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GLOBAL ANALYSIS OF THIRD-ORDER COSTAS LOOP: PULL-IN RANGE AND LOCK-IN RANGE

This work is devoted to problems of BPSK Costas loop's key characteristics estimations. The main approaches are based on the global stability theory for cylindrical phase space and similar estimates for phase-locked loops models. The global stability estimates of the third-order BPSK Costas loop model and the lock-in range estimate are obtained via Lyapunov functions.

Introduction. Costas loops are phase-locked loops based on circuits for carrier recovery and signal demodulation [1-3]. Costas loops are widely used in global positioning systems [4], wireless communication [5], and other applications [6, 7]. There are a lot of types of Costas loops which are used in binary phase-shift keying, quadrature phase-shift keying and also m-ary phase-shift keying [8]. In this paper we consider modified binary phase shift keying (BPSK) Costas loop having the second order proportionally integrating loop filter and discuss its key characteristics: the lock-in range defining fast-locking conditions and the pull-in range ensuring acquisition process.

Such concepts as the hold-in range, the pull-in range, and the lock-in range were originally introduced in classical monographs in 1966 [9-11]. The following problem was stated by American engineer, IEEE Fellow, Floyd M. Gardner in the second edition of his monograph [12]: to determine the lock-in range corresponding to system acquiring lock within at most one beat between carrier frequency and initial voltage-controlled oscillator (VCO) frequency. Strict mathematical definitions of these concepts which were introduced in [13] and extension of Lyapunov theory developed for cylindrical phase space [14, 15] allow one to obtain analytical estimates of key characteristics of Costas loops.

Mathematical model of BPSK Costas loop. Consider modified BPSK Costas loop (see Fig.1).

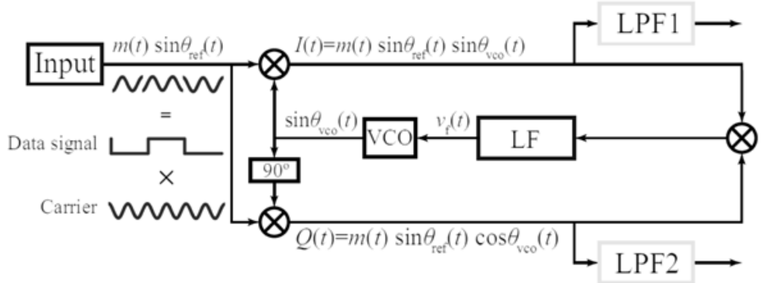

Fig. 1 Modified BPSK Costas loop.

The input signal is

$$m(t) \sin \theta_{ref}(t), \tag{1}$$

where $m(t) \in \{+1, -1\}$ is data signal, $\sin \theta_{ref}(t)$ is carrier, and $\theta_{ref}(t)$ is its phase. The output of VCO block is

$$\sin \theta_{vco}(t), \tag{2}$$

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where $\theta_{vco}(t)$ is corresponding phase. Block *90°* shifts the phase of VCO by *90°*. Two multipliers produce $I(t)$ and $Q(t)$ signals (in phase and quadrature components), which are inputs of the third multiplier. Its output is the input of loop filter (LF) with transfer function

$$F(s)=\frac{(1+s\tau_{z_1})(1+s\tau_{z_2})}{s(1+s\tau_{p_1})}, \tag{3}$$

where $\tau_{z_1} > 0, \tau_{z_2} > 0, \tau_{p_1} > 0$, and initial state $x(0)$. The output of LF block $v_f(t)$ is connected to the control input of the VCO.

Define a phase error $\theta_e(t)$ as

$$\theta_e(t) = \theta_{ref}(t) - \theta_{vco}(t), \tag{4}$$

and frequency detuning ω_e^{free} as

$$\omega_e^{\text{free}} = \omega_{\text{ref}} - \omega_{\text{vco}}^{\text{free}}, \tag{5}$$

where ω_{ref} is the carrier frequency which supposed to be constant, and $\omega_{\text{vco}}^{\text{free}}$ is initial VCO frequency.

The so-called baseband model is widely used for the study of acquisition processes of circuits based on phase locked-loops [3, 9, 10]. The baseband model of modified BPSK Costas loop having the second-order loop filter with transfer function (3) is given by [16]:

$$\begin{cases} \dot{x}_1 = \frac{1}{8}\sin(2\theta_e), \\ \dot{x}_2 = -\frac{1}{\tau_{p_1}}x_2 + \frac{1}{8}\frac{(\tau_{z_1}-\tau_{p_1})(\tau_{p_1}-\tau_{z_2})}{\tau_{p_1}^2}\sin(2\theta_e), \\ \dot{\theta}_e = \omega_e^{\text{free}} - K_{\text{vco}}\left(x_1 + x_2 + \frac{1}{8}\frac{\tau_{z_1}\tau_{z_2}}{\tau_{p_1}}\sin(2\theta_e)\right), \end{cases} \tag{6}$$

where $K_{\text{vco}} > 0$ is the VCO gain, $x(t) = \begin{pmatrix} x_1(t), \\ x_2(t) \end{pmatrix}$.

Application of Lyapunov function of the Lurie-Postnikov form for model (6) provides the global stability domain in the parameters' space. Using the Sylvester's criterion for Lyapunov function's derivative along the trajectories of system (6) allows to extend the global stability domain obtained by D. Abramovitch for the equivalent model [17]. Strict mathematical definition of the lock-in range [13] and geometric interpretation of level sets of the Lyapunov function provide the lock-in range estimate [18].

Conclusion In this work the problems of finding the pull-in range and the lock-in range estimates for modified BPSK Costas loop are considered. The study of the pull-in range for model (6) was held by searching for the Lyapunov function of the Lurie-Postnikov form. The lock-in range of third-order model (6) was estimated by the geometric interpretation of level sets of the Lyapunov function.

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