Nonlinear Analysis of SRF-PLL: Hold-In and Pull-In Ranges

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Abstract A Synchronous Reference Frame Phase-Locked Loop (SRF-PLL) is a nonlinear control circuit widely used in power engineering for the synchronization and control of three-phase grid-connected converters. New applications of power converters in sustainable energy generators have led to the problem of nonlinear analysis of SRF-PLL stability. In this paper, a continuous nonlinear model of SRF-PLL with a first-order loop filter is studied and the hold-in range of the model is analysed. Using the qualitative theory of dynamical systems and classical methods of control theory, we conduct a nonlinear analysis of the SRF-PLL model and suggest an analytical estimate for the pull-in range of the considered model. The obtained estimate is compared with known engineering estimates of the pull-in range. MATLAB Simulink is used to perform computer simulation which confirms theoretical results.

1 Introduction

Control problems related to synchronization in electrical networks are important for developing green energy and sustainable future. One of these problems is connection of a solar or wind generator, which produces direct current (DC) to a grid [1, 2]. The grid is usually an alternating current (AC) network, therefore it is necessary to use DC-AC power converter as part of the generator. It is important to measure grid AC voltage and synchronize the output voltage of the power converter to the main grid before connection to avoid power surges and related damage to the equipment. After

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connection, monitoring of the frequency and phase of the grid allows controlling the balance of active and reactive power supplied to the network by the generator [3]. The most frequently used solution is Synchronous Reference Frame Phase-Locked Loop (SRF-PLL). This nonlinear control device monitors the grid's phase and frequency to control the inverter.

In the present paper, we study SRF-PLL model with a first-order loop filter. There are also several modifications of the SRF-PLL, which are based on some pre-filtering (adding filter before the loop) and in-loop filtering (adding filter inside the loop), such as the DDSRF-PLL [4], moving average filter (MAF) PLLs [5, 6], delayed signal cancellation [7], and numerous other methods (see, e.g., [8–10]). Adding pre-loop filter does not affect the performance (speed, noise, and stability) of the SRF-PLL itself, therefore it is omitted for simplicity. However, additional in-loop filtering requires to redo the whole study for every modification. Nonlinear stability analysis is especially difficult to carry over to SRF-PLLs with additional in-loop filtering and is beyond the scope of this article.

The paper is organized in the following way. In Sect. 2, a continuous time mathematical model of SRF-PLL is derived. Synchronization properties and stability of the corresponding system is studied in Sect. 3. An analytical estimate of global stability is obtained and plotted on a two-dimensional diagram for all SRF-PLL parameters. In Sect. 4, the analytical formula is compared with engineering estimates.

2 Mathematical Model of the SRF-PLL

Consider a simplified model of an inverter (Fig. 1) connected to a three-phase electrical grid [2, 3, 11] and corresponding phase (line-to-neutral) voltages

$$\begin{aligned} u_{\mathrm{a}}(t) &= u\sin(\omega_{\mathrm{ref}}t), \\ u_{\mathrm{b}}(t) &= u\sin(\omega_{\mathrm{ref}}t - \frac{2}{3}\pi), \\ u_{\mathrm{c}}(t) &= u\sin(\omega_{\mathrm{ref}}t + \frac{2}{3}\pi). \end{aligned} \tag{1}$$

Here ω_{ref} is the grid (reference) frequency and u is the Root-Mean-Squared voltage.[1] Denote the reference phase $\omega_{\mathrm{ref}}t$ by $\theta_{\mathrm{ref}}(t) = \omega_{\mathrm{ref}}t$. The measured line voltages $u_{\mathrm{abc}} = \left(u_{\mathrm{a}}\ u_{\mathrm{b}}\ u_{\mathrm{c}}\right)^T$ are converted into dq0 reference frame $u_{\mathrm{dq0}} = \left(u_{\mathrm{d}}\ u_{\mathrm{q}}\ u_0\right)^T$ by the Park transformation $u_{\mathrm{dq0}} = P(\theta_{\mathrm{PLL}})u_{\mathrm{abc}}$ where

[1] In the present work, we consider a SRF-PLL model, which assumes that the reference signal has constant frequency and amplitude. When the reference signal is unbalanced or has DC component or is distorted, some modifications of SRF-PLL can be considered, e.g., a three-phase EPLL (3EPLL) [2].

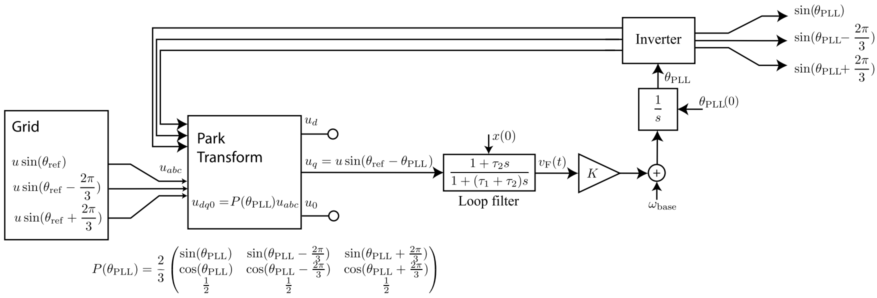


Fig. 1 Nonlinear model of SRF-PLL [2]

$$
P(\theta_{\rm PLL}) = \frac{2}{3}\begin{pmatrix} \sin(\theta_{\rm PLL}) & \sin(\theta_{\rm PLL}-\frac{2\pi}{3}) & \sin(\theta_{\rm PLL}+\frac{2\pi}{3}) \\ \cos(\theta_{\rm PLL}) & \cos(\theta_{\rm PLL}-\frac{2\pi}{3}) & \cos(\theta_{\rm PLL}+\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{pmatrix}
$$

and $\theta_{\rm PLL} = \theta_{\rm PLL}(t)$ is the phase of the inverter output voltage.

The q component of the transformed set of signals is the error signal

$$
\begin{aligned}
u_{\rm q}(t) = &\frac{2u}{3}\Big(\sin(\theta_{\rm ref})\cos(\theta_{\rm PLL}) + \sin\left(\theta_{\rm ref}-\frac{2}{3}\pi\right)\cos\left(\theta_{\rm PLL}-\frac{2}{3}\pi\right) \\
&+ \sin\left(\theta_{\rm ref}+\frac{2}{3}\pi\right)\cos\left(\theta_{\rm PLL}+\frac{2}{3}\pi\right)\Big) \\
= &\; u\sin(\theta_{\rm ref}-\theta_{\rm PLL}).
\end{aligned} \tag{2}
$$

Denote by $\theta_e(t)$ the phase error:

$$
\theta_e(t) = \theta_{\rm ref}(t) - \theta_{\rm PLL}(t). \tag{3}
$$

The error signal $u_{\rm q}(t)$ is an input of the first-order loop filter which transfer function has the form [12–14]

$$
F(s) = \frac{1+\tau_2 s}{1+(\tau_1+\tau_2)s}, \quad \tau_1 > 0,\ \tau_2 > 0. \tag{4}
$$

Denote by $x(t) \in \mathbb{R}$ the state of the loop filter. The output of the loop filter $v_{\rm F}(t) = \frac{1}{\tau_1+\tau_2}x + \frac{\tau_2}{\tau_1+\tau_2}u\sin\theta_e$ is used to control the frequency $\omega_{\rm PLL}(t)$ of the inverter output voltage, which is proportional to the control voltage:

$$
\omega_{\rm PLL}(t) = \dot{\theta}_{\rm PLL}(t) = \omega_{\rm base} + K v_{\rm F}(t), \tag{5}
$$

where $K > 0$ is a gain and $\omega_{\rm base}$ is a base frequency.

Combining (2)–(5) we get equations of the SRF-PLL:

$$\begin{aligned}\dot{x} &= -\frac{1}{\tau_1+\tau_2}x + \frac{\tau_1}{\tau_1+\tau_2}u\sin\theta_e,\\ \dot{\theta}_e &= \omega_e - K\Big(\frac{1}{\tau_1+\tau_2}x + \frac{\tau_2}{\tau_1+\tau_2}u\sin\theta_e\Big),\end{aligned} \tag{6}$$

where $\omega_e = \omega_{\text{ref}} - \omega_{\text{base}}$ is a frequency error.

3 SRF-PLL Stability Analysis

3.1 Local Stability (Small-Signal Analysis)

Observe that system (6) is 2π-periodic in θ_e. If $\omega_e < uK$, then it has an infinite number of equilibria $(x^{\text{eq}},\ \theta_e^{\text{eq}})$ which satisfy

$$x^{\text{eq}} = \frac{\tau_1\omega_e}{K}, \quad \sin\theta_e^{\text{eq}} = \frac{\omega_e}{uK}.$$

Equilibria of system (6) correspond to locked states of SRF-PLL (the phase error $\theta_e(t)$ is constant when the PLL is locked). In engineering literature, a *hold-in range* concept is widely used in order to characterize the ability of the loop to maintain phase-locked conditions when the frequency error ω_e varies slowly. Strict mathematical definition of the hold-in range can be found in [15–17].

System (6) has asymptotically stable equilibria $\big(\frac{\tau_1\omega_e}{K},\ \arcsin\frac{\omega_e}{uK} + 2\pi m\big)$ and unstable equilibria $\big(\frac{\tau_1\omega_e}{K},\ \pi - \arcsin\frac{\omega_e}{uK} + 2\pi m\big)$, and the hold-in range is $[0,\ \omega_h) = [0,\ uK)$.

3.2 Global Stability (Large-Signal Analysis)

To study the hold-in range it is sufficient to analyse the PLL system in vicinities of equilibria (local analysis). In PLL literature, concept of pull-in range is widely used in order to characterize global stability properties of the corresponding system. A *pull-in range* is the largest interval of frequency errors $|\omega_e| \in [0, \omega_p)$ from the hold-in range for which any trajectory of system (6) tends to an equilibrium (for brevity, we shall call such systems *globally stable*).

Notice that analysis of global stability requires consideration of the whole phase space and nonlinearity cannot be neglected. For global analysis of PLL models and estimation of the pull-in range various nonlinear methods can be used such as the direct Lyapunov method (see, e.g., [18, 19]), the method of two-dimensional comparison systems (see, e.g., [20]), phase portrait analysis (see, e.g., [21]), the harmonic balance method (see, e.g., [22, 23]), and others [24–26].

Theorem 1 *Pull-in range of model (6) satisfies the inequality $\omega_p \geq \omega_p^{\text{est}}$ where $\omega_p^{\text{est}} > 0$ is the unique solution of equation*

$$\arcsin \frac{\omega_p^{\text{est}}}{uK} + \sqrt{\left(\frac{uK}{\omega_p^{\text{est}}}\right)^2 - 1} = \frac{\pi \tau_1}{4(\sqrt{\tau_2(\tau_1 + \tau_2)} - \tau_2)}. \tag{7}$$

Proof To analyse the pull-in range of system (6), we apply the direct Lyapunov method and the corresponding theorem on global stability for the cylindrical phase space (see, e.g., [27, 28]). If there is a continuous function $V(x, \theta_e) : \mathbb{R}^2 \to \mathbb{R}$ such that

(i) $V(x, \theta_e + 2\pi) = V(x, \theta_e) \quad \forall x \in \mathbb{R}, \ \forall \theta_e \in \mathbb{R}$,
(ii) for any solution $(x(t), \theta_e(t))$ of system (6) the function $V(x(t), \theta_e(t))$ is nonincreasing,
(iii) if $V(x(t),\ \theta_e(t)) \equiv V(x(0),\ \theta_e(0))$, then $(x(t), \theta_e(t)) \equiv \ (x(0), \theta_e(0))$,
(iv) $V(x,\ \theta_e) + \theta_e^2 \to +\infty$ as $|x| + |\theta_e| \to +\infty$,
then any trajectory of system (6) tends to an equilibrium.

For system (6) we consider the following Lyapunov function satisfying condition (iv):

$$V(x,\ \theta_e) = \frac{1}{2}(x - \frac{\tau_1 \omega_e}{K})^2 + \frac{\tau_1}{K} \int\limits_0^{\theta_e} \Big(\sin\sigma - \frac{\omega_e}{uK} + \beta_0 |\sin\sigma - \frac{\omega_e}{uK}| \Big) d\sigma$$

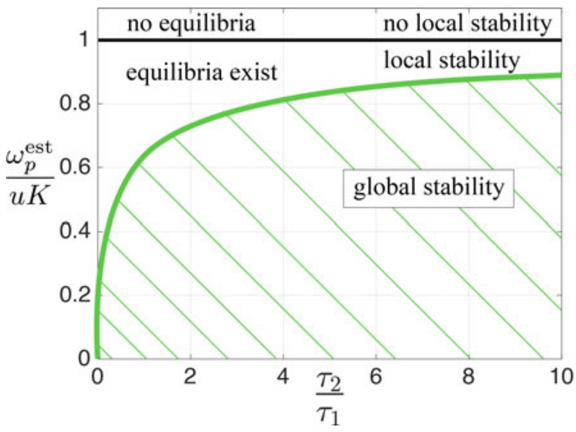
where

$$\beta_0 = - \frac{\int_0^{2\pi} (\sin\sigma - \frac{\omega_e}{uK})\, d\sigma}{\int_0^{2\pi} |\sin\sigma - \frac{\omega_e}{uK}|\, d\sigma} > 0.$$

The special form of coefficient β_0 allows the Lyapunov function to be 2π-periodic and, hence, to satisfy the first condition of the theorem. Computation of the Lyapunov function derivative along the trajectories of system (6), considering it as a quadratic form, and providing its negative-definiteness lead to fulfilling conditions (ii), (iii) and, hence, to global stability of the system. □

Notice that the left-hand side of (7) is a monotonous function and ω_p^{est} can be evaluated numerically. Moreover, Eq. (7) can be considered as an equation in two variables: $\frac{\tau_2}{\tau_1}$ and $\frac{\omega_p^{\text{est}}}{uK}$. Figure 2 shows stability regions observed in the second-order SRF-PLL. The shaded area bounded by curve (7) corresponds to global stability estimate by the Lyapunov function approach. Above the line $\frac{\omega_e}{uK} = 1$ no equilibria exist in system (6). For SRF-PLL parameters from the gap between the line $\frac{\omega_e}{uK} = 1$ and curve (7), system (6) has asymptotically stable equilibria, but the global stability is not guaranteed.

Fig. 2 Stability regions observed in the second-order SRF-PLL. The solid line $\frac{\omega_e}{uK} = 1$ corresponds to equilibria existence, the curve which bounds the global stability domain is Eq. (7) from Theorem 1



Remark 1 Shaded area in Fig. 2 is a domain in the parameters space such that system (6) with parameters from this domain is globally stable, i.e., a trajectory with initial data from any point of the phase space tends to an equilibrium. Notice that many works studying PLL-based systems conduct local analysis only and estimate equilibria's domains of attraction, not analyzing the whole phase space (see, e.g., recent papers [29–33]). For global analysis of systems in the cylindrical phase space the corresponding theorem from [27, 28] should be used.

4 Comparison with Known Results

In this section, we compare estimate (7) for the pull-in range of SRF-PLL with engineering results from [34–38]. Whereas conservative estimate (7) from Theorem 1 is strictly mathematically justified, the rest rely on approximations.

One of the first equations used in engineering design was derived by D.Richman [34]:

$$\omega_p^{\text{Richman}} \approx uK\sqrt{\frac{2\tau_2}{\tau_1+\tau_2} - \left(\frac{\tau_2}{\tau_1+\tau_2}\right)^2}. \tag{8}$$

In [34], Richman uses phase plane descriptions and derives an approximate formula for the pull-in time T_F assuming $uK\tau_2 \gg 1$. Equation (8) for the pull-in frequency was introduced in [38], [13, p. 168] by setting $T_F \to +\infty$.

Another formula based on estimations was derived by Viterbi [35, 38, 39] for $uK\tau_2 \gg 1$:

$$\omega_p^{\text{Viterbi}} \approx uK\sqrt{\frac{2\tau_2}{\tau_1+\tau_2}} > \omega_p^{\text{Richman}}. \tag{9}$$

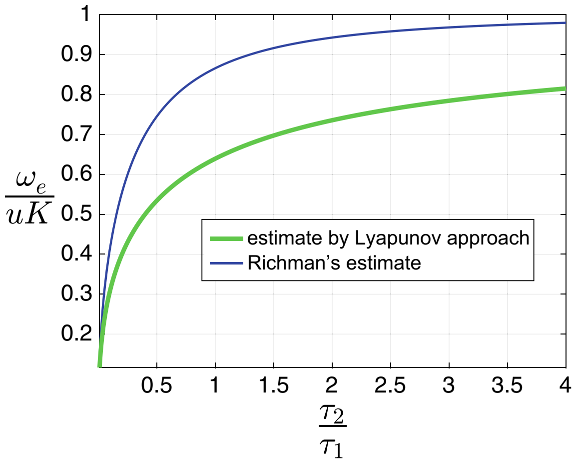


Fig. 3 Comparison of the pull-in range conservative estimate ω_p^{est} from Theorem 1 with the Richman's estimate (8) $\omega_p^{\text{Richman}} \approx uK\sqrt{\frac{2\tau_2}{\tau_1+\tau_2} - \left(\frac{\tau_2}{\tau_1+\tau_2}\right)^2}$

Later Lindsey and Mengali derived approximate formulae for the case of arbitrary periodic nonlinearity in equation (6) [36, 37]. Their formulae for sinusoidal nonlinearity are identical to each other and to the Viterbi's formula (9).

Notice that formula (9) is not valid for $\tau_2 > \tau_1$: this case leads to $\frac{\omega_e}{uK} > 1$, whereas equilibria do not exist in this case and the system is not globally stable. In contrast, $\sqrt{\frac{2\tau_2}{\tau_1+\tau_2} - \left(\frac{\tau_2}{\tau_1+\tau_2}\right)^2} \in (0,\ 1)$ and the pull-in range estimate (8) is less than the hold-in range.

The global stability estimate ω_p^{est} provided by Theorem 1 is conservative, i.e., $\omega_p^{\text{est}} \leq \omega_p$. Although the global stability of SRF-PLL model (6) is guaranteed for any $\omega_e < \omega_p^{\text{est}}$, the exact global stability domain can be wider. Figure 3 shows that the approximate formula (8) provides the wider global stability domain for system (6) than estimate (7). Consequently, the following inequality is valid for the considered estimates:

$$\omega_p^{\text{est}} < \omega_p^{\text{Richman}} < \omega_p^{\text{Viterbi}}.$$

To analyse estimates (8) and (9), we simulate SRF-PLL model (6) in MATLAB Simulink, which is widely used for the study of PLL-based circuits [14, 40]. Simulation of PLL models in SPICE can be found in [41] and [42].

Let us consider SRF-PLL model (6) in MATLAB Simulink with the following parameters:

$$\tau_1 = 0.0448,\ \tau_2 = 0.4,\ K = 2500,\ u = 1.$$

Theorem 1 guarantees that the pull-in frequency $\omega_p \geq \omega_p^{\text{est}} \approx 2208$, whereas equation (8) provides the following pull-in range estimate $\omega_p^{\text{Richman}} \approx 2487.3$.

In Fig. 4, simulation of trajectory of system (6) with initial data $x(0) = -\tau_1,\ \theta_e(0) = 0$ is shown. In the left subfigure, the trajectory tends to an equilibrium point, what

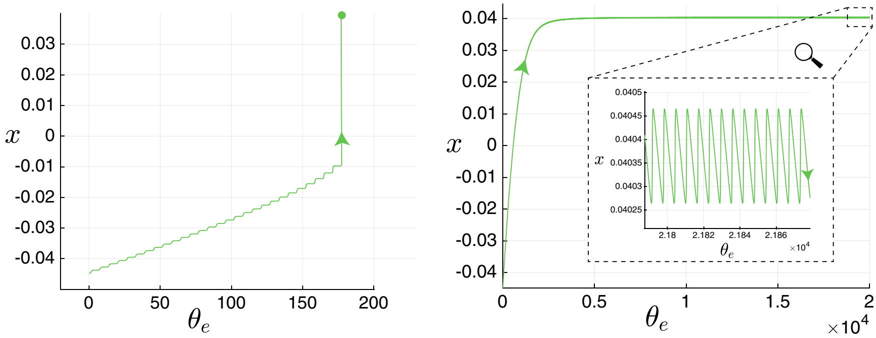


Fig. 4 Simulation of system (6). Initial data: $x(0) = -\tau_1$, $\theta_e(0) = 0$. Parameters: $\tau_1 = 0.0448$, $\tau_2 = 0.4$, $K = 2500$, $u = 1$. In the left subfigure, $\omega_e = 2208 \approx \omega_p^{\text{est}} \leq \omega_p$ and the trajectory tends to an equilibria, what corresponds to theoretical results (the system is globally stable). In the right subfigure, $\omega_e = 2487.3 \approx \omega_p^{\text{Richman}}$ and the system experiences a persistent oscillation (see the magnified domain), which indicate that the Richman's estimate may yield an unstable behavior

fits the theoretical results. However, oscillations appear in the right subfigure (see the magnified domain), hence, there is no global stability. The simulation shows that estimate (8) should be used carefully.

5 Conclusion

In this work, global analysis of SRF-PLL stability was conducted. As a result, the analytical formula for the pull-in range for SRF-PLL was derived and compared with estimates of Viterbi, Richman, Lindsey, and Mengali, which rely on approximations. Simulation results show that those estimates known from the literature may lead to an unstable behaviour and should be used carefully.

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