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Сборник носит междисциплинарный характер, содержит работы математиков, логиков, философов, специалистов по информатике и лингвистов. Сборник представляет интерес для широкого круга читателей, интересующихся вопросами взаимодействия философии, математики, лингвистики.

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Wednesday, May 23

Room	
10:00 - 10:55 Plenary Lecture	**Jaakko Hintikka** IF Logic and Linguistic Theory
Break - 5 min.	
11:00 - 11:55 Plenary Lecture	**Gabriel Sandu** IF Logic and Foundations of Mathematics
Break - 15 min.	
12:10 - 13:05 Plenary Lecture	**Edward F. Karavaev** Belief and Knowledge:Kant's Heritage in Philosophy and Logic Today
Break - 5 min.	
13:10 - 14:05 Plenary Lecture	**Andrey Patkul** The Kant's Treatment of Logic in Historical Context
Break - 5 min.	
14:10 - 15:05 Plenary Lecture	**Andrei Rodin** Univalence and Constructive Identity
Break 15:05 - 16:30	

Room	Hall 1
16:30 - 18:00 Panel Discussion	**The Heritage of Kant and Contemporary Formal Logic** Discutants: **T. Achourioti, J. Hintikka, E. Karavaev, M. van Lambalgen, G. Mints, A. Patkul, G. Priest** Discussion is led by: **A. Rodin**
Break - 15 min.	

Rooms	Hall 1	Hall 2
18:15 - 18:40	**Jos Tellings** On the relation between language and mathematics	**Yury V. Nechitaylov** Basic Principles for the Dynamic Elimination of the Logical Omniscience
Break - 5 min.		
18:45 - 19:10	**Hans M. Dietz, Janna Rohde** Adventures in Reading Maths	**Marcos Cramer** Implicit dynamic function introduction and its connections to the foundations of mathematics
Break - 5 min.		
19:15 - 19:40	**Paniel Osberto Reyes Cárdenas** Mathematical Structuralism, Continuity and Peirce'sDiagrammatic Reasoning	**Vladimir Stepanov** The dynamic model of a language allowing the mechanism of a self-reference

Thursday, May 24

Room	
10:00 - 10:55 Plenary Lecture	**Francis Jeffry Pelletier** The Effect of Applying Mathematical Logic to MetaphysicalConcepts of Vagueness
Break - 5 min.	
11:00 - 11:55 Plenary Lecture	**Vladimir L. Vasyukov** Categorical Ontology of Non-Classical Mathematics
Break - 15 min.	
12:10 - 13:05 Plenary Lecture	**Isidora Stojanovic, Oliver Kutz** Generalized Quantifiers and Ontological Commitments
Break - 5 min.	
13:10 - 14:05 Plenary Lecture	**Markus Werning** Making Quotation Transparent
Break 14:05 - 15:30	
15:30 - 16:25 Plenary Lecture	**Bertrand Boisvert, Louis Féraud, Sergei Soloviev** Graph Transformations, Proofs, and Grammars
Break - 5 min.	
16:30 - 17:25 Plenary Lecture	**Mark Colyvan** From Notation to Knowledge
Break - 5 min.	
17:30 - 18:25 Plenary Lecture	**Igor D. Nevvazhay** Semiotics of Mathematical Thinking Culture

Making Quotation Transparent

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Abstract: Quotation is traditionally regarded as an opaque context. This claim is often illustrated by the observation that, within quotation marks, two synonymous expressions cannot be substituted for each other without changing the semantic value of the quotation and its embedding context. Since the substitutability of synonyms *salva significatione* is logically equivalent to the principle of compositionality, the view that quotation is opaque is tightly linked to the claim that it constitutes an irrevocable exception to the principle of compositionality. This principle demands that the semantic value of a syntactically complex expression be a syntax-dependent function of the semantic values of its parts. However, aside from the general postulate that any semantic analysis of natural language constructions should abide by the principle of compositionality, there are a number of linguistic phenomena that challenge the view that quotation is non-compositional. We will see that any compositionality-friendly analysis of quotation faces a dilemma: In order to be compositional, any analysis of quotation, on the one hand, must avoid that the quoted expression be a syntactic part of the quotation. Only thus one can escape the substitutability objection. On the other hand, in order for the semantic value of the quoted expression to contribute to the semantic value of the greater linguistic context, the quoted expression must be a syntactic part of the greater linguistic context. In the paper a solution to this dilemma and a fully compositional analysis of quotation will be developed. The solution covers both the obviously opaque and the apparently transparent aspects of quotation. The analysis stays fully in the realm of semantics and does neither appeal to any pragmatic use-mention shifts nor to extra-linguistic context parameters.

Оглавление

procedures. Isomorphism is a translation object lenguage on interpretative lenguage. Mathematician always sears a way to make that translation. So, for example, Rene Descartes put connection between numbers and geometrical forms with the help of concept of coordinate frame. This was a way to construct analitical geometry.

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The Kant's Treatment of Logic in Historical Context

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Abstract: The reconstruction of Kant's classification of kinds of logic is given in this presentation. It is shown also that the forming of contemporary understanding of so-called "formal logic" was impossible without the Kant's notion of pure general logic, which was formed in the framework of mentioned classification and in context of Kant's critique of reason. Such critique led to the change of treatment of logic as organon to its treatment as canon of finite cognition. The question of status of Kant's transcendental logic and of possibility of its formalization in account of Kant's idea of pure general logic is also put on in the presentation.

Key words: critique of reason, logic as organon, logic as canon, logic of general use of understanding, logic of particular use of understanding, pure general logic, applied logic, transcendental logic

Introduction

It is generally accepted today that the logic in the shape, in which it was created by Aristotle, is the science of forms of thought. It seems that the logic understood in such way have to be titled as "formal logic". This logic is the doctrine of pure structures of any finite thought, as so far as it is absolutely free from a specific content. But there is the question: is this understanding of logic as science of empty thought's forms self-evident?

1. The Aristotelian treatment of a form and the possibility of logic as science

The reason to put on the mentioned question is in the circumstance that μορφή as εἶδος, treated in Aristotelian way, is always something rich in content. And therefore, logic can't be the science of pure forms of thinking, which would be independent of any content. Also, it is doubtful that the logic – the title that wasn't used by Aristotle himself – is an ἐπιστήμη in general for this philosopher. We couldn't find something as logic in his list of sciences both theoretical and practical. For example, logic absents in the Aristotle's set of theoretical sciences in the same line that first philosophy, physics and mathematics. It indicates that the logic don't have its own subject, which principles would be necessary and invariable, from the point of view of the logic's creator. And therefore there is no specific domain of beings (including the domain of mathematical objects, which are immovable and dependent upon our mind), which could be the subject field of logical investigations as such. Rather, logic is a kind of τέχνη, which deals with the rules of any correct cognition and, thus, which we should learn before we will cognize any object domain defined as itself, in accordance with Aristotle.

2. The modern treatment of Aristotle's logic

But now we are convinced that the logic – even in its Aristotelian shape – is the science in the most rigorous sense. And this conviction has the presupposition that the logic – according to the general notion of science – should have a specific domain which should be cognized by the logic as a science in that, what and how it as such is. This domain is the empty forms of thinking which should be already defined as such, and then, should be only investigated by the logic as science. In the fact, nothing is changed when we go from the "formal logic" in its traditional understanding, which go back to Aristotle's shape of this discipline, to the variety of contemporary shapes of logical science, which are titled as "symbolic" or "mathematical" logic in general. As well as in the case of Aristotelian logic, the contemporary logic deals with the empty forms, although they shouldn't be treated as the forms of thinking by all means. But even considering its revolutionary character, the contemporary logic remains only specification of idea of science of pure forms, but technically more sophisticated.

3. Exclusion of so-called "dialectical logic" from this discussion

There is an exclusion from such tendency in the case of so-called "dialectical logic", which has its origin in the Hegel's philosophy of absolute idealism and its materialistic reinterpretation in ideology of Marxism. This kind of philosophical thinking has the claim to elaborate "the logic of content". But it is out of place to discuss this project in actual context. Although it is permissible notice that the idea of "logic of content", which is based on the Hegel's premise of speculative identity of logic and ontology, shouldn't be identified with Aristotelian understanding of form as a something rich in content. Therefore, the "dialectical logic" is situated outside of the main path of the development of logic as science of empty forms, whereas the Aristotelian shape of logic lies in the beginning's point of this path.

4. The Kant's treatment of logic as the main point on the way to its formal understanding

And now we should put on the following question. Where is the origin of transformation of logic's idea from its Aristotelian understanding to its contemporary treatment as the sciences of the pure thought's forms? Perhaps, such transformation has begun very long ago. But one of possible answers of this question is: the origin of transformation of understanding of logic lies in the reinterpretation of this science given by I. Kant in his "Critique of Pure Reason".

4.1. The transformation of logic's status in the context of the separation between the things in themselves and phenomena

It is very well known that the Kant's transformation of logic's idea is connected with change of treatment of this science as ὄργανον of cognition – as it was in Aristotle – to its treatment as only κανών of any possible cognition. Such change is based in the Kant's position, according to which logic may be employed only to the things as phenomena, but not to the things in themselves, to the beings as beings in the terms of Aristotle. Thus, the logic and the structures of thought described by this science are not a sufficient condition of an actual cognition, but only its necessary condition (sine qua non). Any cognition has to be measured by logic, but logic as itself can't give any new knowledge. The science of logic acquires the meaning of science of pure forms, which are independent of concrete content of actual situation in the area of phenomena given to us by the experience. These actual situations of

phenomena emerge in an absolutely accidental way, but forms of thought are most general and necessary. They have to be given to us *a priori* before any accidental experience and only then they have to be applied to the things as phenomena, which make up a content of such experience.

On the one hand, the famous division of logic's types made by Kant in his "Critique of Pure Reason" can be understood only in the context of this transformation of logic's role for the human cognition. On the other hand, this division can show the Kant's reduction of logic to the science of pure forms of thought in the best way.

4.2 The division of the logic of general and the logic of particular use of understanding

So firstly Kant divides the general notion of logic into *logic of general* and *logic of particular* use of our understanding. He writes:

> Now, logic in its turn may be considered as twofold, – namely, as logic of general [universal], or of the particular use of understanding. (Kant, 1855, p.46 – 47)

The last mentioned type of logic deals always with a particular object domain and the main rules of its knowledge. Logic of our understanding's particular use is always related to the *matter* of one of the object domains. It isn't pure formal logic. Thus, it is very noteworthily that a logic, which would be related to any matter, could be possible only as particular but not as universal discipline. It could be assumed that this thesis goes back to Aristotle's thesis "τὸ ὂν λέγεται πολλαχῶς" (Met. VII, 1, 1028 a, 10), or – in scholastic formula – to the "analogia entis".

Kant himself describes logic of particular use of understanding in following words:

> The logic of the particular use of the understanding contains the laws of correct thinking upon a particular class of objects. (Kant, 1855, p. 47)

As opposed to the logic of general use of understanding, the logic of its particular use may be *organon* of cognition of a *specific object domain* in accordance with Kant. He states:

> The former (the logic of general use of understanding – A.P.) may be called elemental logic, – the latter, (the logic of particular use of understanding – A.P.) the organon of this or that particular science. The latter is for the most part employed in the schools, as a propaedeutic to the sciences, although, indeed, according to the course of human reason, it is the last thing we arrive at, when the science is already matured, and needs only in finishing touches toward its correction and completion; for our knowledge of the objects of our attempted science must be tolerably extensive and complete before we can indicate the laws by which a science of these objects can be established. (Kant, 1855, p. 47)

Thereby, the logic of particular use of our understanding could be identified with the *methodology* of a particular science in contemporary word usage. It deals with the rules of cognition of specified object domain but after the maturity of particular science corresponding to this object domain.

4.2.1. The comparison of the Kant's idea of the logic of particular understanding's use and the idea a regional ontology in phenomenology

NB: It would be of very interesting to compare Kant's idea of logic of particular use of understanding with the concept of a *regional ontology* elaborated in the phenomenological

philosophy (e.g., Husserl, 1989, §§ 9 – 11 & §§ 147 – 150). It is very important for interpretation of status of logic as science, because they use in phenomenological philosophy another title for marking of regional ontology with the same terms as Kant's "logic" in its particular meaning. This title is – in the Heidegger's words – a "productive logic of science". It is obvious that such *productive logic of science* may be always only a productive logic of *a particular science* but not of the science in general. It seems so as if Kant legitimized the usage of term "logic" in the meaning of particular employment of understanding to particular object domains (i.e. regions of beings) in German philosophy after him. But we should remember that the productive logic of a particular science not only corrects and completes such science, as methodology makes it, but also – as ontology – grounded it. Therefore it differs from the logic of particular use of understanding as only methodology, which follows already matured particular science.

It could be illustrated by the Heidegger's sentences form his "History of Concept of Time". This philosopher says:

> Because reality – as well nature as history – can be reached only by leaping over the sciences to some extent, this prescientific – actually philosophical – disclosure of them becomes what I call a *productive logic*, an anticipatory disclosure and conceptual penetration of potential domains of objects for the sciences. Unlike traditional philosophy of science, which proceeds after the fact of accidental, historically given science in order to investigate its structure, such a logic leaps ahead into the primary field of subject matter of a potential science and first makes available the basic structure of the possible object of the science by disclosing the constitution of being of that field. This is the procedure of *original logic* put forward by Plato and Aristotle, of course within very narrow limits. Since then, the idea of logic lapsed into obscurity and was no longer understood. Hence phenomenology has the task of making the domain of the subject matter comprehensible *before* its scientific treatment and, on this basis, the latter as well. (Heidegger, 1985, p. 4)

The Heidegger's use of word "logic" is very significant regarding to phenomenological project of regional ontologies. But it is clear that the way to the logic as science of empty forms of thought don't go via the point of logic of understanding's particular logic, via the point of productive logic of a particular science in its phenomenological meaning. Hence, now we should turn back to the mentioned Kant's division between the logic of general use of understanding and the logic of its particular use and look at the first part of such division.

4.2.2. The repetition concerning the Kant's division of logic's notion

Let us remind that the philosopher opposes the logic of particular use of understanding to the logic of its general use defined by him as discipline, which deals with general rules of thought without regard to the specific matter of this thought's employment.

Namely, he states that the logic of general use of understanding

> …contains the absolutely necessary laws of thought, without which no use whatever of the understanding is possible, and gives laws therefore to the understanding, without regard to the difference of objects on which it may be employed. (Kant, 1855, p. 47)

As it was said earlier, the logic of general use of understanding can't be general organon of our finite cognition but only its canon. And Kant calls it "elemental logic".

4.3 The further division of logic of general use of understanding into the pure logic and the applied logic

Unlike the notion of logic of particular use of understanding the notion of logic of general use of understanding is divisible further. Kant believes that logic of general use of understanding can have two parts; namely, the part of *pure logic* and of *applied logic*.

The first of mentioned types of logic as general discipline is dedicated to the rules of thought regardless to the concrete conditions of its realization by an empirical subject. On the contrary, the general but applied logic takes into account such empirical conditions of thinking.

This difference could be very well clarified by some Kant's sentences from the text of "Critique of Pure Reason":

> General logic is again either pure or applied. In the former, we abstract all the empirical conditions under which the understanding is exercised; for example, the influence of the senses, the play of the phantasy or imagination, the laws of the memory, the force of habit, of inclination etc, consequently also, the sources of prejudice, – in a word, we abstract all causes from which particular cognitions arise, because these causes regard the understanding under certain circumstances of its application, and, to the knowledge of them experience is required. (Kant, 1855, p. 47 – 48)

The thinker states concerning to general applied logic the following:

> "General logic is called applied, when it is directed to the laws of the use of the understanding, under the subjective empirical conditions which psychology teaches us. It has therefore empirical principles, although, at the same time, it is in so far general, that it applies to the exercise of the understanding, without regard to difference of objects. On this account, moreover, it is neither a canon of the understanding in general, nor an organon of a particular science, but merely a cathartic of the human understanding". (Kant, 1855, p. 48)

But *pure general logic* has no need to allow such empirical conditions, and therefore, to have any relation to our actual but accidental experience:

> Pure general logic has to do, therefore, merely with pure *a priori* principles, and is a canon of understanding and reason, but only in respect of the formal part of their use, be the content what it may, empirical or transcendental. (Kant, 1855, p. 48)

Hence, Kant concludes:

> In general logic, therefore, that part which constitutes pure logic must be carefully distinguished from that which constitutes applied (though still general) logic. The former alone is properly science, although short and dry, as the methodical exposition of an elemental doctrine of the understanding ought to be. (Kant, 1855, p. 48)

So, the philosopher gets the notion of logic, which is very near to our contemporary term of "formal logic". In Kant's words, this is the pure general logic, i.e. the logic, which is absolutely free as well from concrete content given by the experience as from concrete conditions of realization of thinking by an empirical subject. Kant points out two requirements for such kind of logic to be:

> 1. As general logic, it makes abstraction of all content of the cognition of the understanding, and of the difference of objects, and has to do with nothing but the mere form of thought.

2. As pure logic, it has no empirical principles, and consequently draws nothing (contrary to the common persuasion) from psychology, which therefore has no influence on the canon of the understanding. It is a demonstrated doctrine, and every thing in it must be certain completely *a priori*. (Kant, 1855, p. 48)

4.3.1. The notion of pure logic as origin of an antiphychologism

This is necessary to emphasize the second Kant's requirement to the pure general logic especially. In the fact, this thinker forms the conditions of arising of so-called *antipsychologism* in the treatment of logic's nature. He makes it by the way of the rigorous distinguishing between the pure formal principles, with which the logic has to do, and subjective empirical conditions of thinking, which are the theme of only psychological but not logical investigations. As it is very well known, that detailed critique of the grounding of logic's laws on the laws of psychical life was developed after Kant in the different schools of philosophy, as well in analytical philosophy (Frege) as in phenomenology (Husserl). It is very representative, because the members of mentioned schools were not agree with Kant's treatment of nature and status of logic and, especially, of its relation to mathematics and its objects. (For example, intuitivism Kant's vs. logicism Frege's in the interpretation of mathematics). But how it is shown by J. Macfarlane, the possible problem in interpretation of the nature of logic in Kant and – for example – in the case of Frege – is in the circumstance that the understanding of logic's function is very different in the both philosophers. Macfarlane states that Kant's

> …picture of logic is evidently incompatible with Frege view that logic can supply us with substantive knowledge about objects. (Macfarlane, 2002, p.29)

5. Kant's transcendental logic and the possibility of its formalization

But the main problem is opposite to the mentioned in our opinion. The question is that Kant introduces also the notion of transcendental logic besides his classification of logic's kinds, to which belongs his notion of pure general logic as logic as itself. And the transcendental logic has to do with the possibility of relation of the thought's forms (categories etc.) to the objects as phenomena. And therefore it takes a part in "substantive knowledge about objects". (In opposition to our earlier hypothesis it may be stated here that the origin of idea of a regional ontology roots not only in the Kant's idea of logic of particular use of understanding, which plays exclusively methodological role for a particular positive science, but also in his concept of a transcendental logic, which can ground a particular science on the basis of categorical structure and its relation to a subject matter.)

Kant writes regarding to his idea of transcendental logic:

> In this case, there would exist a kind of logic, in which we should not make abstraction of all content of cognition; for that logic which should comprise merely the laws of pure thought (of an object), would of course exclude all those cognitions which were of empirical content. This kind of logic would also examine the origin of our cognitions of objects, so far as that origin cannot be ascribed to the objects themselves; while, on the contrary, general logic has nothing to do with the origin of our cognitions…
> (Kant, 1855, p. 48)

And he adds concerning to pure general logic in its relation to the transcendental logic:

> Consequently, general treats of the form of understanding only, which can be applied to representations, from whatever source they may have arisen. (Kant, 1855, p. 48)

Conclusion

Thus, Kant gives the room in his architectonic of logic for a kind of logic, which would be general but couldn't be absolutely independent of content; insofar as– at the same time – this content does not be of empirical source. This logic could be titled as properly "philosophical logic", which deals with the origin of our cognitions and their possible relation to the objects. And the final question is, whether this philosophical logic, to which belongs Kant's doctrine of kinds of judgment and categories, could be formalized, as so far as Kant's notion of pure general logic can play role of formalization's ideal for contemporary attempts of, so to say, "symbolic formalization" of "formal logic" in its ordinary meaning. Is it possible to formalize this kind of logic, even allowing the semantic character of contemporary logicism, which is underlined by J. Macfarlane?

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