Schwinger mechanism of pair production in rapidly-varying electric fields

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Plan of the talk

- Introduction. Schwinger effect
- Theoretical techniques
- Enhancement mechanisms

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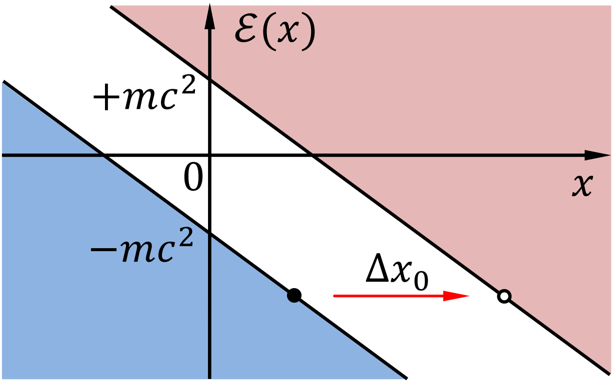
  Dynamically assisted Schwinger effect and fast-switching effects

Schwinger mechanism

Tunneling process from the Dirac sea (vacuum + electric field E_0):



Probability: $T \sim \exp(-\text{const}/E_0)$ $\qquad$ $(\Delta x_0 \sim 1/E_0)$

Schwinger mechanism

Pair-production probability:

$$P_{e^+e^-} \sim \exp\left(-\frac{\pi E_\mathrm{c}}{E_0}\right).$$

Schwinger critical field strength:

$$E_\mathrm{c} = \frac{m^2c^3}{|e|\hbar} \approx 1.3 \times 10^{16}\ \mathrm{V/cm}.$$

Schwinger mechanism

Scales in QED (QM):
e, $\hbar$, mc^2,
Compton wavelength of the electron $\lambda_{\mathrm{C}} = \frac{\hbar}{mc} \approx 3.86 \times 10^{-13}$ m,
Compton time $\tau_{\mathrm{C}} = \frac{\hbar}{mc^2} \approx 1.29 \times 10^{-21}$ s.

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$$\Delta E \Delta t \geqslant \frac{\hbar}{2}.$$

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Heisenberg's uncertainty principle "energy–time":

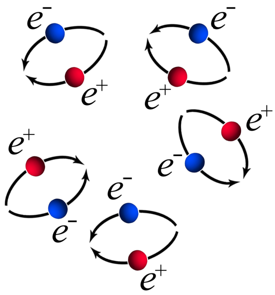
$$\Delta E \Delta t \geqslant \frac{\hbar}{2}.$$

For $\Delta E = mc^2$, one obtains $\Delta t \sim \frac{\hbar}{mc^2} = \tau_{\mathrm{C}}$.

$\Longrightarrow$ virtual particles with lifetime τ_{C}

Schwinger mechanism

QED vacuum $\neq$ "empty space"



e^+e^- pair can be separated by the external force:

$$|e|E_0\lambda_{\mathrm{C}} \sim mc^2 \quad \Longrightarrow \quad E_0 \sim E_{\mathrm{c}} \equiv \frac{m^2c^3}{|e|\hbar} \sim 10^{16} \ \mathrm{V/cm}.$$

Schwinger mechanism in time-dependent fields

What should be taken into account first:

- preexponential (volume) factor

Characteristic value of the laser wavelength is $\lambda \sim 1\ \mu\text{m}$

$\Longrightarrow$ huge volume factor $(\lambda/\lambda_{\text{C}})^4 \ \sim\ 10^{25}$

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- temporal dependence of the external (laser) field

Keldysh parameter:

$$\gamma = \frac{mc\omega}{|eE_0|}.$$

L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1945 (1964)

Schwinger mechanism in time-dependent fields

Pair-production probability:

$$P_{e^+e^-} \sim \begin{cases} \exp\left(-\frac{\pi E_c}{E_0}\right) & (\gamma \ll 1), \\ \left(\frac{1}{\gamma^2}\right)^{2mc^2/\hbar\omega} & (\gamma \gg 1). \end{cases}$$

$\gamma \gg 1 \iff$ multiphoton (perturbative) regime

$\gamma \ll 1 \iff$ tunneling regime (Schwinger effect)

Schwinger mechanism in time-dependent fields

Once we incorporate

- preexponential (volume) factor,
- temporal dependence in the regime $\gamma \ll 1$,

the pair-production threshold amounts to $E_0 \sim 0.1 E_{\mathrm{c}}$.

It corresponds to $I_0 \sim 10^{27}~\mathrm{W/cm^2}$.

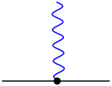
Modern intensity record: $I_0 = 1.1 \times 10^{23}~\mathrm{W/cm^2}$.

J. W. Yoon et al., Optica 8, 630 (2021)

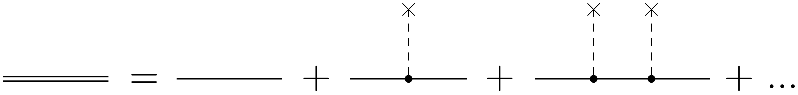
Nonperturbative treatment of the external field

QED Lagrangian ($\hbar = c = 1$):

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\hat{A}^\mu\bar{\psi}\gamma_\mu\psi.$$

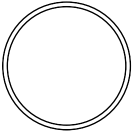
Interaction vertex: 

Consider only a classical background, $\hat{A}^\mu \to \mathcal{A}^\mu$:



Nonperturbative treatment of the external field

All of the connected vacuum diagrams are given by



$$Z\left[\mathcal{A}^{\mu}\right] = \mathcal{N}\int\mathcal{D}\bar{\psi}\int\mathcal{D}\psi\,\mathrm{e}^{iS} \equiv \mathrm{e}^{iS_{\mathrm{eff}}[\mathcal{A}^{\mu}]},$$

$$S_{\mathrm{eff}}[\mathcal{A}^{\mu}] = S_{\mathrm{Max}}[\mathcal{A}^{\mu}] + S^{(1)}[\mathcal{A}^{\mu}],$$

where $S^{(1)}[\mathcal{A}^{\mu}]$ is the one-loop effective action.

Nonperturbative treatment of the external field

For a purely electric field E_0,

$$\mathrm{Im}\ \mathcal{L}_{\mathrm{eff}} = \frac{(eE_0)^2}{8\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2}\, \mathrm{e}^{-n\pi E_{\mathrm{c}}/E_0}.$$

Vacuum persistence amplitude $= \exp(iS_{\mathrm{eff}})$.

Probability of vacuum decay:

$$P_{\mathrm{decay}} = 1 - |\mathrm{e}^{iS_{\mathrm{eff}}}|^2 = 1 - \mathrm{e}^{-2\,\mathrm{Im}\,S_{\mathrm{eff}}} \approx 2\,\mathrm{Im}\,S_{\mathrm{eff}}.$$

Modern theoretical techniques

- Dirac equation + in-out formalism,
- Dirac-Heisenberg-Wigner approach (kinetic theory),
- Quantum kinetic equations (QKE),
- Locally-constant field approximation (LCFA),
- WKB approximation,
- Worldline instanton formalism.

Enhancement mechanisms in time-dependent fields

The particle yield may be substantially enhanced by the following mechanisms:

- Dynamically assisted Schwinger effect,
- Fast-switching effects.

Dynamically assisted Schwinger effect (DASE)

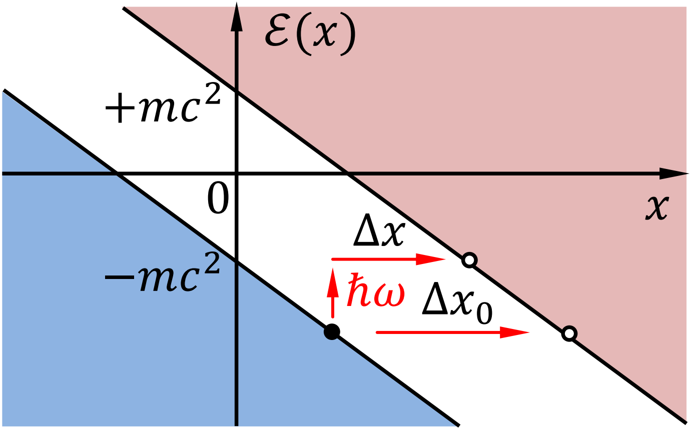
The idea is to superimpose a strong (but slow) background by a rapid (but weak) pulse:

$$\{E, \Omega\} \quad + \quad \{\varepsilon, \omega\}$$

$\gamma_E = m\Omega/|eE| \ll 1$, $\gamma_\varepsilon = m\omega/|e\varepsilon| \gg 1$.

R. Schützhold, H. Gies, and G. Dunne, PRL 101, 130404 (2008)

Dynamically assisted Schwinger effect



$$T \sim \exp(-\mathrm{const}/E_0)\, \exp[(\omega/2m)(\mathrm{const}/E_0)]$$

Dynamically assisted Schwinger effect

$$E(t) = \frac{E}{\cosh^2 \Omega t} + \frac{\varepsilon}{\cosh^2 \omega t},$$

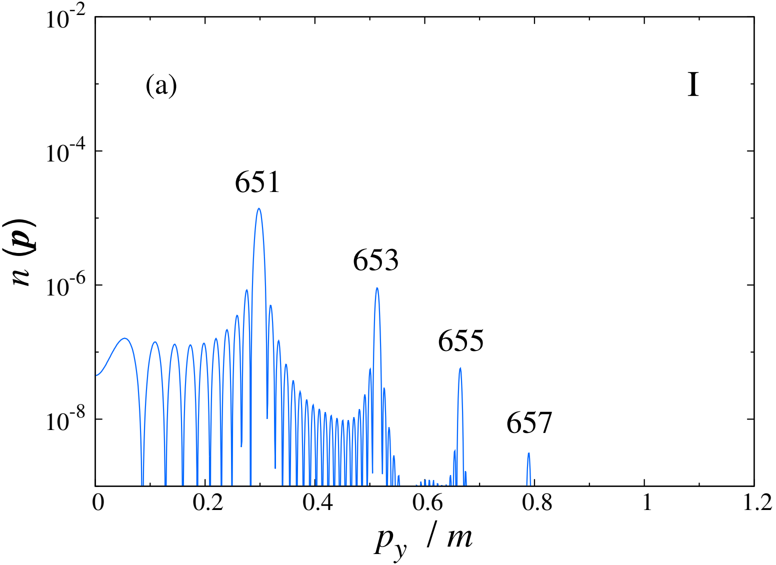
where $\varepsilon \ll E$, $\omega \gg \Omega$, $\gamma_E \ll 1$, and $\gamma_\varepsilon \gg 1$.
The "combined" Keldysh parameter:

$$\gamma_{\mathrm{c}} = \frac{m\omega}{|eE|}.$$

It turns out that for $\gamma_{\mathrm{c}} > \pi/2$ the particle yield drastically increases.

R. Schützhold, H. Gies, and G. Dunne, PRL 101, 130404 (2008)

Typical momentum distribution for $\gamma_c = 2.5$. Strong pulse



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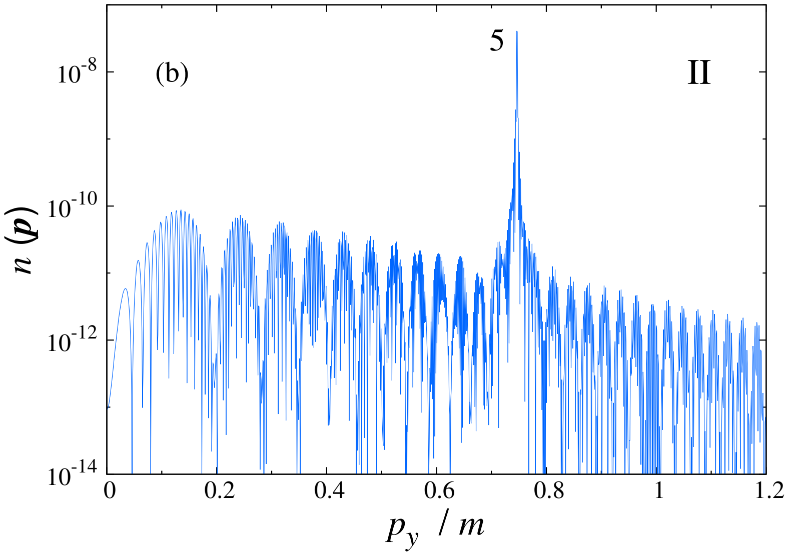
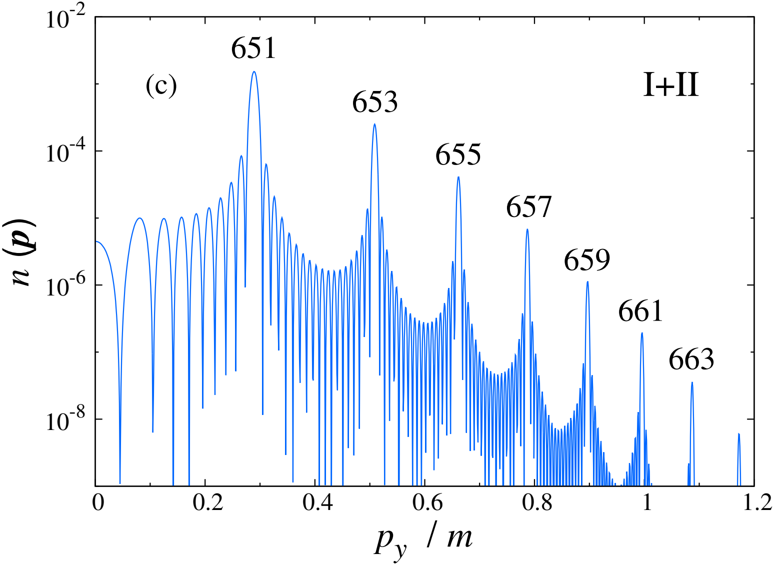


Typical momentum distribution for $\gamma_c = 2.5$. Two pulses



Fast-switching effects

Adiabatic number of particles

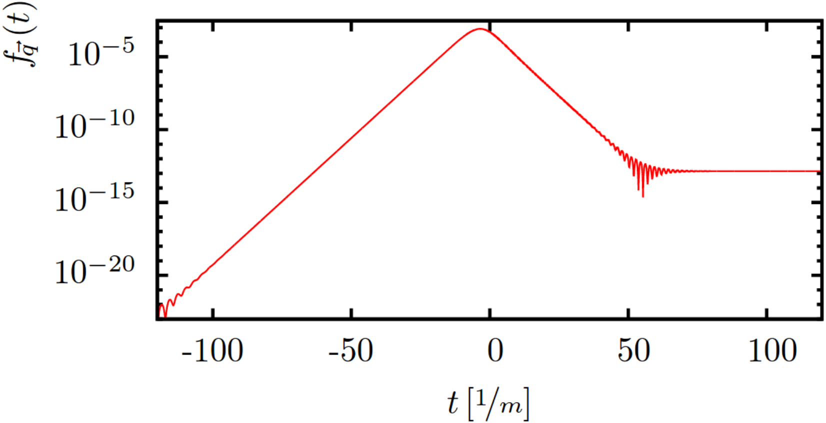
Within many theoretical approaches, one evolves the adiabatic number density $f(\boldsymbol{p}, t)$ corresponding to the basis of the instantaneous Hamiltonian eigenfunctions.

$f(\boldsymbol{p}, t)$ is basis dependent at intermediate times t.

$f(\boldsymbol{p}, t)$ can exhibit values orders of magnitude larger than the final, unambiguous, number of pairs.

Adiabatic number of particles

typical example from [A. Blinne, G. Gies, PRD 89, 085001 (2014)]



Adiabatic number of particles

Can we take advantage of large $f(\boldsymbol{p}, t)$?

One can try to [A. Ilderton, PRD 105, 016021 (2022)]

1) find time instant t_* corresponding to max $f(\boldsymbol{p}, t)$,
2) smoothly switch the field off within $[t_*, t_* + \tau]$ with sufficiently small τ.

By shaping the external-field profile, one may optimize it and obtain a larger number of pairs.

DASE vs. fast switching

- Dynamically assisted Schwinger effect

Strong-field amplitude: E
Weak-field period: τ
Pair-production enhancement for $\gamma_{\mathrm{c}} = m/|eE\tau| \gtrsim 1$

- Fast switching

Field amplitude: E
Duration of the switching part: τ

What is the onset of the PP enhancement?

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Duration of the switching part: τ

What is the onset of the PP enhancement?

It is $\gamma_\mathrm{c} = m/|eE\tau| \gtrsim 1$!

[IAA, D. G. Sevostyanov, V. M. Shabaev, arXiv:2210.15626]
(exact numerical calculations + worldline instanton approach)

DASE vs. fast switching. Experimental prospects

Threshold values of the wavelength $\lambda \approx 2\tau$ ($\gamma_c = 1$):

E/E_c	I (W/cm^2)	λ (μm)
0.1	2.3×10^{27}	8×10^{-6}
0.05	5.8×10^{26}	2×10^{-5}
0.02	9.2×10^{25}	4×10^{-5}
0.01	2.3×10^{25}	8×10^{-5}
0.005	5.8×10^{24}	2×10^{-4}
0.002	9.2×10^{23}	4×10^{-4}
0.001	2.3×10^{23}	8×10^{-4}

These are much smaller than the typical laser wavelengths!

Conclusions

- The two enhancement mechanisms have the same onset in terms of γ_{c}.
- Laser-pulse shaping seems completely unrealistic.
- DASE is feasible! Even $\lambda = 10^{-6}$ μm corresponds to the photon energy ~ 1 MeV.

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- DASE is feasible! Even $\lambda = 10^{-6}$ μm corresponds to the photon energy ~ 1 MeV.

Thank you for your attention!

Our recent papers on strong-field QED

- Pair production in inhomogeneous fields
  IAA, D. Sevostyanov, V. Shabaev, arXiv:2210.15626
  IAA, C. Kohlfürst, PRD 101, 096009 (2020)
- Pair production and approximate methods
  IAA, D. Sevostyanov, V. Shabaev, Symmetry 14, 2444 (2022)
  D. Sevostyanov, IAA, G. Plunien, V. Shabaev, PRD 104, 076014 (2021)
  IAA, G. Plunien, V. Shabaev, PRD 99, 016020 (2019)
- Vacuum photon emission
  IAA, A. Di Piazza, G. Plunien, V. Shabaev, PRD 105, 116005 (2022)
  IAA, A. Panferov, S. Smolyansky, PRA 103, 053107 (2021)
  IAA, G. Plunien, V. Shabaev, PRD 100, 116003 (2019)
- Vacuum birefringence
  IAA, V. Shabaev, arXiv:2303.16273
  IAA, V. Shabaev, Optics and Spectroscopy 129, 890 (2021)
- Positron generation in laser plasma
  IAA, A. Andreev, PRA 104, 052801 (2021)