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# ANALYTICAL STUDY OF GENERAL MECHANOCHEMICAL CORROSION OF THE PIPE UNDER THE AXIAL FORCE AND PRESSURE

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Abstract. *This paper is concerned with uniform mechanochemical corrosion, often observed in practice. The linearly elastic thick-walled cylindrical long tube subjected to a longitudinal force, constant internal and external pressure of active environments is investigated. It is known that the rate of corrosion depends on many factors. In neutral and alkalescent environments or when tension is less than certain threshold, mechanical stress has no influence on corrosiveness. When this is the case, the values of stress-components at some instant are determined by Lame's formulae with given temporal laws of corrosive wear. In other situations, according to most experimental data, the rate of uniform corrosion is linear with stress. Furthermore, corrosion rate is inversely as the exponent of time when closed oxide layer leads to the inhibition of corrosion. The problem is then reduced to the ordinary differential equation in either circumferential or longitudinal stress as the situation requires. In the general way, basic equations can be solved by computational methods. In some cases (e.g. when corrosion is one-sided or axial force is relatively small) analytical solutions of the equations are found. Taking into account changes of mechanical characteristics of a pipe, the method for a durability prediction is developed. It is obvious that failure can be caused by the several reasons. To specify the cause and the instant of failure, different estimating functions are suggested. Calculations performed have shown that increasing the exponent of inhibition of corrosion leads to considerable prolongation of the service life of a tube.*

#### 1 INTRODUCTION

Most damage terminations in metal structures are due to material degradation induced by an operating environment. It is well known that corrosion activity may be intensified by mechanical stresses. When this is the case, corrosion is said to be a mechanochemical one. The research in this area has been conducted by numerous authors. This paper is concerned with surface mechanochemical corrosion. A comprehensive review of models and calculations of structures taking into account corrosive wear was given e.g. in [1, 2]. One of the first in the field is the article [3] concerned with mechanochemical corrosion of a thin-walled cylindrical shell under a longitudinal force. In the work [4] the lifetime of a loaded pipe has been assessed under the assumption of the exponential dependence of corrosion rate on the mean stress. According to [2, 5], corrosion rate depends on stress linearly and is inversely as the exponent of time. Using this relation, the corrosive wear of a nonlinearly elastic cylinder subjected to pressure and temperature has been simulated in book [2]. In this paper the equal-rate mechanochemical corrosion of a linearly elastic thick-walled cylindrical tube subjected to a longitudinal force, internal and external pressure is discussed [6, 7].

### 2 PROBLEM STATEMENT

The uniform surface corrosion of an elastic cylindrical tube subjected to a longitudinal force *Q*, constant internal presure  $p_r$  and external pressure  $p_R$  is investigated. The inner and outer tube radii at the initial instant  $t = 0$  are denoted by  $r_0$  and  $R_0$  ( $r_0 < R_0$ ). The action of the ends of the cylinder is not taken into account. Changes of the tube radii are assumed to be quasi-static. Corrosion rates at the internal ( $\rho = r$ ) and external ( $\rho = R$ ) boundaries are given by the expressions [2, 5]:

$$
v_r = \frac{dr}{dt} = \frac{d[r_0 + \delta_r]}{dt} = [a_r + m_r \sigma_1(r)] \exp(-bt) \text{ when } |\sigma_1(r)| \ge |\sigma_r^{th}|,
$$
 (1)

$$
v_R = -\frac{dR}{dt} = -\frac{d[R_0 - \delta_R]}{dt} = [a_R + m_R \sigma_1(R)] \exp(-bt) \text{ when } |\sigma_1(R)| \ge |\sigma_R^{th}|
$$
 (2)

correspondingly. Here  $b, a_r, a_R, m_r, m_R$  are observable quantities, and  $a_r = v_r^0 - m_r \sigma_r^{th}$ ,  $a_R = v_R^0 - m_R \sigma_R^{th}$ ;  $\sigma_r^{th}$ ,  $\sigma_R^{th}$  are the threshold stresses (as a matter of fact, which are different for traction and compression);  $v_r^0$ ,  $v_R^0$  are the initial corrosion rates at  $|\sigma_1(r)| < |\sigma_r^{th}|$ ,  $|\sigma_1(R)| < |\sigma_R^{th}|$ ;  $\sigma_1$  is the maximum principal stress.

It is necessary to assess the life-time of the tube concerned.

### 3 BASIC EQUATIONS

The problem of a tube under pressure has been disscused by numerous writers including G. Lame. The stress-components in this case are expressed, by reference to cylindrical coordinates  $\rho$ ,  $\theta$ , z by the equations

$$
\sigma_{\theta\theta}(\rho) = \frac{p_r r^2 - p_R R^2}{R^2 - r^2} + \frac{p_r - p_R}{R^2 - r^2} \frac{r^2 R^2}{\rho^2},\tag{3}
$$

$$
\sigma_{\rho\rho}(\rho) = \frac{p_r r^2 - p_R R^2}{R^2 - r^2} - \frac{p_r - p_R}{R^2 - r^2} \frac{r^2 R^2}{\rho^2},
$$

$$
\sigma_{zz}^{w}(\rho) = 2\nu \frac{p_r r^2 - p_R R^2}{R^2 - r^2}, \quad \sigma_{zz}^{pQ}(\rho) = \frac{p_r r^2 - p_R R^2 + Q/\pi}{R^2 - r^2},
$$

$$
\sigma_{zz}^{Q}(\rho) = \frac{Q}{\pi (R^2 - r^2)},
$$

$$
r \le \rho \le R, \quad 0 \le \theta < 2\pi,
$$
 (4)

where *ν* is Poisson's ratio.

When the length of the cylinder is maintained constant, then there is longitudinal tension of amount  $\sigma_{zz}^w$ . If a closed cylindrical vessel is under an axial force *Q*, internal pressure  $p_r$ and external pressure  $p_R$ , then the resultant longitudinal tension is  $\sigma_{zz} = \sigma_{zz}^{pQ}$ . If the vessel is unclosed, then  $\sigma_{zz} = \sigma_{zz}^Q$ .

When  $r = 0$ ,  $p_r = 0$ ,  $p_R = p$  or  $p_r = p_R = p$ , there is a homogeneous stress  $\sigma_{\theta\theta} \equiv \sigma_{\theta\theta} \equiv -p$  in a tube irrespective of corrosion. Moreover, in neutral and alkalescent environments or when load is less than the threshold  $\sigma^{th}$ , stress has no influence on corrosion rate. In that cases the stress-components at any instant are determined by the above equations (3)–(4) with given lows of  $r(t)$ ,  $R(t)$ . We shall discuss other situations.

The maximum principal stress is the circumferential tension or the longitudinal one as the case may be.

#### 3.1 Case (a)

At first, let the maximum principal stress be the circumferential tension  $\sigma_1 = \sigma_{\theta\theta}$ . The case  $\sigma_1 = \sigma_{zz}^w$  is a particular case of this situation. The greatest tension is at the inner surface  $\rho = r$ . So we are to observe its amount  $\sigma_{\theta\theta}(r) = \sigma_1$ . It will be convenient to rewrite the formula (3) in the form

$$
\sigma_1(r) = \sigma_{\theta\theta}(r) = p_r \frac{\eta^2 + 1}{\eta^2 - 1} - 2p_R \frac{\eta^2}{\eta^2 - 1}
$$
\n(5)

and

$$
\sigma_1(R) = p_r \frac{2}{\eta^2 - 1} - p_R \frac{\eta^2 + 1}{\eta^2 - 1},\tag{6}
$$

where

$$
\eta = \frac{R}{r} = \frac{R_0 - \delta_R}{r_0 + \delta_r}.\tag{7}
$$

If we eliminate  $\sigma_1$  from the formulae (1) and (2) by using (5), (6) we obtain the relationship

$$
Rm_r + rm_R = m_R \left( r_0 + \frac{a_r}{-b} \left[ \exp(-bt) - 1 \right] \right) + m_r \left( R_0 - \frac{A_R}{-b} \left[ \exp(-bt) - 1 \right] \right) \tag{8}
$$

where

$$
A_R = a_R - m_R (p_r - p_R). \tag{9}
$$

On differentiating the expression  $(5)$  with respect to *t*, and using  $(1)$ ,  $(2)$ ,  $(7)$ – $(9)$ , we can deduce the ordinary differential equation for  $\sigma_1 = \sigma_{\theta\theta}(r)$  [6]

$$
\frac{d\sigma_1}{dt} = \frac{\sqrt{[\sigma_1 + p_r][\sigma_1 - p_r + 2p_R]}}{p_r - p_R} \left[ m_r \sqrt{\sigma_1 + p_r} + m_R \sqrt{\sigma_1 - p_r + 2p_R} \right] \times \qquad (10)
$$
\n
$$
\times \frac{[A_R + m_R \sigma_1]\sqrt{\sigma_1 - p_r + 2p_R} + [a_r + m_r \sigma_1]\sqrt{\sigma_1 + p_r}}{m_R \left( r_0 - \frac{a_r}{-b} \right) + m_r \left( R_0 + \frac{A_R}{-b} \right) \left[ \exp(bt) + m_R \frac{a_r}{-b} - m_r \frac{A_R}{-b} \right]}.
$$

The initial conditions to be satisfied at  $t = 0$  are

$$
\sigma_1|_{t=0} = \sigma_{\theta\theta}^0(r) = p_r \frac{\eta_0^2 + 1}{\eta_0^2 - 1} - 2p_R \frac{\eta_0^2}{\eta_0^2 - 1}, \qquad \eta_0 = \frac{R_0}{r_0}.
$$
\n(11)

### 3.2 Case (b)

Now let the maximum principal stress be the longitudinal tension  $\sigma_1 = \sigma_{zz}^Q$ . In this case instead of relationship (8) we may derive the formula

$$
Rm_r + rm_R = m_R \left( r_0 + \frac{a_r}{-b} \left[ \exp(-bt) - 1 \right] \right) + m_r \left( R_0 - \frac{a_R}{-b} \left[ \exp(-bt) - 1 \right] \right) \tag{12}
$$

On combining the expression (4) and (12) we find that the following equation must hold at every moment *t*

$$
r = \left[ -\frac{m_R}{m_r} \left\{ \frac{m_R}{m_r} \left( r_0 + \frac{a_r}{-b} \left[ \exp(-bt) - 1 \right] \right) + R_0 - \frac{a_R}{-b} \left[ \exp(-bt) - 1 \right] \right\} + \left( \left\{ \frac{m_R}{m_r} \left( r_0 + \frac{a_r}{-b} \left[ \exp(-bt) - 1 \right] \right) + R_0 - \frac{a_R}{-b} \left[ \exp(-bt) - 1 \right] \right\}^2 + \left( \left( \frac{m_R}{m_r} \right)^2 - 1 \right] \frac{Q}{\pi \sigma_1} \right)^{1/2} \right] \frac{1}{1 - \left( \frac{m_R}{m_r} \right)^2}.
$$
\n(13)

On differentiating the expression  $(4)$  with respect to  $t$ , and using  $(1)$ ,  $(2)$ ,  $(4)$ ,  $(12)$ ,  $(13)$  we can obtain the ordinary differential equation

$$
\frac{d\sigma_1}{dt} = -\frac{2\pi\sigma_1^2}{Q\left[1 - \left(\frac{m_R}{m_r}\right)^2\right] \exp(bt)} \left[ \left(-a_R + \frac{m_R}{m_r} a_r\right) \times \left(\frac{m_R}{m_r} \left(r_0 + \frac{a_r}{-b} \left[\exp(-bt) - 1\right]\right) + R_0 - \frac{a_R}{-b} \left[\exp(-bt) - 1\right] \right] + \left(\frac{m_R}{m_r} (a_R + m_R \sigma_1) - a_r - m_r \sigma_1\right) \left(\left[\left(\frac{m_R}{m_r}\right)^2 - 1\right] \frac{Q}{\pi\sigma_1} + \left\{\frac{m_R}{m_r} \left(r_0 + \frac{a_r}{-b} \left[\exp(-bt) - 1\right]\right) + R_0 - \frac{a_R}{-b} \left[\exp(-bt) - 1\right] \right\}^2\right)^{1/2},
$$
\n(14)

where  $\sigma_1 = \sigma_{zz}^Q$ .

 $+$ 

The initial conditions are of the form

$$
\sigma_1|_{t=0} = \sigma_{zz}^0 = \frac{Q/\pi}{R_0^2 - r_0^2}.
$$
\n(15)

### 3.3 Case (c)

The case  $\sigma_1 = \sigma_{zz}^{pQ}$  is investigated in detail in the article [7]. There the problem is also reduced to the ordinary differential equations.

### 4 SOLUTIONS OF THE BASIC EQUATIONS

### 4.1 Case (a)

In the case  $\sigma_1 = \sigma_{\theta\theta}$  we are to solve the equation (10). The integral of this equation, satisfying the conditions (11), is

$$
t = -\frac{1}{b} \ln \left\{ 1 - b \frac{m_R r_0 + m_r R_0}{m_R a_r - m_r A_R} \left( \exp \left[ (m_R a_r - m_r A_R) F(\sigma_1) \right] - 1 \right) \right\},\tag{16}
$$

where

$$
F(\sigma_1) = (p_r - p_R) \int_{\sigma_1^0}^{\sigma_1} \frac{1}{\sqrt{[\sigma_1 + p_r][\sigma_1 - p_r + 2p_R]}} \times
$$
  

$$
\times \frac{1}{[A_R + m_R \sigma_1[\sqrt{\sigma_1 - p_r + 2p_R} + [a_r + m_r \sigma_1]\sqrt{\sigma_1 + p_r}]} \times
$$
  

$$
\times \frac{d\sigma_1}{m_r \sqrt{\sigma_1 + p_r} + m_R \sqrt{\sigma_1 - p_r + 2p_R}}.
$$

When the corrosion is one-sided the solution can be simplified. For example, if  $p_R = p > 0$ ,  $p_r = 0$ ,  $a_r = m_r = 0$  (external corrosion) the result may be written in the form

$$
t = -\frac{1}{b} \ln \{1 - bF(\sigma_1)\} \quad \text{when} \quad b \neq 0,
$$
  

$$
t = F(\sigma_1) \quad \text{when} \quad b = 0,
$$
 (17)

where

$$
-\operatorname{if} \quad 2p > |A_R/m_R|
$$

 $-$  if  $2p < |A_R/m_R|$ 

$$
F(\sigma_1) = -\frac{r_0}{m_R (A_R/m_R - 2p)} \left( \sqrt{\frac{\sigma_1}{\sigma_1 + 2p}} - \eta_0 + \frac{p}{\sqrt{A_R/m_R (A_R/m_R - 2p)}} \times \right)
$$
  
 
$$
\times \left[ \ln \frac{\sqrt{A_R/m_R (A_R/m_R - 2p) \sigma_1 [\sigma_1 + 2p]} + \sigma_1 (p - A_R/m_R) - p A_R/m_R}{\sigma_1 + A_R/m_R} - \right]
$$
  
- 
$$
\ln \frac{\sqrt{A_R/m_R (A_R/m_R - 2p) \sigma_1^0 [\sigma_1^0 + 2p]} + \sigma_1^0 (p - A_R/m_R) - p A_R/m_R}{\sigma_1^0 + A_R/m_R} \right) \right),
$$

$$
F(\sigma_1) = -\frac{r_0}{m_R (A_R/m_R - 2p)} \left( \sqrt{\frac{\sigma_1}{\sigma_1 + 2p}} - \eta_0 + \frac{p}{\sqrt{A_R/m_R (A_R/m_R + 2p)}} \times \right)
$$
  
\$\times \left[ \ln \frac{\sqrt{A\_R/m\_R (A\_R/m\_R + 2p) \sigma\_1 [\sigma\_1 + 2p]} + \sigma\_1 (p - A\_R/m\_R) - pA\_R/m\_R - \frac{\sigma\_1 + A\_R/m\_R}{\sigma\_1 + A\_R/m\_R} - \ln \frac{\sqrt{A\_R/m\_R (A\_R/m\_R + 2p) \sigma\_1^0 [\sigma\_1^0 + 2p]} + \sigma\_1^0 (p - A\_R/m\_R) - pA\_R/m\_R - \frac{\sigma\_1^0 + A\_R/m\_R}{\sigma\_1^0 + A\_R/m\_R} \right] \right).

Here it is necessary to take the real branch of the function  $\ln(x)$ . It is to be noted that in that case  $m_R < 0$  since  $\sigma_1 < 0$ .

In the case of internal corrosion when  $p_R = 0$ ,  $p_r = p > 0$ ,  $a_R = m_R = 0$ , the solution is of the form (17) where

 $\frac{1}{p}$  if  $p > |a_r/m_r|$ 

 $\int$ *—* **if**  $p < |a_r/m_r|$ 

$$
F(\sigma_1) = \frac{R_0}{a_r - m_r p} \left( \sqrt{\frac{\sigma_1 - p}{\sigma_1 + p}} - \frac{1}{\eta_0} + \frac{p}{\sqrt{p^2 - a_r^2/m_r^2}} \right)
$$

$$
+ \frac{p}{\sqrt{p^2 - a_r^2/m_r^2}} \left[ \arcsin \frac{\frac{p^2 - a_r^2/m_r^2}{\sigma_1 + a_r/m_r} + \frac{a_r}{m_r}}{p} - \arcsin \frac{\frac{p^2 - a_r^2/m_r^2}{\sigma_1 + a_r/m_r} + \frac{a_r}{m_r}}{p} \right] \right)
$$

$$
F(\sigma_1) = \frac{R_0}{a_r - m_r p} \left( \sqrt{\frac{\sigma_1 - p}{\sigma_1 + p}} - \frac{1}{\eta_0} + \frac{p}{\sqrt{a_r^2/m_r^2 - p^2}} \right) \ln \frac{p^2 + \sigma_1 a_r/m_r - \sqrt{(a_r^2/m_r^2 - p^2)([\sigma_1]^2 - p^2)}}{\sigma_1 + a_r/m_r} - \frac{p_r}{\sqrt{a_r^2/m_r^2 - p^2}} \ln \frac{p^2 + \sigma_1^0 a_r/m_r - \sqrt{(a_r^2/m_r^2 - p^2)([\sigma_1^0]^2 - p^2)}}{\sigma_1^0 + a_r/m_r} \right),
$$

$$
\sigma_1=\sigma_{\theta\theta}(r).
$$

Here it is necessary to take the real branch of the function  $\ln(x)$  and the increasing branch of the function  $arcsin(x)$ .

### 4.2 Case (b)

When  $\sigma_1 = \sigma_{zz}^Q$  we are to solve the equation (14). In the general way, the equation (14) with the initial conditions (15) can be solved by computational methods. When the corrosion is one-sided the solution may be written in the form (17) where

— if  $a_r = m_r = 0$  (external corrosion)

$$
F(\sigma_1) = \frac{1}{a_R} \left( \sqrt{r_0^2 + \frac{Q}{\pi \sigma_1}} - R_0 + \frac{Q}{2\pi r_0 \sqrt{\frac{-Q a_R}{\pi r_0^2 m_R} + \frac{a_R^2}{m_R^2}}} \times \frac{2\pi r_0 \sqrt{\frac{-Q a_R}{\pi r_0^2 m_R} + \frac{a_R^2}{m_R^2}}}{2\pi r_0 \sqrt{\frac{-Q a_R}{\pi r_0^2 m_R} + \frac{a_R^2}{m_R^2}}} \right)
$$

$$
\times \left[ \ln \frac{2 \sqrt{\left(\frac{-Q a_R}{\pi r_0^2 m_R} + \frac{a_R^2}{m_R^2}\right) \left(\sigma_1^2 + \frac{Q \sigma_1}{\pi r_0^2}\right)} + \left(\frac{Q}{\pi r_0^2} - \frac{2 a_R}{m_R}\right) \sigma_1 - \frac{Q a_R}{\pi r_0^2 m_R}}{\frac{a_R}{m_R} + \sigma_1} \right]
$$

$$
-\ln \frac{\frac{2QR_0}{\pi r_0(R_0^2 - r_0^2)} \sqrt{\frac{-Qa_R}{\pi r_0^2 m_R} + \frac{a_R^2}{m_R^2} + \left(\frac{Q}{\pi r_0^2} - \frac{2a_R}{m_R}\right) \sigma_1^0 - \frac{Qa_R}{\pi r_0^2 m_R}}{\frac{a_R}{m_R} + \sigma_1^0}
$$

 $\overline{\phantom{a}}$  =  $m_R$  = 0 (internal corrosion)

$$
F(\sigma_1) = \frac{1}{a_r} \left( \sqrt{R_0^2 - \frac{Q}{\pi \sigma_1}} - r_0 + \frac{Q}{2\pi R_0 \sqrt{\frac{Q a_r}{\pi R_0^2 m_r} + \frac{a_r^2}{m_r^2}}} \times \right) \tag{18}
$$
\n
$$
\times \left[ \ln \frac{2 \sqrt{\left(\frac{Q a_r}{\pi R_0^2 m_r} + \frac{a_r^2}{m_r^2}\right) \left(\sigma_1^2 - \frac{Q \sigma_1}{\pi R_0^2}\right)} - \left(\frac{Q}{\pi R_0^2} + \frac{2a_r}{m_r}\right) \sigma_1 + \frac{Q a_r}{\pi R_0^2 m_r}}{\frac{a_r}{m_r} + \sigma_1} - \frac{2Q r_0}{\pi R_0 (R_0^2 - r_0^2)} \sqrt{\frac{Q a_r}{\pi R_0^2 m_r} + \frac{a_r^2}{m_r^2}} - \left(\frac{Q}{\pi R_0^2} + \frac{2a_r}{m_r}\right) \sigma_1^0 + \frac{Q a_r}{\pi R_0^2 m_r}}{\frac{a_r}{m_r} + \sigma_1^0} \right) \right],
$$
\n(18)

$$
\sigma_1 = \sigma_{zz}^Q.
$$

#### 4.3 Case (c)

The analytical solutions of the basic equations in the case  $\sigma_1 = \sigma_{zz}^{pQ}$  are given in [7]. The example of the lifetime determination in the case concerned is worked out there.

#### 5 LIFETIME ASSESSMENT

#### 5.1 Estimating functions

Taking into account synergetic interaction of general corrosion with mechanical stresses, lifetime of a tube may be assessed. It is obvious, that failure can be due to a variety of reasons. To determine the reason and the instant of failure, estimating functions are suggested. Following the L. Kachanov approach [8], different kinds of damage are represented by scalar functions changing in the interval [0, 1] (or  $[-\infty, 1]$ ) and mounting to 1 in the moments of fault related to concrete criteria. To assess the strength margin, functions of the type  $\Pi_s(t) = \frac{f(\sigma, \epsilon)}{t}$ *σs ≤* 1 may be used. For the maximum stress criterion we can write  $\Pi_s(t) = \frac{\sigma_1(t)}{2\sigma_1(t)}$  $\sigma_s(t)$ , where  $\sigma_s(t)$  is limiting stress that may change in time. In that situation and if  $\sigma_s = \text{const}$ , the time to rupture is evaluated by the formulae (16)–(18) with the  $\sigma_s$  for  $\sigma_1$ .

Functions to assess the stability factor may be of the form  $\Pi_{cr}(t) = \frac{\sigma_{zz}(t)}{\sigma_{zz}^{cr}(t)}$  $+\frac{\sigma_{\theta\theta}(t)}{cr(t)}$ *σ cr θθ*(*t*) *≤* 1, where  $\sigma_{zz}^{cr}$  is buckling stress depending on the tube sizes and mechanical quantities for only

longitudinal tension (under no pressure) and  $\sigma_{\theta\theta}^{cr}$  is buckling stress for only circumferential tension (under no axial force) [9]. Stability of thin-walled shells under conditions of the corrosive action have been investigated by many scientists, e.g. [10], [11].

Damage being due to different reasons, to assess damage accumulation numerous functions can be proposed. For instance, according to Bailey's principle, the time to destruction *t ∗* is

determined by the equation  $\Pi_d(t_d^*) =$ *t ∗* ∫ *d* 0 *dt*  $\frac{du}{\tau[\sigma(t)]} = 1$ , where  $\tau[\sigma]$  is the working life of the

material under stress *σ*.

Furthermore, failure may be determined apparently by accidental circumstances. For such assessment we can introduce estimating function as being equal to probability i.e. accident risk  $\Pi_p(t) = P(x_1, \ldots, x_n, t/y_1, \ldots, y_n)$ . It is to be emphasized that unreliability function depends on other estimating functions.

#### 5.2 Lifetime determination

The graphs of all estimating functions are plotted and compared with each other. The curve being the first to run up to 1 determines the most probable reason of breakdown and the durability of an item  $t^* = \min\{t_i^*: \Pi_i(t_i^*) = 1\}.$ 

Typical curves for the functions Π*<sup>s</sup>* and Π*cr* are showm in Fig. 1.



Figure 1: Durability prediction.

The exponent  $b_2$  of inhibition of corrosion corresponding to the curves  $\prod_s^2$  and  $\prod_{cr}^2$ , is greater than the exponent  $b_1$  corresponding to the curves  $\prod_s^1$  and  $\prod_{cr}^1$ , so that  $b_2 = 2b_1$ , other conditions being equal. In the case  $b = b_1$  the curve  $\prod_s^1$  is the first to rich up to 1, therefore the most probable cause of failure is fracture. So the lifetime  $t^*$  is the moment of fracture  $t_s^*: \Pi_s^1(t_s^*) = 1$ . As it can be seen in Fig. 1, increasing the exponent *b* of inhibition of corrosion leads to considerable prolongation of the service life of a tube. When *b* is relatively high (as in the case  $b = b_2$ ) corrosion can practically stop before any critical state is reached.

# 6 CONCLUSIONS

- The problem of a tube under a longitudinal force and pressure may have two version. The first version take place when the greatest stress is eqal to the circumferential tension. The second one take place when the maximum stress is the longitudinal tension.
- For the case of mechanochemical corrosion the problem is reduced to the ordinary differential equation.
- When it is possible the analytical solutions of this equation are found.
- Various functions assessing the strength margin, stability factor, damage accumulation, accident risk are introduced for the lifetime prediction.
- Lifetime is determined by the estimating function being the first to run up to 1.

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