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On the Deformation of a Round Three-layer Plate under the Action of a Surface Charge

Alexey A. Tikhonov, Natalia V. Naumova, Andrey B. Yakovlev, Denis N. Ivanov, Nikita P. Dorofeev

Dedicated to the memory of Professor Ćemal Dolićanin (1945–2023)

Abstract: In this work there are the charging model and the results of calculations of electric charge density on the surface of the solar sail in the space. These results are used to estimate change of a sail form as a result of its electrifying in space plasma of solar system.

Keywords: three-layer plate, deformation, solar sail.

1 Introduction

The development of space research, especially for the implementation of long-range flights into deep space, requires the study of the possibility of using alternative propulsion systems. The scientific literature describes the use of solar sails proposed by F.A. Tsander for interplanetary space flights for the implementation of these tasks [7], [4]. Today, many problems of trajectory and control of spacecraft with a sail are solved taking into account various factors of outer space, in particular, the influence of the Earth's shadow. The implementation of this propulsion system requires the study of other space factors that may affect its performance. The effect of degradation and change in the reflectivity of the solar sail film on its dynamics was studied in [8].

It is known that all bodies in space plasma acquire an electric charge, which depends on the density and temperature of the plasma, the flux of sunlight, the electrophysical characteristics of the body, and hence on the position and orientation of the body during its motion [11], [9]. But the charging of thin films has a number of features that differ from the known

Manuscript received March 5, 2024; accepted March 26, 2024
Alexey A. Tikhonov (ORCID 0000-0003-0838-5876) is with Saint Petersburg State University, Russia;
Natalia V. Naumova (ORCID 0000-0003-1918-093X) is with Saint Petersburg State University, Russia;
Andrey B. Yakovlev (ORCID 0000-0002-8744-0783) is with Saint Petersburg State University, Russia;
Denis N. Ivanov is with Saint Petersburg State University, Russia;
Nikita P. Dorofeev (ORCID 0000-0002-9094-7956) is with Saint Petersburg State University, Russia
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processes of charging in space plasma [1], [13], [5]. The most important of them is the partial film permeability for charged particles of space plasma [3].

If the surface of the solar sail remains flat during interplanetary flights, then the obtained estimates of the influence of the electric charge induced on the surfaces of the sail on the motion of the spacecraft during flights from the Earth's orbit to the orbit of Mars and from the Earth's orbit to the orbit of Jupiter showed that this influence is very small [12]. The reasons for this are the very small value of the interplanetary magnetic field and the normal orientation of the sail relative to the direction to the Sun. However, an electrical charge induced on the surfaces of the sail can cause the surface to deform, which in turn changes the effective surface area of the sail. This work is an attempt to estimate the change in the effective area of the sail as a result of the sail electrification in the cosmic plasma of the solar system.

2 Mathematical model of the charging process

Actually for a solar sail, the kapton film with aluminum coating on both sides is usually used. Note that in this case, to describe the motion of a solar sail with accessible thickness and other characteristics necessary for interplanetary flights, it is possible to use a double-sided, infinite, aluminum plate that completely absorbs particles with a given sunlight reflection coefficient as a solar sail model. Aluminum is a material with a high photoemission yield, so near the surface illuminated by sunlight, there is a double layer with a nonmonotonic potential distribution. Since the sizes of the solar sail are larger than the values of the radius of gyration and the Debye length, the surface can be assumed to be infinitely large in modeling. The following assumptions are also used in the model:
1) spaceship body does not affect sail charging, 2) coronal gas expansion is adiabatic (the assumption used to calculate the solar wind speed), 3) the speed of the body is much less than the speed of the solar wind, so the plate can be considered motionless, 4) the influence of the film surface boundaries is not taken into account, 5) the velocity distribution functions of charged particles at infinity and photoelectrons on the surface of the sail are known, 6) on the upper (looking from the Sun) side of the plate, the distribution of the electrostatic potential is nonmonotonous, 7) on the reverse side of the plate, the distribution of the electrostatic potential is monotonous, 8) potentials on both surfaces of the plate are equal.

As shown in [2], the distribution of photoelectrons leaving an aluminum plate should be Maxwellian with concentration on a surface $N_{\nu 0} = 1.891 \cdot 10^9 m^{-3}$ and a temperature $T_\nu = 0.9eV$ for the case of complete absorption of sunlight and normal incidence of brightness on the surface of the plate. For other values of the polar angle and reflection coefficient, the value of $N_{\nu 0}$ is recalculated accordingly. For electrons and ions of the solar wind, the distributions are Maxwellian with $T_e = 10eV$ and $N_{e0} = 9 \cdot 10^6 m^{-3}$ (calm solar wind) [5]. Thus, for small angles of incidence θ of solar rays, the condition for high photoemission $N_{\nu 0} \gg N_{e0}$ is satisfied. Therefore, for particles above the plate, one can expect the existence of a nonmonotonic potential distribution.

The potential of the electric field and the distribution of particles in the double layer are found by solving the system of Vlasov–Poisson equations with the appropriate boundary conditions. Since the size of the sail is much greater than the Debye length of the solar wind plasma, the Poisson equation becomes one-dimensional. For a two-sided plate, the solution of the system of Vlasov–Poisson equations is considered separately in two areas — above the plate (region 1) and below the plate (region 2):

$$\frac{d^2\Phi}{d^2z} = -\frac{e}{\varepsilon_0}(N_i - N_{e1} - N_v) \tag{1}$$

for region 1 and

$$\frac{d^2\Phi}{d^2z} = \frac{e}{\varepsilon_0}N_{e2} \tag{2}$$

for region 2.

Here Φ is the electric field potential, z is the height above the sail surface, N_i, N_{e1}, N_{e2}, N_v are the concentrations of ions and electrons (in the regions 1 and 2) of the solar wind and photoelectrons, respectively, e is the electric charge of the proton, ε_0 is the dielectric constant.

As in the papers [1], [13], [5], we use four parameters to determine the nonmonotonic potential: Φ_0 is the potential on the plate surface, Φ_m is the minimum value of Φ at height z_m, Φ_1 is the value of the potential at the outer boundary of the double layer. The following conditions are used to find the parameters mentioned above:

– zero total current to the surface

$$j_i - j_{e1}(\psi_m) - j_{e2}(\psi_m) - j_v(\psi_0, \psi_m) = 0$$

– neutrality condition on external border of a double layer ($\zeta \ll 1$)

$$N_{e1}(\psi_1) + N_v(\psi_1) = N_i$$

– no electric field on the outer boundary of the double layer

$$\left.\frac{d\psi}{d\zeta}\right|_{\zeta\to\infty} = -\left|Z_2(\psi_1)\right|^{1/2} = 0$$

Here

$$\zeta = \frac{z}{D},\ D = \left[\frac{\varepsilon_0 kT_e}{N_{e0}e^2}\right]^{1/2},\ w = \frac{V_0}{V_e}\cos\theta,\ \zeta_m = \frac{z_m}{D},\ \tau = \frac{T_e}{T_v},\ V_e = \sqrt{\frac{2kT_e}{m_e}},\ \psi = -\frac{e\Phi}{kT_e},$$

$$\psi_0 = -\frac{e\Phi_0}{kT_e}, \quad \psi_m = -\frac{e\Phi_m}{kT_e}, \quad \psi_1 = -\frac{e\Phi_1}{kT_e}, \quad \Psi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt,$$

$$N_{e1} = \frac{N_{e0}}{2\sqrt{\pi}}e^{-w^2}\left[\int_\psi^\infty \frac{exp(-t+2w\sqrt{t})}{\sqrt{t-\psi}}\,dt \mp \int_\psi^{\psi_m} \frac{exp(-t+2w\sqrt{t})}{\sqrt{t-\psi}}\,dt\right],$$

$$N_{e2} = \frac{N_{e0}}{2\sqrt{\pi}} e^{-w^2} \int_{\psi}^{\infty} \frac{exp(-t - 2w\sqrt{t})}{\sqrt{t - \psi}} dt,$$

$$N_{\nu}(\psi) = \cos\theta \frac{N_{\nu 0}}{2} exp(\tau\psi_0 - \tau\psi) \left[1 \pm \Psi(\sqrt{\tau\psi_m - \tau\psi})\right],$$

$$j_{e1}(\psi_m) = N_{e0} \frac{V_e}{2\sqrt{\pi}} \left\{ exp\left[-(w - \sqrt{\psi_m})^2\right] + \sqrt{\pi} w \left[1 + \Psi(w - \sqrt{\psi_m})\right] \right\},$$

$$j_{e2}(\psi_m) = N_{e0} \frac{V_e}{2\sqrt{\pi}} \left\{1 - \sqrt{\pi} w\right\},$$

$$j_{\nu}(\psi_0, \psi_m) = -\cos\theta N_{\nu 0} \frac{V_e}{2\sqrt{\pi}\sqrt{\tau}} exp(\tau\psi_0 - \tau\psi_m), \; j_i = N_i V_0 \cos\theta,$$

$$Z_{1,2}(\psi) = -2(\psi_m - \psi) + e^{-\psi}\left[1 \mp \Psi(\sqrt{\psi_m - \psi})\right] - exp(-\psi_m)\left\{1 + \frac{A}{\tau} exp[(1-\tau)\psi_m]\right\} +$$

$$+ \frac{A}{\tau} exp[-\tau\psi] \times \left[1 \pm \Psi(\sqrt{\tau\psi_m - \tau\psi})\right] \pm \frac{2}{\sqrt{\pi}} \sqrt{\psi_m - \psi} exp(-\psi_m) \times$$

$$\times \left\{1 - \frac{A}{\sqrt{\tau}} exp[(1-\tau)\psi_m]\right\} + w \int_{\psi}^{\psi_m} F_{1,2}(\psi, \psi_m)\, d\psi,$$

$$A = \frac{N_{\nu 0}}{N_{e0}} \cos\theta exp(\tau\psi_0), \; F_{1,2}(\psi, \psi_m) =$$

$$= \frac{4}{\sqrt{\pi}} e^{-\psi} \left[\int_0^{\infty} e^{-x^2} \sqrt{x^2 + \psi}\, dx \mp \int_0^{\sqrt{\psi_m - \psi}} e^{-x^2} \sqrt{x^2 + \psi}\, dx \right]$$

Here, the upper sign refers to the region $0 \le \zeta \le \zeta_m$, the lower sign refers to the region $\zeta_m < \zeta < \infty$ of region 1, V_0 is the velocity of the solar wind, ion density $N_{i0} = N_{e0}$, m_e is the electron mass, k is Boltzmann constant. In the new variables, the equation (1) is solved by integrating

$$\zeta = \zeta_m \mp \int_{\psi}^{\psi_m} \frac{1}{\sqrt{Z_{1,2}(\psi)}} d\psi$$

where

$$\zeta_m = \int_{\psi_0}^{\psi_m} \frac{1}{\sqrt{Z_1(\psi)}} d\psi$$

The equation (2) is solved similarly. If we know the potential distribution near each of the plate surfaces, we can find the electric field strength near the surfaces. We then determine the electric charge density on both surfaces using the Gauss theorem.

3 Modeling the sail deformation process

In the ANSYS [6] software package, we created a finite element model of a solar sail. In the study, the solar sail was approximated by a round three-layer plate. The inner layer of the plate is a mylar film d thick. The enveloping layers are a very thin aluminum film with a thickness of one tenth of d. On the lower and upper sides of the surface, an electric charge of σ_1 and σ_2 densities is uniformly distributed, respectively. We believe that under the action of a given surface force, the plate under consideration is distorted as part of the surface of a sphere, which fairly corresponds to the expression for the surface force [10]:

$$T = 2\pi k_0 \sigma_1 \sigma_2 R, \tag{3}$$

where $k_0 = 9 \times 10^9 m^2/Q^2N$, R is the radius of curvature of the surface. The problem was solved, as already noted, by approximating the plate with a finite-element model in the ANSYS package. For this, the area occupied by the plate was divided into 10,000 elements. The program uses a three-layer eight-node elemental shell91 designed for designing thin multilayer shells.

The radius of curvature of the plate was determined, which is retained after the application of a surface force. To do this, first set the inner and outer radii, as well as the initial radius of curvature. To determine the radius of curvature of a curved plate after applying surface forces, there are coordinates of 4 points of the plate. Finally, a new radius is determined from these points, which is compared with the one that was originally adopted. An iterative process is obtained, which is interrupted when the required accuracy is reached.

4 Calculation results

The calculations were carried out for two panels of surface densities $\sigma_1 = 1.719 \cdot 10^{-10}$ and $\sigma_2 = 3.41278 \cdot 10^{-13}$ (for the Earth's vicinity) and $\sigma_1 = 1.1507 \cdot 10^{-9}$ and $\sigma_2 = 2.7941 \cdot 10^{-12}$ (for the vicinity of the Sun).

The results obtained for sails with a radius of up to 50 meters, with $d = 2$ microns, are presented in the table. An analysis of the calculation results presented in Tables 1 and 2 shows that when considering the stress state of a space sail, its deflection due to the distributed surface charge cannot be neglected. With an increase in the radius of a round plate, such surface deformation only increases.

Table 1. Deformation of the plate depending on the radius in the vicinity of the Sun.

Plate's radius, m	Initial radius of curvature, m	Final radius of curvature, m	Maximum value of deflection , m
10	120.9	120.99	0.42
20	121.1	121.14	1.65
30	121.1	121.15	3.72
40	121.2	121.19	6.62
50	121.3	121.27	10.34

Table 2. Deformation of the plate depending on the radius in the vicinity of the Earth.

Plate's radius, m	Initial radius of curvature, m	Final radius of curvature, m	Maximum value of deflection , m
10	809.6	809.63	0.06
20	810.8	810.71	0.25
30	810.7	810.79	0.56
40	811.2	811.22	0.99
50	811.7	811.80	1.55

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