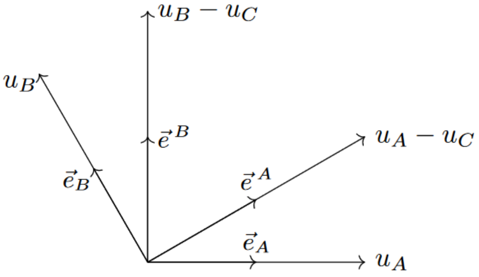

Fig..2 line voltage axes are codirectional to the dual axes

units of basic voltage vectors, accepted in mathematics, along the dual axes $\overrightarrow{e^k}$ differs from the dimension of units of basic voltage vectors $\overrightarrow{e_k}$ along the basic axes.

$$\left|\overrightarrow{e^A}\right| \neq |\overrightarrow{e_A}| = 1$$

However, the instant coordinate of the current i_A appears to be measured in the correct dimension $|\overrightarrow{e_A}|$=1 and vector $i_A \overrightarrow{e^A}$ is the vector projection vector $\vec{\imath}$ to the conjugate axis i^A. Vector projection phase voltage $u_A \overrightarrow{e^A}$ and covariant coordinates u_A and the vector projection are depicted in Fig. 3.

Contravariant coordinates can be obtained from covariant:

$$u^k = \sum_{j=A,B}^{2} g^{kj}\, u_j$$

here g_{kj} – inverse metric tensor:

$$g^{kj} = \begin{pmatrix} (\overrightarrow{e^A}, \overrightarrow{e^A}) & (\overrightarrow{e^A}, \overrightarrow{e^B}) \\ (\overrightarrow{e^B}, \overrightarrow{e^A}) & (\overrightarrow{e^B}, \overrightarrow{e^B}) \end{pmatrix} = \begin{pmatrix} 4/3 & 2/3 \\ 2/3 & 4/3 \end{pmatrix}$$

$$u^A = g^{AA} u_A + g^{AB} u_B = \frac{4}{3} u_A + \frac{2}{3} u_B =$$

$$= \frac{2}{3} u_A - \frac{2}{3} u_C + \frac{2}{3} u_A + \frac{2}{3} u_B + \frac{2}{3} u_C$$

Here the sum of the last three terms could be made equal to 0 due to the freedom in the choice of the origin of phase's voltage (3). We get:

$$u^A = \frac{2}{3}(u_A - u_C) = \frac{2}{\sqrt{3}} \frac{u_A - u_C}{\sqrt{3}} \qquad (6)$$

The contravariant coordinate u^A is the decomposition of the representing vector $\vec{u}$ in the basis of unit vectors $\overrightarrow{e_k}$. The vector projection of the vector $u^A \overrightarrow{e_A}$ onto the dual axis U^A is equal to:

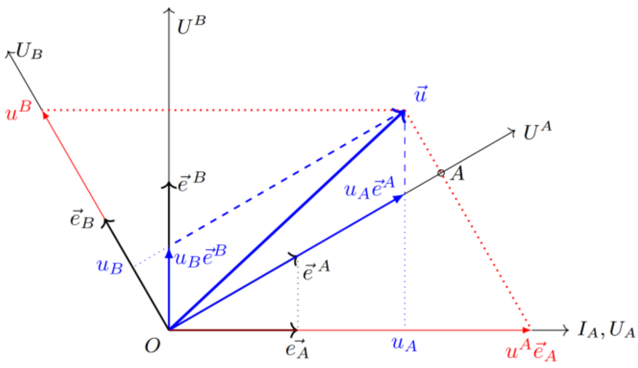

Fig.3 Geometric relations with contravariant coordinate u^A

$$|OA| = \frac{u_A - u_C}{\sqrt{3}}$$

whence for the contravariant coordinate u^A we get:

$$u^A = \frac{\left|\overrightarrow{e^A}\right|}{\left|\overrightarrow{e_A}\right|}|OA| = \frac{2}{\sqrt{3}}\frac{u_A - u_C}{\sqrt{3}}$$

which agrees with the expression (6).

For the scalar product $(\vec{\imath}, \vec{v})$ following (4) and (6):

$$(\vec{\imath}, \vec{v}) = i_A u^A + i_B u^B = \frac{2}{\sqrt{3}}\left(i_A \frac{u_A - u_C}{\sqrt{3}} + i_B \frac{u_B - u_C}{\sqrt{3}}\right)$$

Comparing the above expression with the formula for power (2) we obtain the rule: power P is scalar product of the current vector measured by phase axes, and line voltage vector, measured along the axes of line voltage:

$$P = (\vec{\imath}, \overrightarrow{v_{line}})$$

II. MODIFYING ALGORITHMS

Typical algorithm for control of an inverter is shown in Fig. 4. We are going to change the PWM algorithms in terms of line voltages and phase currents, and to be specific: to eliminate the direct and the inverse Park-Gorev and Clarke transform and algorithm for duty cycles of switches in half-bridge phases T_a, T_b, T_c. All changed blocks are colored red.

A. *Elimination of direct and reverse Park-Gorev transformation*

Park-Gorev's direct transformation is a transition from stationary frame in rotating frame. Reverse the Park-Gorev transformation makes the reverse transition from rotating to stationary frames.

Instead of reverse Park-Gorev transform to Cartesian with axes α,β we convert to stationary normalized line voltages u_{ac} and u_{bc} as shown on Fig.5. The same substitution could be done with direct Park-Gorev transform.

$$\begin{pmatrix} \frac{u_{AC}}{\sqrt{3}} \\ \frac{u_{BC}}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} \cos\left(\theta - \frac{\pi}{6}\right) & -\sin\left(\theta - \frac{\pi}{6}\right) \\ \sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} u_d \\ u_q \end{pmatrix}$$

Here u_d and u_q are the coordinates of the representing voltage vector in a rotating frame, ϑ is the vector between the rotating and stationary frames. uac is the normalization of the line voltage to unity.

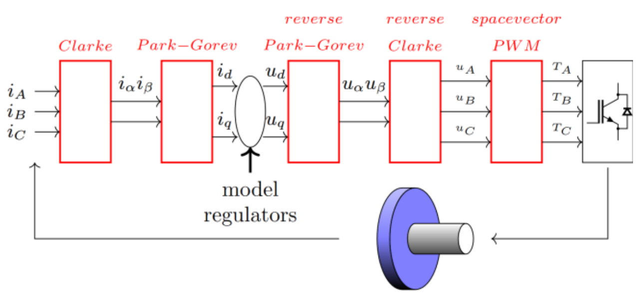

Fig.4 Typical algorithm for control of an inverter

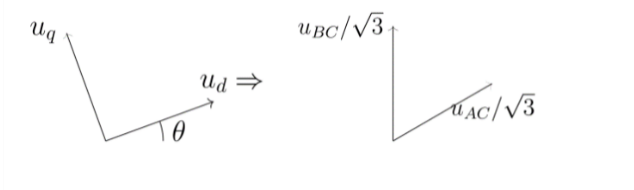

Fig.5 Transform form rotating frame to stationary frame

B. *Developing of the algorithm for duty cycles for continuous and discontinuous mode*

Let's find the coordinates of the vector as the center of mass of the scales m_0, m_1, m_2 in sector I. Maximum amplitude line voltage vector is chosen so that u_{acmax} = 1, u_{bcmax}= 1. Line voltage axis U_{BC} selected as shown in Fig. 6 from point O' to point O". The weights of vectors $\overrightarrow{U_0}, \overrightarrow{U_1}, \overrightarrow{U_2}$ for sector I are depicted in Fig. 6.

$$\vec{u} = m_0\overrightarrow{U_0} + m_1\overrightarrow{U_1} + m_2\overrightarrow{U_2}.$$

From Archimedes law for leverage:

$$|u_{ac}|m_0 = (u_{ac\,max} - |u_{ac}|)(m_1 + m_2)$$

we get a system of equations:

$$\begin{cases} u_{ac}m_0 = (u_{ac\,max} - u_{ac})(m_1 + m_2) \\ u_{bc}(m_0 + m_1) = (u_{ac\,max} - u_{ac})m_2 \\ m_0 + m_1 + m_2 = 1 \end{cases} \quad (3)$$

Here the sum $m_0 + m_1 + m_2 = 1$ expresses the fact that we are in sector I. The results from system 10 are summarized in Table I

TABLE I. WEIGHTS OF VECTORS $U_i, i = 0, \ldots, 7$

	I	II	III	IV	V	VI
m_1	u_{ab}	u_{ac}	u_{bc}	u_{ba}	u_{ca}	u_{cb}
m_2	u_{bc}	u_{ba}	u_{ca}	u_{cb}	u_{ab}	u_{ac}

Although these results are presented in the literature [5], we derived these results without using the Cartesian system or complex plane which is also Cartesian coordinates.

For PWM in continuous mode for sector I duty cycles T_a, T_b, T_c obey the system of equations:

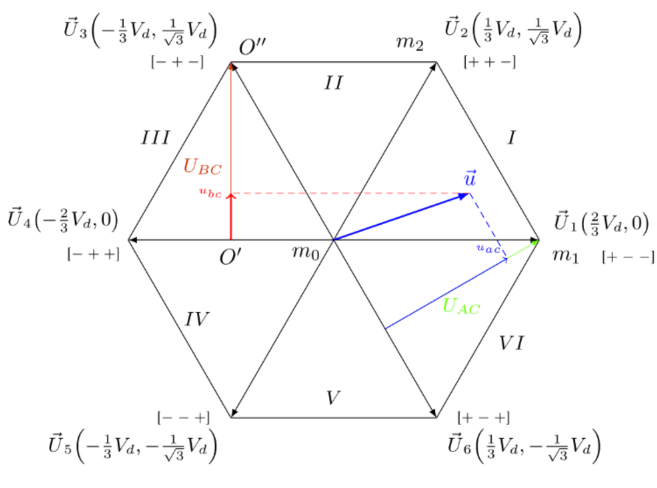

Fig.6 Representing vector of line voltage in sector I

$$\begin{cases} 1 - T_a + T_c = 1 - (m_1 + m_2) \\ T_a - T_b = m_1 \\ T_b - T_c = m_2 \end{cases} \quad (4)$$

The first equation is linearly dependent and could be replaced by an equation that expresses the fact that part zero vector U_7 when all half-bridges are connected to positive bus is equal to the zero vector U_0 when all half bridges are connected to the negative bus. Timer settings for sector I for continuous mode are shown in Fig. 7

$$1 - T_a = T_c$$

We get a system of equations in matrix form:

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} T_a \\ T_b \\ T_c \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \\ 1 \end{pmatrix} \quad (5)$$

By introducing variables T_i' for the PWM duty cycle, where T_i' varies from -1 to 1 while T_i varies from 0 to 1.

We finally get for sector I:

$$T_a' = u_{ac}$$

$$T_b' = -u_{ac} + 2u_{bc}$$

$$T_c' = -u_{ac}$$

PWM duty cycles in continuous mode for all sectors are given in Table 2

TABLE II. DUTY CYCLES IN CONTINUOUS MODE $T_i' \in [-1,1]$

	I	II	III	IV	V	VI
T_a'	u_{ac}		$-u_{ba}$	$-u_{ca}$		u_{ab}
T_b'		u_{bc}	u_{ba}		$-u_{cb}$	$-u_{ab}$
T_c'	$-u_{ac}$	$-u_{bc}$		u_{ca}	u_{cb}	

We could replace the first equation in (4) by different equations, we could also replace second or third equation. By replacing the first equation in (4) by the condition that all half-bridges are predominantly connected to the negative bus $T_c = 0$ in sector I. The phase C half-bridge will never be connected to positive bus in sector I. Timer settings for sector I are shown in Fig. 8.

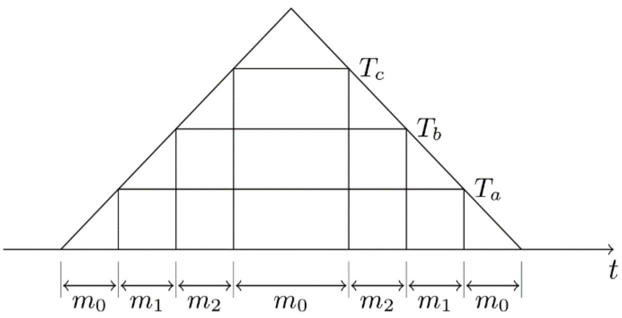

Fig.7 Timer settings in discontinuous mode, sector I, $1 - T_a = T_c$

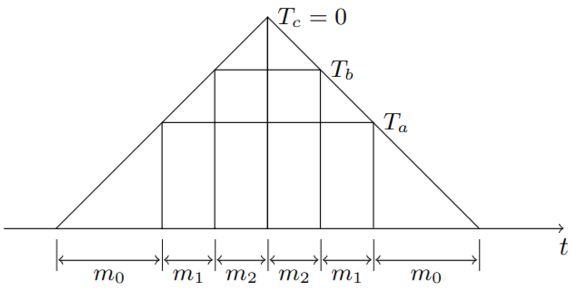

Fig.8 Timer settings in discontinuous mode, sector I, $T_c = 0$

Duty cycles $T_i \in [0,1]$ are presented in Table 3.

TABLE III. DUTY CYCLES IN DISCONTINUOUS MODE $T_i \in [-1,1]$

	I.II	III,IV	V,VI
T_a	u_{ac}	-1	u_{ab}
T_b	u_{bc}	u_{ba}	-1
T_c	-1	u_{ca}	u_{cb}

In discontinuous mode with amplitude modulation much less than one in the case when the keys half bridges predominantly on the negative bus we will apply sequentially relatively small voltage to phases U_A, U_B, U_C, to start the motor up to the nominal mode.

Graphs for discontinuous and continuous modes with a modulation amplitude equal to unit, are presented in Fig. 9-11.

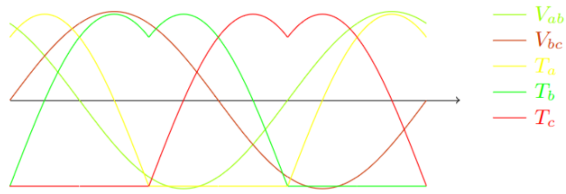

Fig.9 Graphs of duty cycles for continuous mode

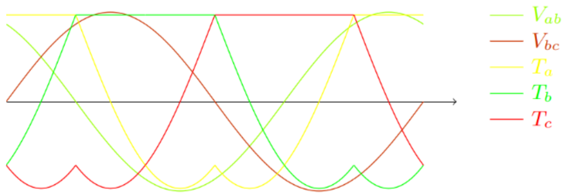

Fig.10 Graphs of duty cycles for discontinuous mode, switches are mainly at minus bus

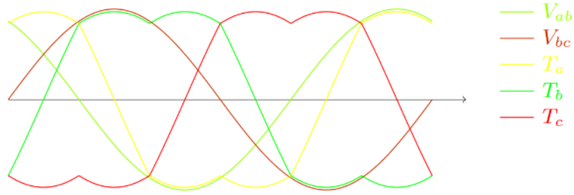

Fig.11 Graphs of duty cycles for discontinuous mode, switches are mainly at plus bus

III. CONCLUSION

Comparison of the proposed algorithm in continuous mode with algorithms offered by Texas Instruments, STMicroelectronics, NPF Mechatronika-Pro, The Russian Electrotechnical Society showed: originality of the algorithm, the algorithm is based on measured physical quantities, proposed the algorithm has half as many operations unless take into account the auxiliary algorithm sector definitions.

The proposed algorithm showed 20% decrease of computation time in comparison with algorithms of Texas Instruments and STMicroelectronics.

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